Relativistic Feynman-Metropolis-Teller Treatment at Finite Temperatures and its Application to WDs

CSQCDIII

Sheyse Martins de Carvalho*

In collaboration with: J. Rueda and R. Ruffini

ICRANet, University of Nice Sophia-Antipolis, Sapienza University of Rome

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Outline

Introduction: Effects of temperature.

The equation of State:
- -Chandrasekhar Approach
- -Lattice Model
- -Salpeter Approach
- -Classical Feynman Metropolis Teller Treatment
- -Relativistic Feynman Metropolis Teller Treatment (T=0)

General Relativistic Equations of Equilibrium

Numerical Results: Application to White Dwarfs

Extending the Relativistic FMT Approach to finite temperatures

The Equation of State: Relativistic Feynman Metropolis Teller Treatment ($T \neq 0$)

Numerical Results:
- -Carbon EoS
- -Mass and Radius of General Relativistic Carbon WD

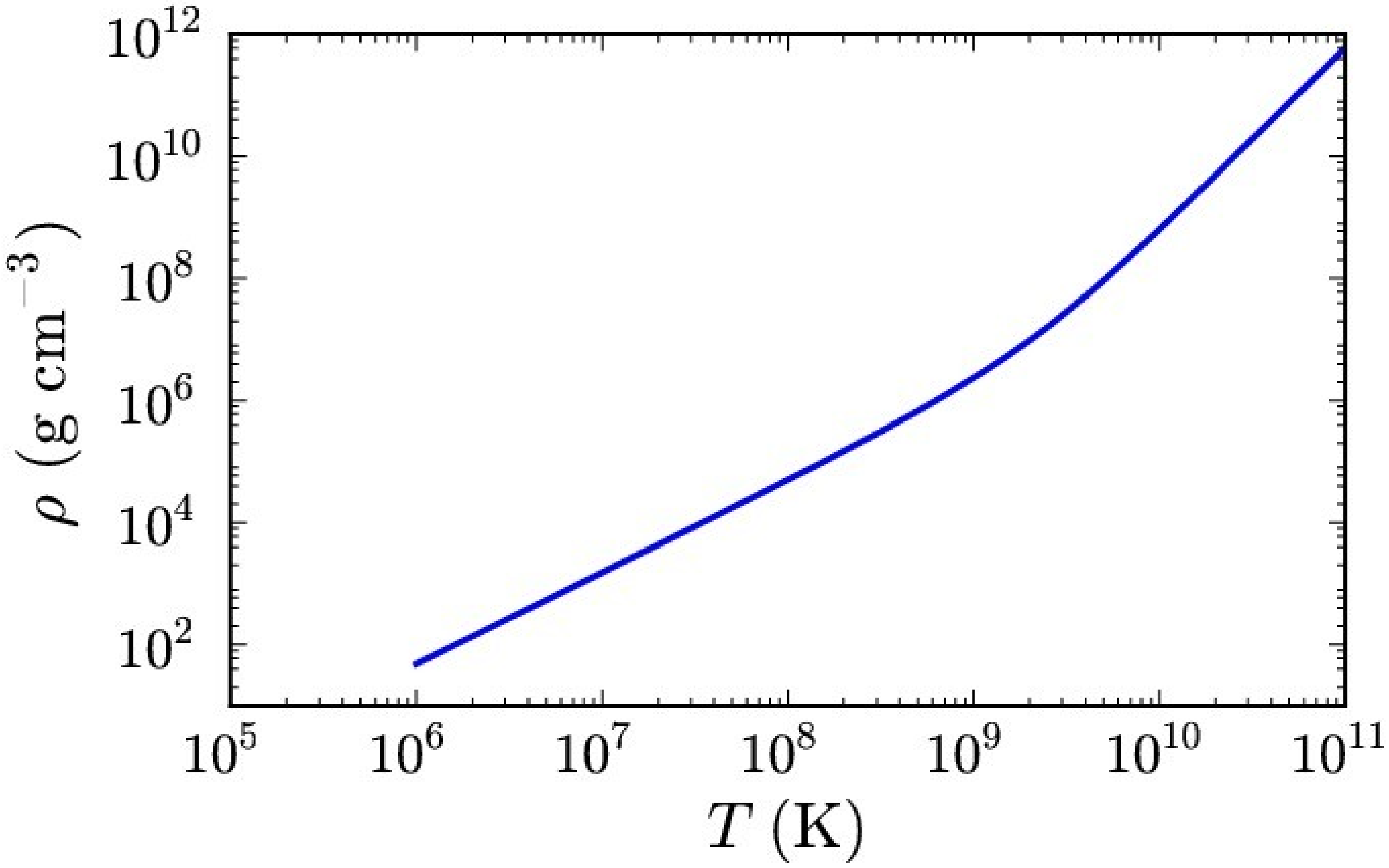
Effects of Temperature

A crude estimate...

$P^{ideal} = P^{deg}$

$P^{ideal} = n_e T$

$$P^{deg} = \frac{1}{3}\frac{2}{(2\pi\hbar)^3}\int_0^{P_e^F}\frac{c^2p^2}{\sqrt{c^2p^2+m_e^2c^4}}4\pi p^2\,dp$$



Equation of State: some known models

Chandrasekhar: Uniform Approximation of Nucleons and Electrons

↓

Lattice Model: Point-like Nucleus + Uniform Electrons

↓

Salpeter: lattice +TF corrections

↓

Feynman-Metropolis-Teller approach based on the Thomas-Fermi model

↓

Relativistic Feynman-Metropolis-Teller: degenerate and finite temperatures

Chandrasekhar Approximation

S.Chandrasekhar, Astrophys. J.74, 81(1931)

- No Coulomb Interaction
- Electrons and Nucleons have a uniform distribution
- Fully degenerate free-gas described by Fermi-Dirac statistics.

$$
\begin{aligned}
\mathcal{E}_e &= \frac{2}{(2\pi\hbar)^3}\int_0^{P_e^F} \sqrt{c^2p^2+m_e^2c^4}\,4\pi p^2 dp \\
&= \frac{m_e^4c^5}{8\pi^2\hbar^3}\left[x_e\sqrt{1+x_e^2}(1+2x_e^2)-\mathrm{arcsinh}(x_e)\right] \\
P_e &= \frac{1}{3}\frac{2}{(2\pi\hbar)^3}\int_0^{P_e^F} \frac{c^2p^2}{\sqrt{c^2p^2+m_e^2c^4}}4\pi p^2 dp \\
&= \frac{m_e^4c^5}{8\pi^2\hbar^3}\left[x_e\sqrt{1+x_e^2}(2x_e^2/3-1)\right. \\
&+ \left.\mathrm{arcsinh}(x_e)\right],
\end{aligned}
$$

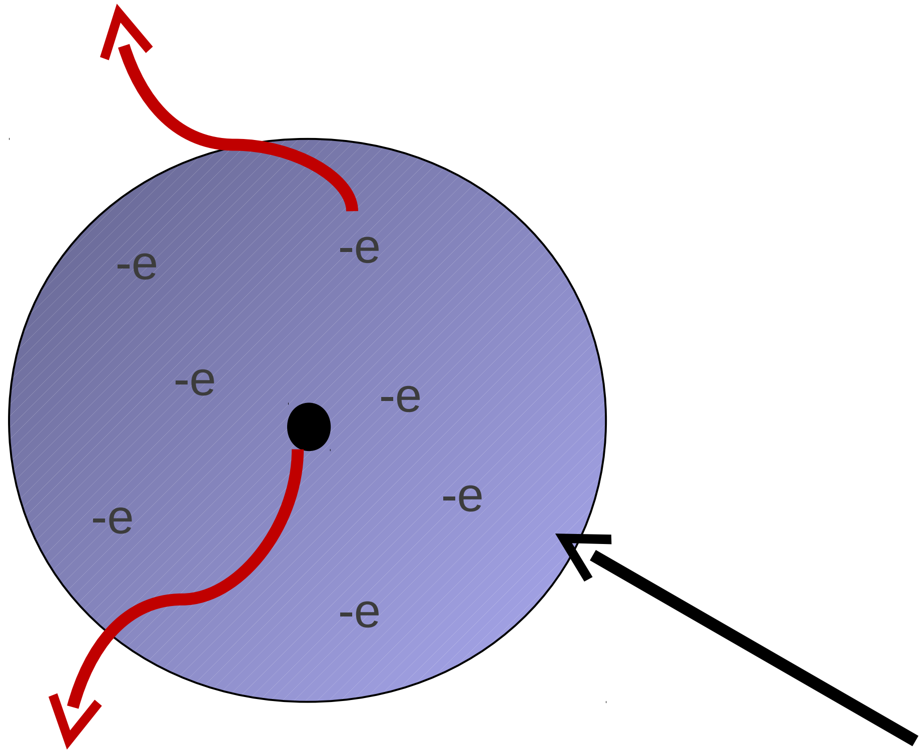
$$
\begin{aligned}
\mathcal{E}_{\text{unif}} &= \mathcal{E}_N+\mathcal{E}_e \approx \frac{A_r}{Z}M_u c^2 n_e+\mathcal{E}_e\,, \\
P_{\text{unif}} &\approx P_e\,,
\end{aligned}
$$

Lattice Model

(Baym, Bethe, Pethick, Nuclear Physics A, 1971)

- Beyond electron-ion fluid approx.: point-like nucleus + background of deg. e-.
- Introduces Wigner-Seitz cell.
- Consider Coulomb Interaction.

Ne = Z non-relativistic electrons



Point-like nucleus +eZ

Wigner-Seitz cell

$$E_{\mathrm{L}} = \mathcal{E}_{\mathrm{unif}} V_{\mathrm{ws}} + E_C \,,$$

$$E_C = E_{e-N} + E_{e-e} = -\frac{9}{10}\frac{Z^2 e^2}{R_{\mathrm{ws}}} \,,$$

$$P_{\mathrm{L}} = -\frac{\partial E_{\mathrm{L}}}{\partial V_{\mathrm{ws}}} = P_{\mathrm{unif}} + \frac{1}{3}\frac{E_C}{V_{\mathrm{ws}}} \,,$$

Salpeter Approach

E.E.Salpeter, Astrophys. J. 134, 669 (1961).

-The generalization of the lattice model came from Salpeter who studied the corrections due to the non-uniformity of the electron distribution inside a Wigner-Seitz cell.

We can write the formula of Salpeter energy as: $E_S = E_{CH} + E_C + E_S^{TF}$

The third contribution is obtained assuming $n_e[1+\varepsilon(r)]$ $\qquad n_e = \dfrac{3Z}{4\pi R_{WS}^3}$

$$E_S^{TF} = -\frac{162}{175}\left(\frac{4}{9\pi}\right)^{2/3}\alpha^2 Z^{7/3}\mu_e$$

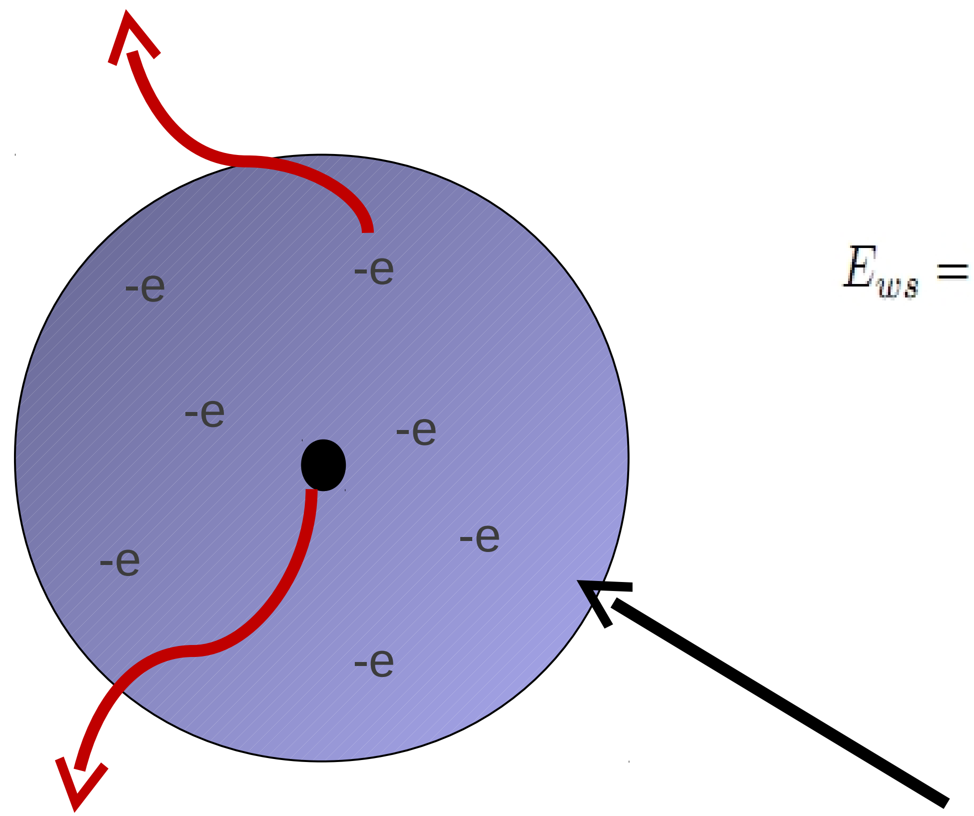
The pressure of the Wigner-Seitz cell is given by: $P_S = P_L + P_{TF}^S$

Where $P_{TF}^S = \dfrac{1}{3}\left(\dfrac{P_e^F}{\mu_e}\right)^2 \dfrac{E_S^{TF}}{V_{WS}}$

Feynman-Metropolis-Teller Treatment: Classic

R. P. Feynman, N. Metropolis, and E.Teller, Phys. Rev. 75,1561(1949)

- FMT showed how to derive the EOS using Thomas-Fermi model
- The profile of electrons changes with distance, is not uniform.



$$E_e^F = \frac{(P_e^F)^2}{2m_e} - eV = \text{constant} > 0\,,$$

$$\frac{d^2\phi(\eta)}{d\eta^2} = \frac{\phi(\eta)^{3/2}}{\eta^{1/2}}\,.$$

$$E_{ws} = E_N + E_k + E_C$$

$$\begin{aligned}
E_N &= M_N(Z,A)c^2\,,\\
E_k^{(e)} &= \int_0^{R_{\rm ws}} 4\pi r^2 \mathcal{E}_e[n_e(r)]dr\\
&= \frac{3}{7}\frac{Z^2e^2}{b}\left[\frac{4}{5}\eta_0^{1/2}\phi^{5/2}(\eta_0) - \phi'(0)\right],\\
E_C &= E_{e-N} + E_{e-e}\\
&= -\frac{6}{7}\frac{Z^2e^2}{b}\left[\frac{1}{3}\eta_0^{1/2}\phi^{5/2}(\eta_0) - \phi'(0)\right],
\end{aligned}$$

$$P_{\rm TF} = \frac{2}{3}\frac{E_k^{\rm unif}[n_e(R_{\rm ws})]}{V_{\rm ws}}\,.$$

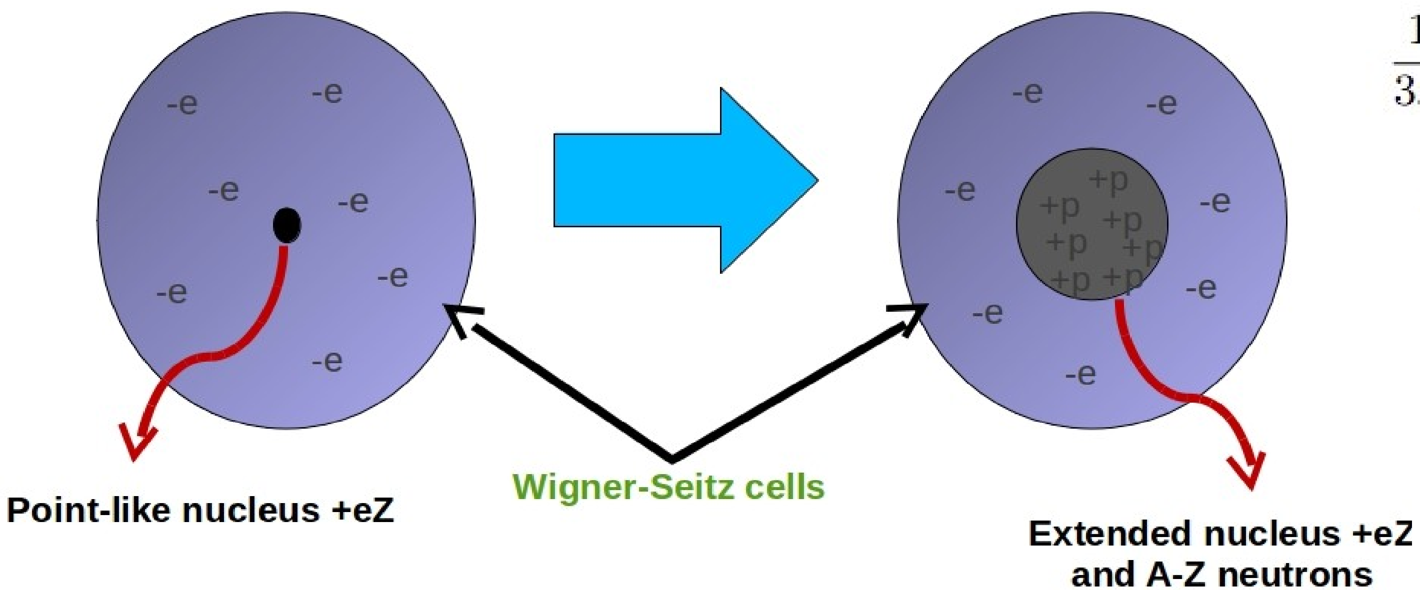
Relativistic Feynman-Metropolis-Teller treatment

M. Rotondo, J. A. Rueda, R. Ruffini, and S. S. Xue, Phys. Rev. C 83, 045805(2011), arXiv:0911.4622.

$$E_e^F = \sqrt{c^2(P_e^F)^2 + m_e^2c^4} - m_e^2c^2 - eV(r) = constant > 0,$$

$$\frac{1}{3x}\frac{d^2\chi(x)}{dx^2} = \frac{-\alpha}{\Delta^3}\Theta(x_c - x) + \frac{4\alpha}{9\pi}\left[\frac{\chi^2(x)}{x^2} + \frac{2m_e}{m_\pi}\frac{\chi(x)}{x}\right]^{\frac{3}{2}},$$

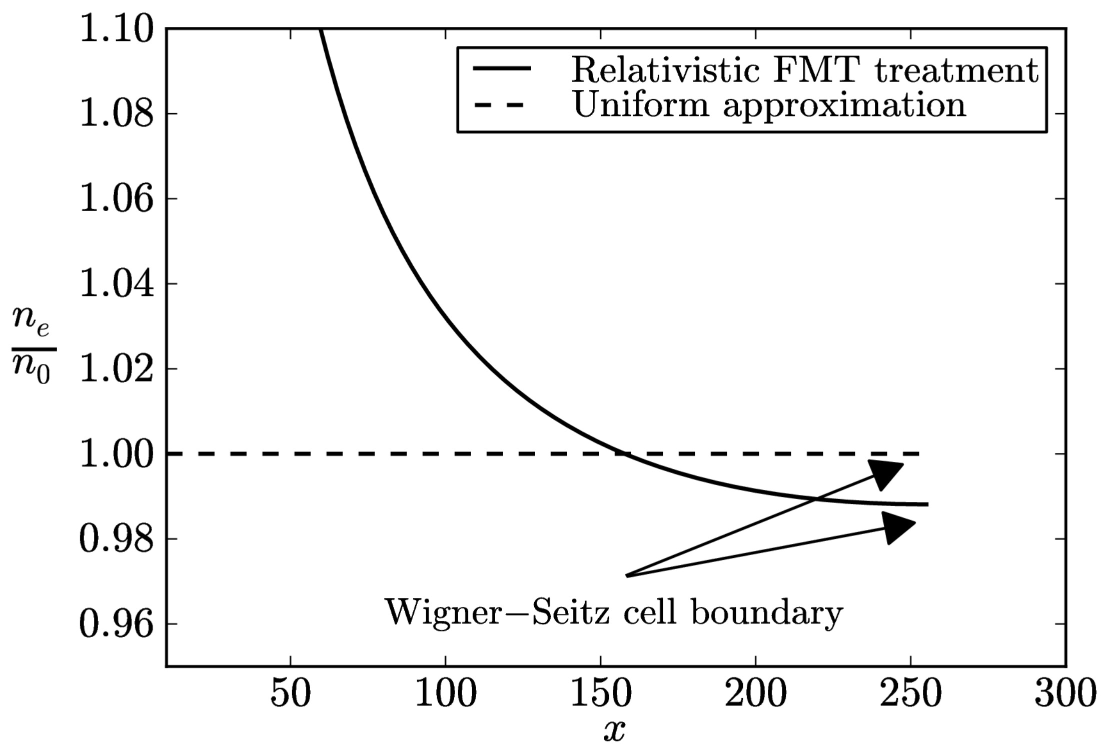
$$E_{ws} = E_N + E_k + E_C \qquad E_N = M_N(A,Z)c^2,$$



$$E_C = \frac{1}{2}\int_{R_C}^{R_{ws}} 4\pi r^2 e[n_p(r) - n_e(r)]V(r)dr.$$

$$E_k = \int_0^{R_{ws}} 4\pi r^2(\mathcal{E}_e - m_e n_e)dr, \quad \mathcal{E}_e = \frac{2}{(2\pi\hbar)^3}\int_0^{P_e^F}\sqrt{c^2p^2 + m_e^2c^4}4\pi p^2 dp,$$

$$P_{FMT}^{rel} = P_e[n_e(R_{ws})], \quad P_e = \frac{1}{3}\frac{2}{(2\pi\hbar)^3}\int_0^{P_e^F}\frac{c^2p^2}{\sqrt{c^2p^2+m_e^2c^4}}4\pi p^2 dp.$$



General Relativistic Equations of Equilibrium

Newtonian → $$\frac{dF(r)}{dr}=\frac{GM(r)}{r^2}$$

$$\frac{dM(r)}{dr}=4\pi r^2\rho(r)$$

$$\frac{dP(r)}{dr}=-\frac{GM(r)}{r^2}\rho(r)$$

General Relativity → $$\frac{dv(r)}{dr}=\frac{2G}{c^2}\frac{4\pi r^3 P(r)/c^2+M(r)}{r^2\left[1-\frac{2GM(r)}{c^2 r}\right]}$$

$$\frac{dM(r)}{dr}=4\pi r^2\frac{\varepsilon(r)}{c^2}$$

$$\frac{dP(r)}{dr}=-\frac{1}{2}\frac{dv(r)}{dr}[\varepsilon(r)+P(r)]$$

Inverse β-decay instability

M. Rotondo, J. A. Rueda, R. Ruffini, and S. S. Xue, Phys. Rev. C 83, 045805(2011), arXiv:0911.4622.

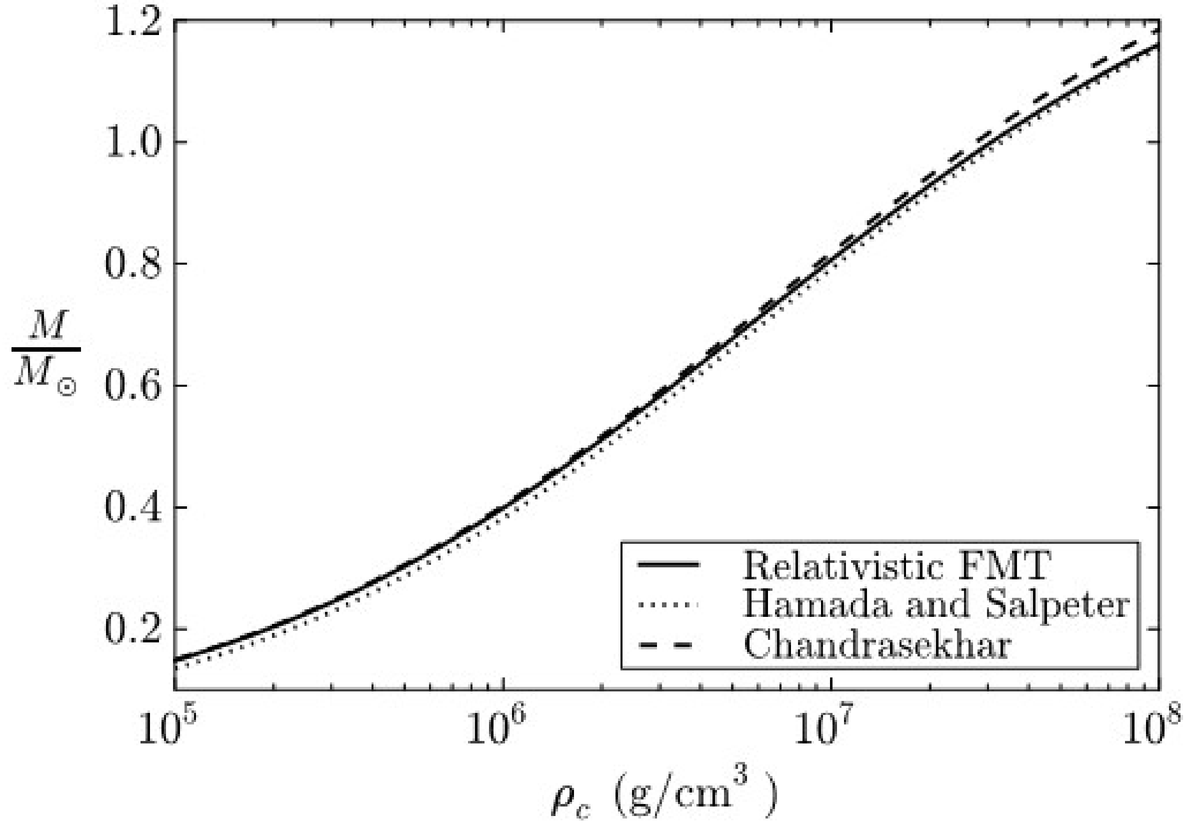
It is known that white dwarfs may become unstable against the inverse β -decay process (Z,A) → (Z-1,A) through the capture of energetic electrons. In order to trigger such a process, the electron Fermi energy must be larger than the mass difference between the initial nucleus (Z,A) and the final nucleus (Z-1,A).

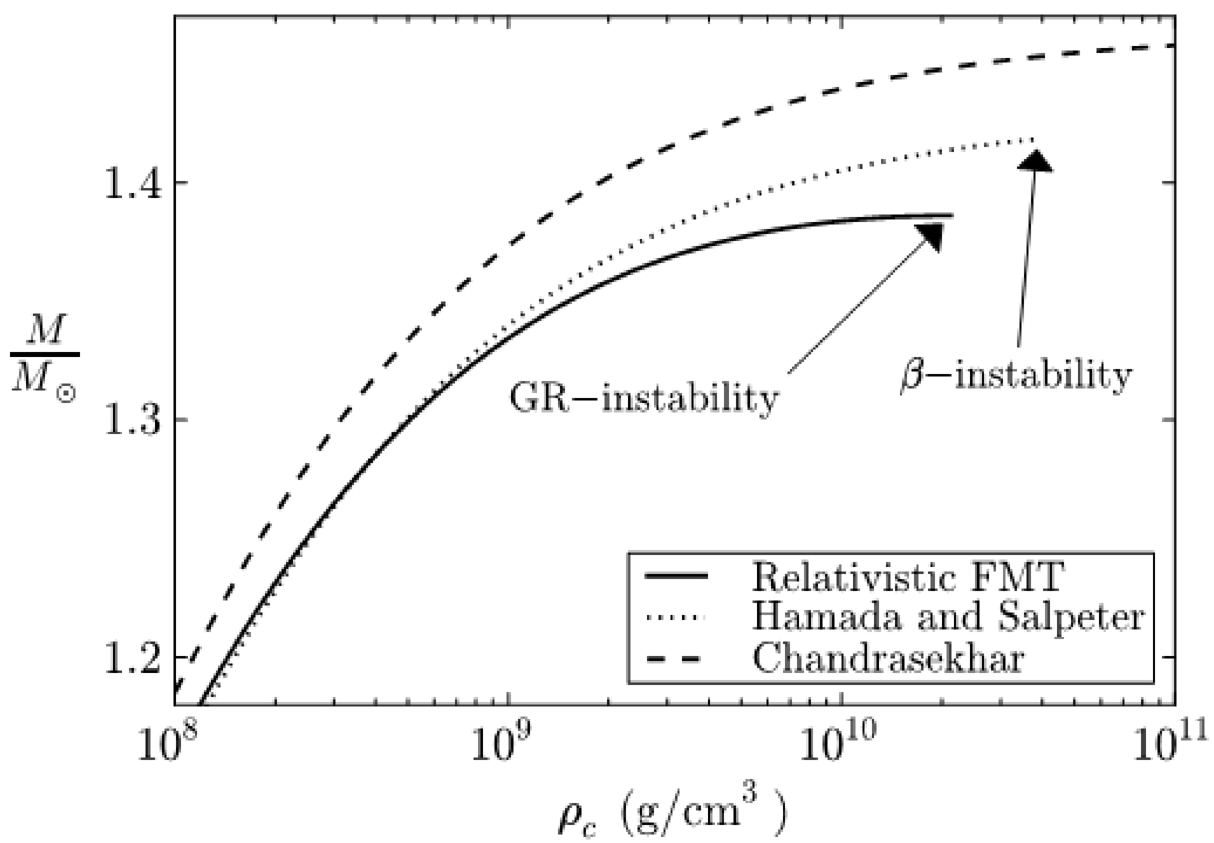
Decay	ϵ_Z^β	$\rho_{crit}^{\beta,relFMT}$	$\rho_{crit}^{\beta,unif}$
$^4He \to ^3H + n \to 4n$	20.596	1.39×10^{11}	1.37×10^{11}
$^{12}C \to ^{12}B \to ^{12}Be$	13.370	3.97×10^{10}	3.88×10^{10}
$^{16}O \to ^{16}N \to ^{16}C$	10.419	1.94×10^{10}	1.89×10^{10}
$^{56}Fe \to ^{56}Mn \to ^{56}Cr$	3.695	1.18×10^9	1.14×10^9

Numerical Results: Application to White Dwarfs

M. Rotondo, J. A. Rueda, R. Ruffini, and S. S. Xue, Phys. Rev. D (2011).

Results for ^{12}C WDs





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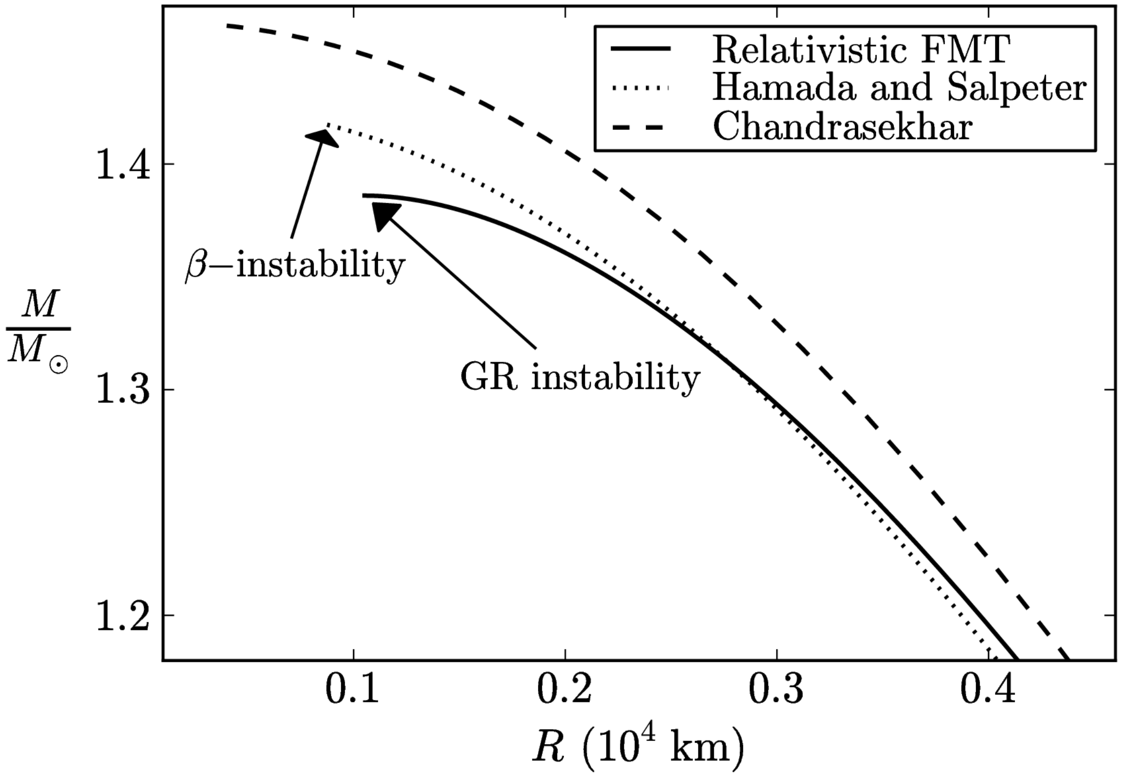

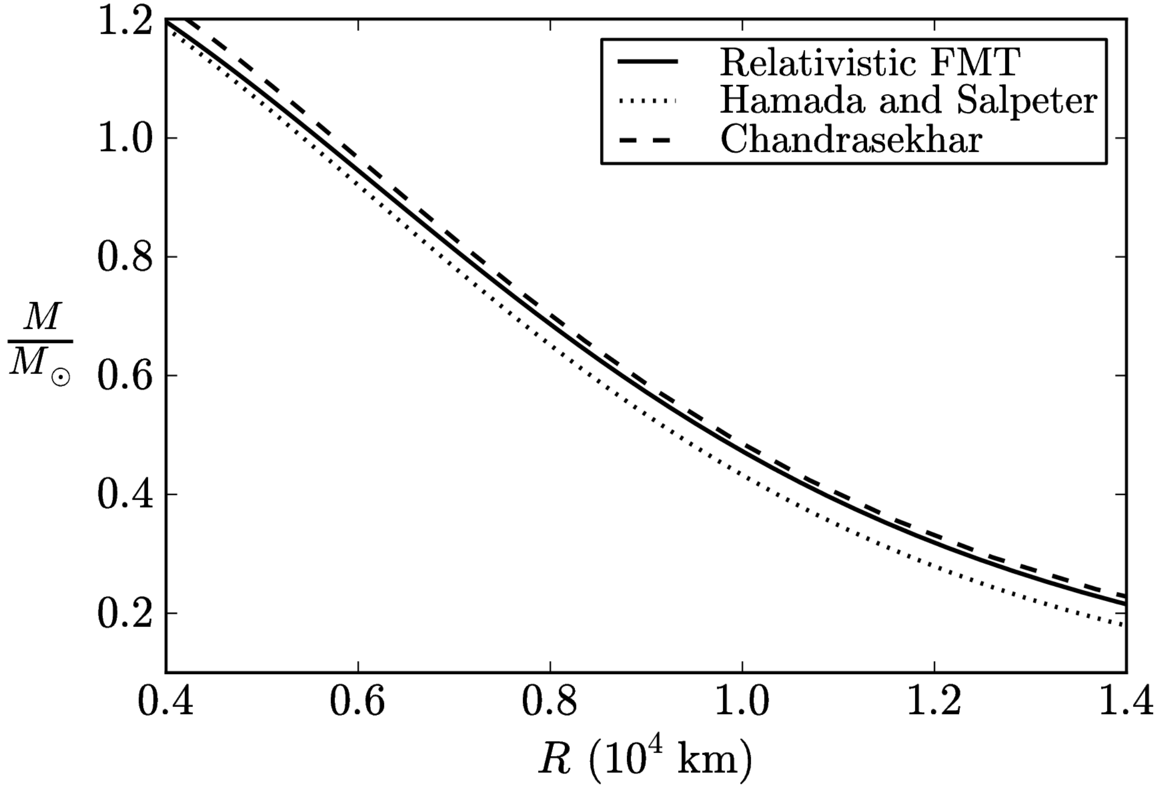

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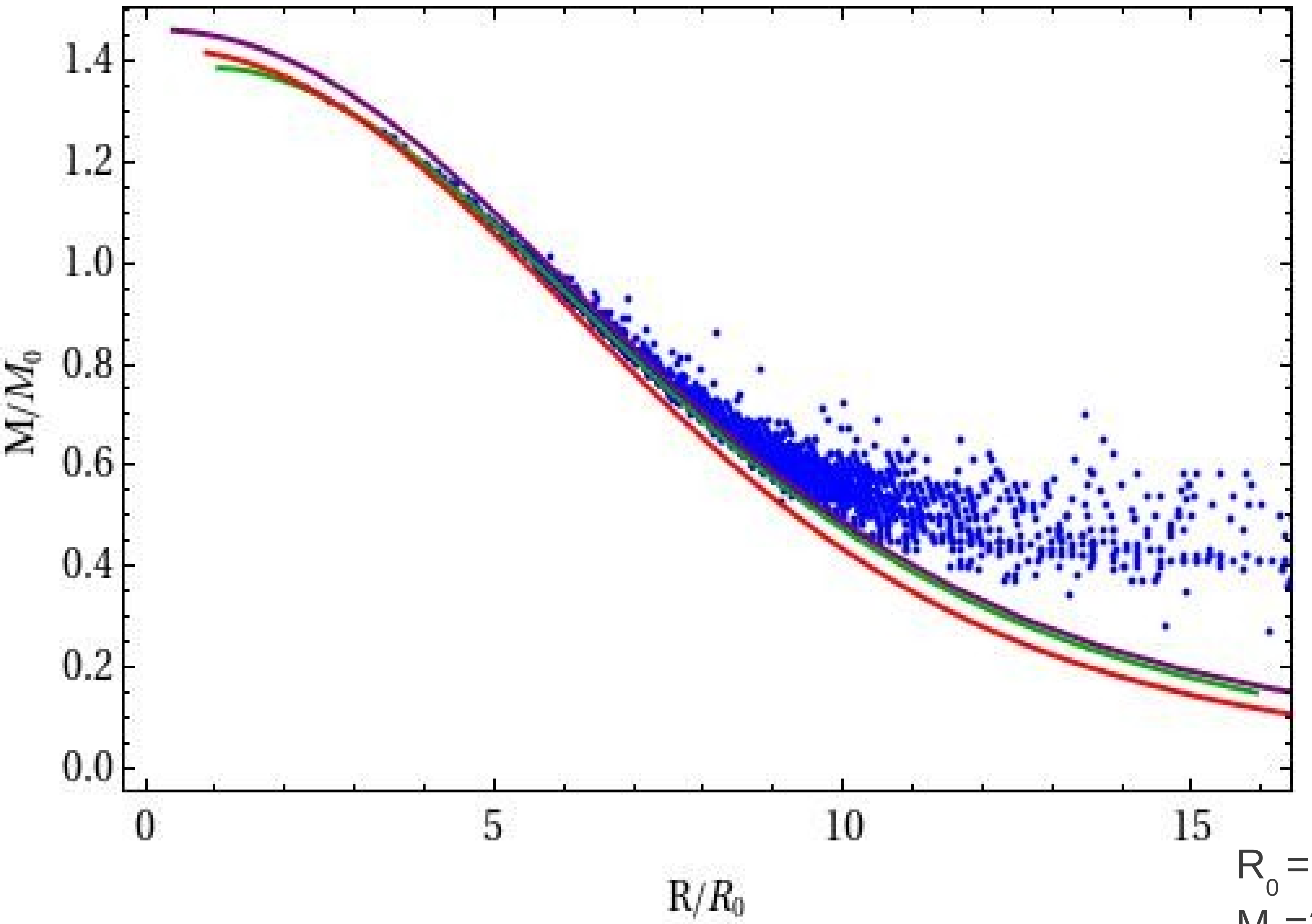


Rel.FMT

Chandrasekhar

Salpeter

WD from the catalog SDSS-E06

R_0 =7.810x10^8 cm= 0.01 R_{sun}

M_0=2.9M_{sun}

(0.3 – 1.3 M_{sun})

Extending the Relativistic FMT Approach to Finite Temperatures

Relativistic and non degenerate gas of electrons at temperature T surrounding a degenerate finite sized and positively charged nucleus.

$$E_e = \tilde{\mu}_e - eV = constant \qquad \mu_e = Mc^2 + \tilde{\mu}_e$$

The electron density in this case is given by:

$$n_e = \frac{2}{(2\pi\hbar)^3}\int_0^\infty \frac{4\pi p^2 dp}{\exp\left(\frac{\tilde{E}(p)-\tilde{\mu}_e(p)}{k_B T}\right)+1}, \qquad n_e = \frac{8\pi\sqrt{2}}{(2\pi\hbar)^3} m^3 c^3 \beta^{3/2}\left[F_{1/2}(\eta,\beta)+\beta F_{3/2}(\eta,\beta)\right],$$

$$F_k(\eta,\beta) \equiv \int_0^\infty \frac{t^k\sqrt{1+(\beta/2)t}}{1+\exp(t-\eta)}dt.$$

Replacing the particle densities into the Poisson equation we obtain the relativistic Thomas-Fermi Equation:

$$\frac{d^2\chi(x)}{dx^2} = -4\pi\alpha x\left[\frac{3}{4\pi\Delta^3}\Theta(x_c - x) - \frac{\sqrt{2}}{\pi^2}\left(\frac{m_e}{m_\pi}\right)^3 \beta^{3/2}\left[F_{1/2}(\eta,\beta)+\beta F_{3/2}(\eta,\beta)\right]\right]$$

Extending the Relativistic FMT Approach to Finite Temperatures

For the case of finite temperature both, the nucleus energy and the pressure, take into account the contribution of the internal energy density.

Then, the total energy of the Wigner-Seitz cell can be written as:

$$E_{ws} = E_N + E_k + E_C$$

$$E_N = M_N(A,Z)c^2 + \frac{3}{2}Ak_BT,$$

$$E_k = \int_0^{R_{ws}} 4\pi r^2(\mathcal{E}_e - m_e n_e)dr, \quad \mathcal{E}_e = c^2 m_e n_e + \frac{8\pi\sqrt{2}}{(2\pi\hbar)^3} m_e^4 c^5 \beta^{5/2}\left[F_{3/2}(\eta,\beta) + \beta F_{5/2}(\eta,\beta)\right],$$

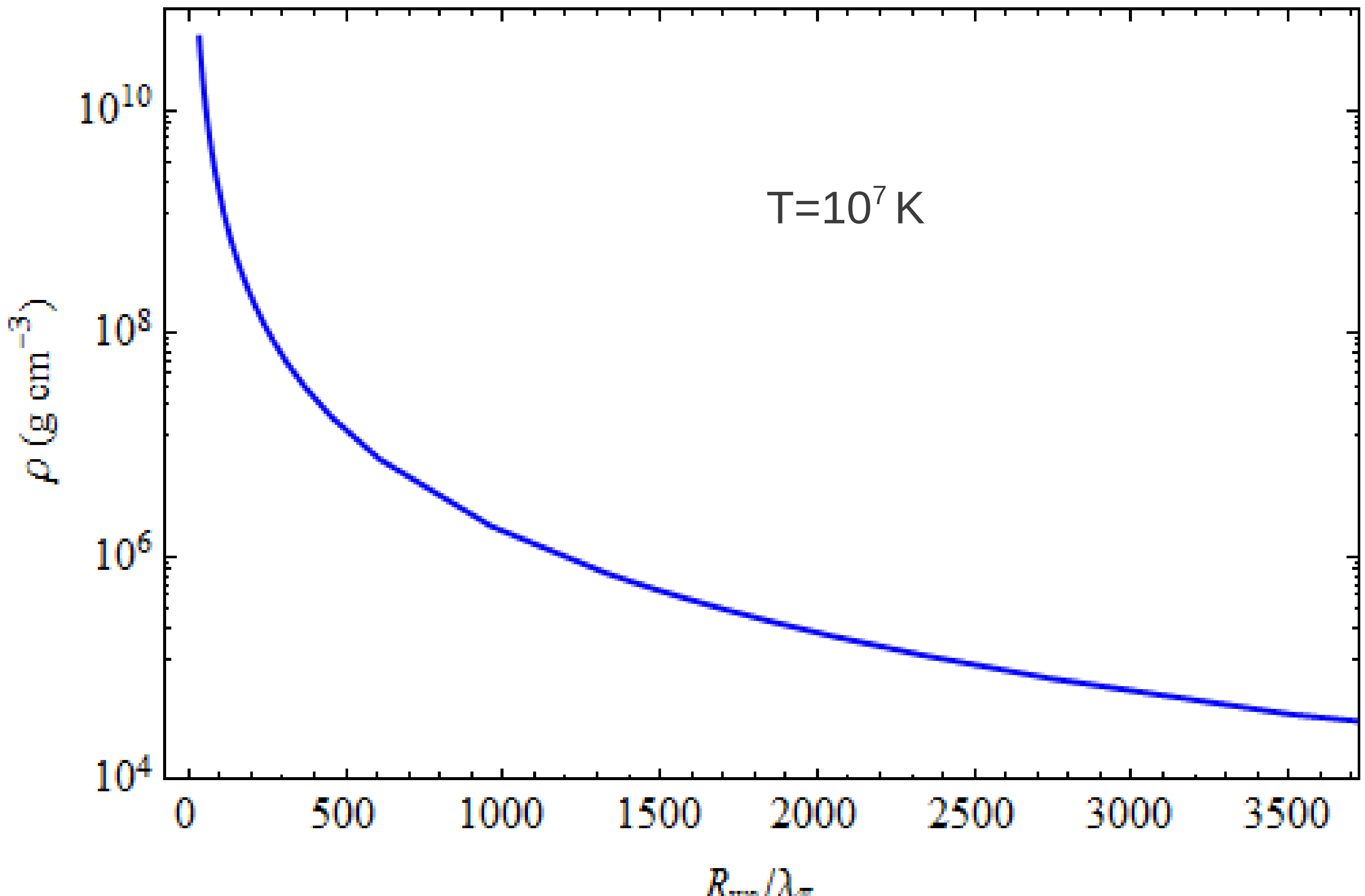
$$E_C = \frac{1}{2}\int_{R_C}^{R_{ws}} 4\pi r^2 e[n_p(r) - n_e(r)]V(r)dr.$$

The total pressure of the Wigner-Seitz cell is given by the sum of the internal energy pressure and the pressure of the uniform model but computed at the boundary of the cell.

$$P = n_N k_B T + P_e = n_N k_B T + \frac{1}{3}\frac{2}{(2\pi\hbar)^3}\int_0^\infty \frac{c^2p^2}{\tilde{E}(p)} \frac{d^3p}{\exp\left(\frac{\tilde{E}(p)-\tilde{\mu}_e(p)}{k_BT}\right)+1}.$$

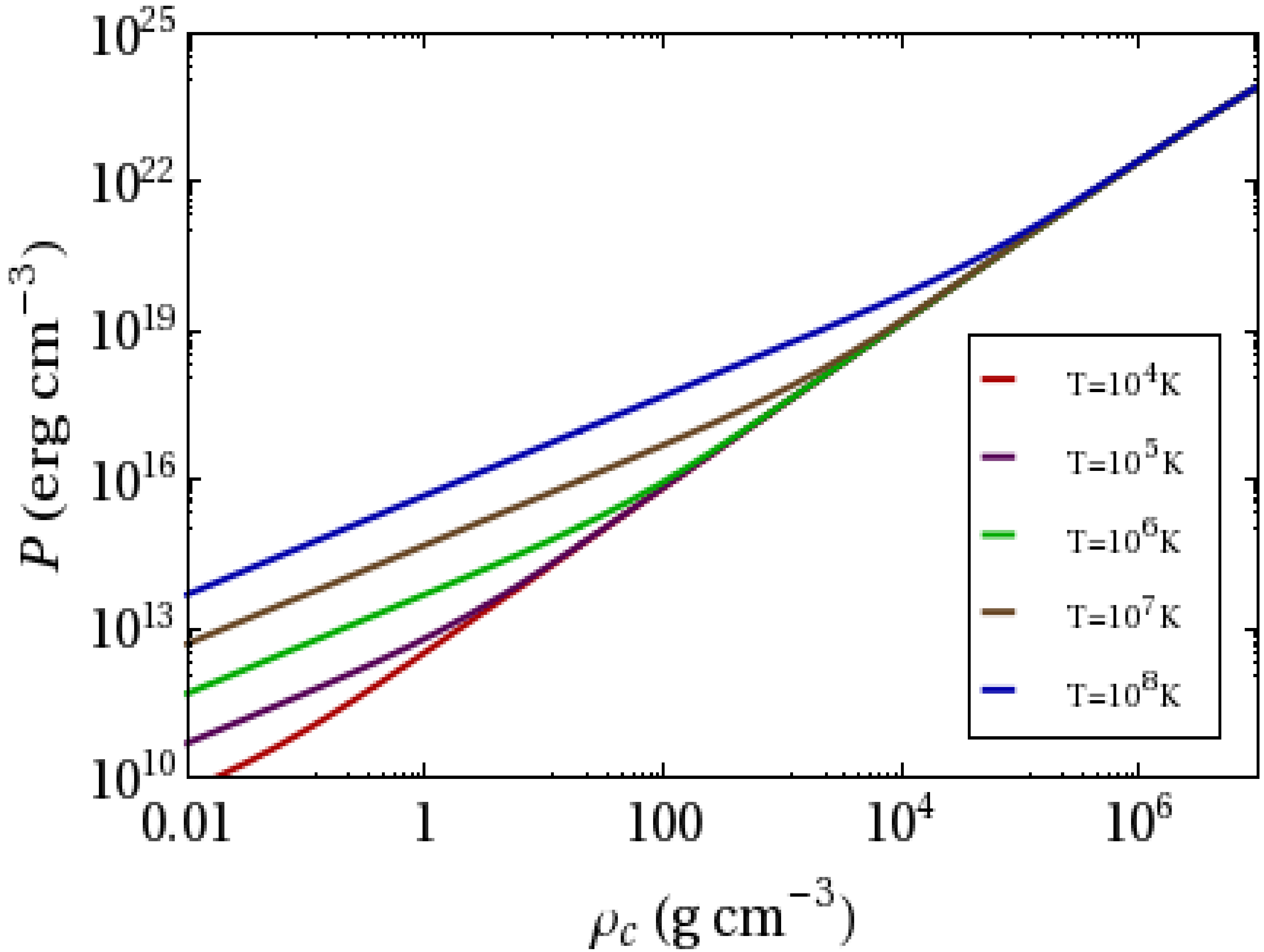
Numerical Results: Application to White Dwarfs

Results for ^{12}C WDs



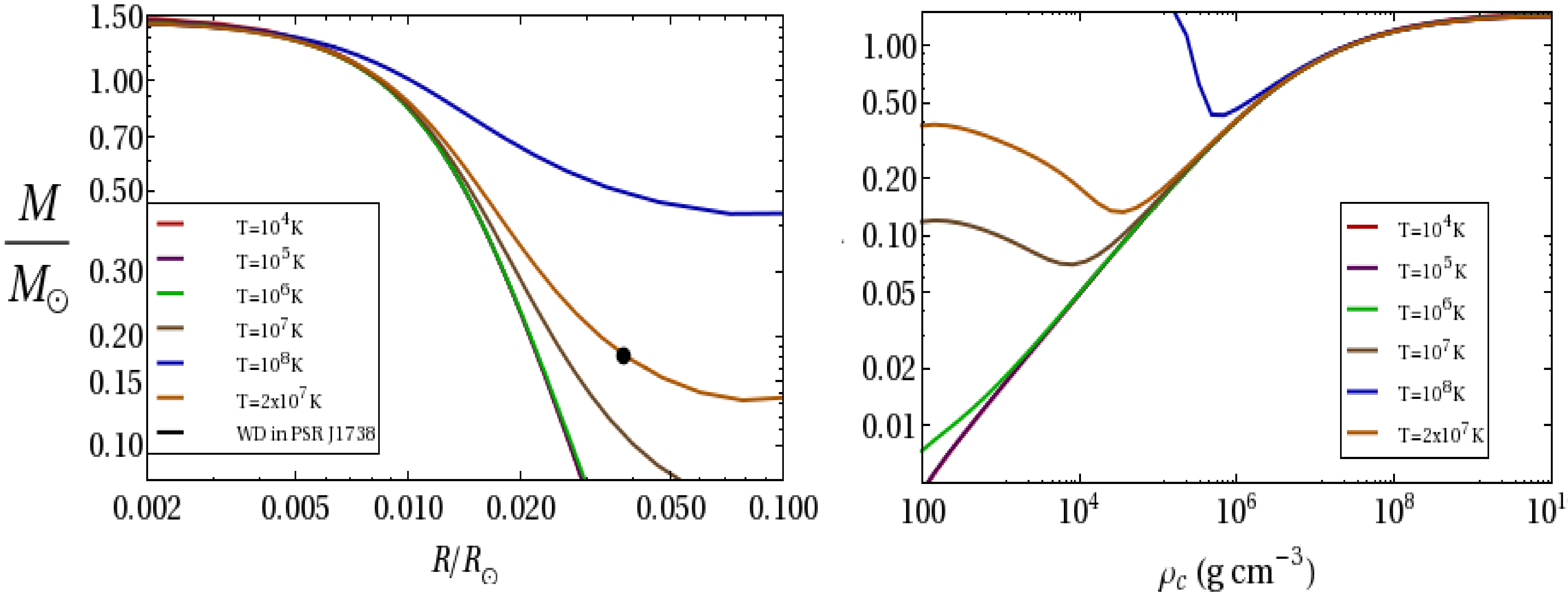
Numerical Results: Application to White Dwarfs

EoS for ^{12}C WDs



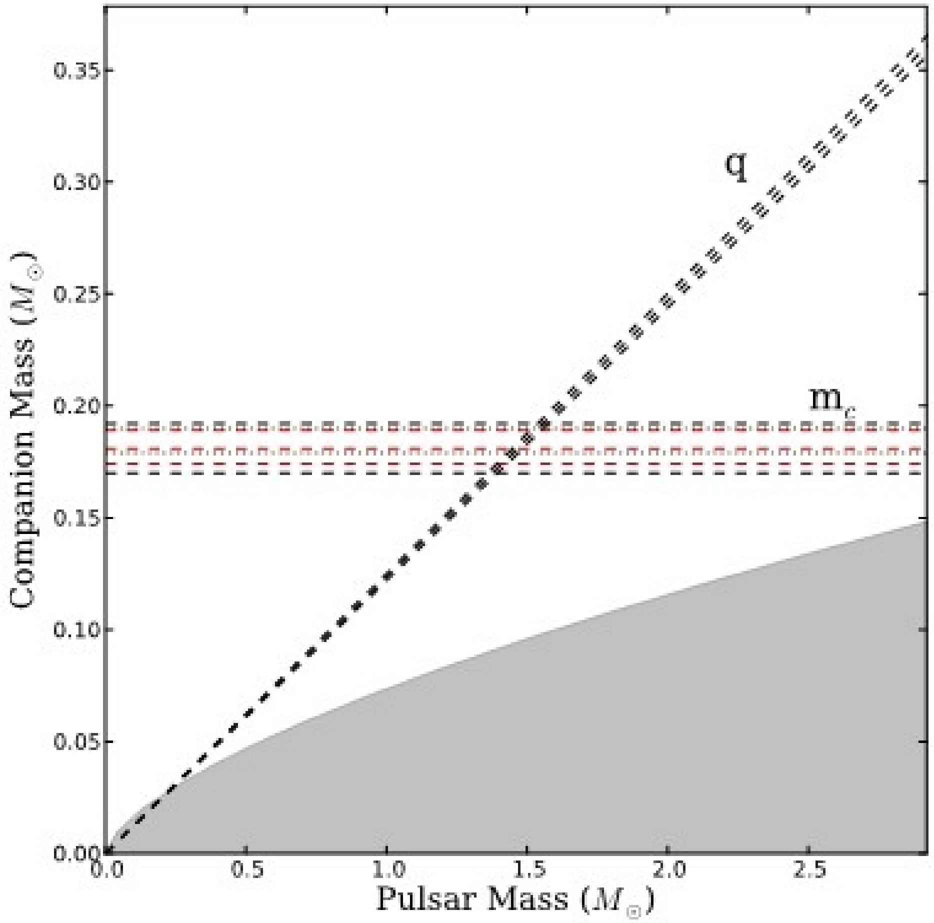
Numerical Results: Application to White Dwarfs

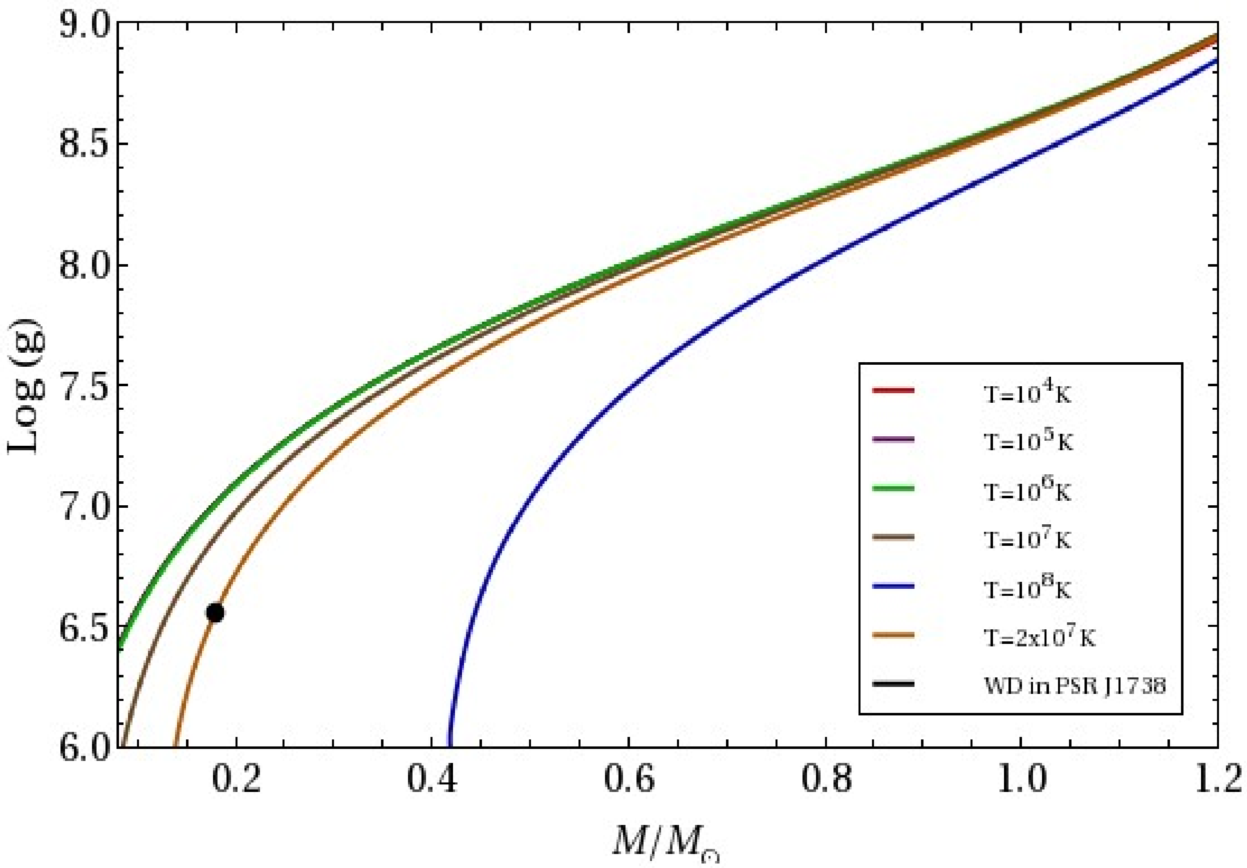
Results for ^{12}C WDs



Numerical Results: Application to White Dwarfs

WD-NS binary: PSR1738+0333





J. Antoniadis et. al (2012)

$M=0.18M_{sun,}$
$R=0.032R_{sun}$ (theoretically inferred)

Conclusions

- The radius of the Wigner-Seitz cell is not changing with increasing temperature;

- For high densities the effects of temperature are not so important;

- The critical density of inverse β -decay is not changing;

- For low densities we have important effects for temperatures over $T=10^7$ K;

Work in Progress

- He, O, Fe white Dwarfs.

- Neutron Star Cooling.

Thank You!