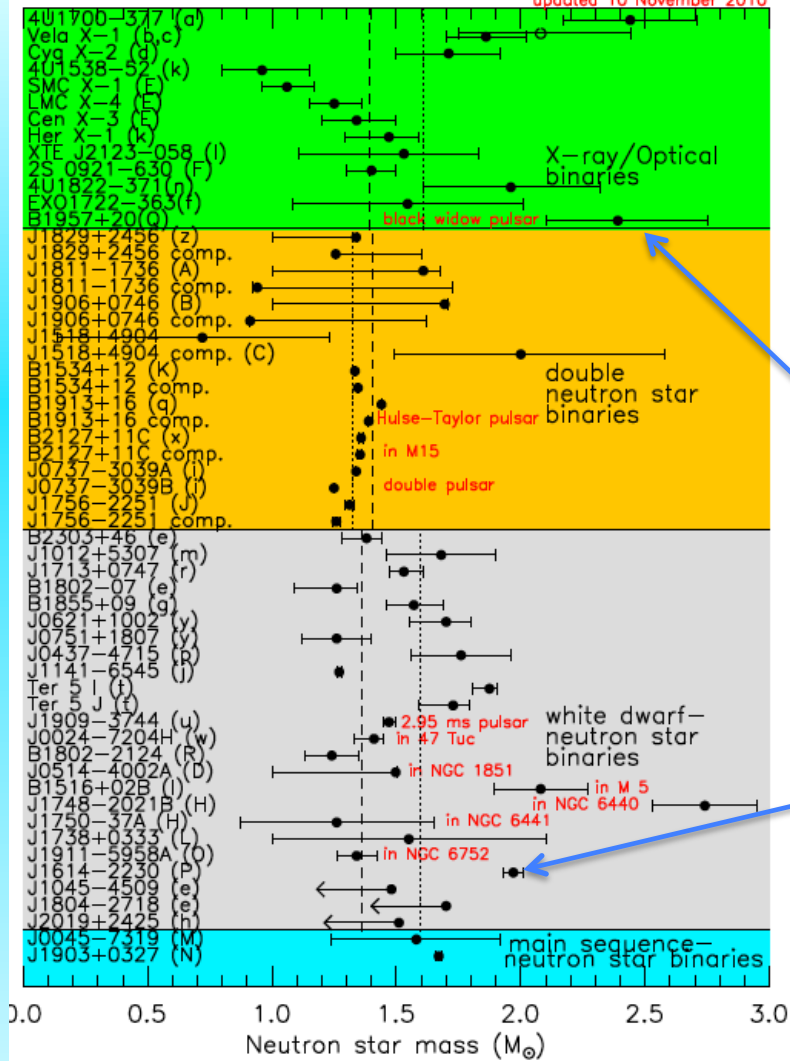


Modeling Hybrid Stars in Quark-Hadron Approaches

OUTLINE

- hadronic SU(3) model
- adding quarks
- alternative chiral formulation

updated 10 November 2010



Masses of Neutron Stars

Masses of radio pulsars

Kiziltan, Kottas, Thorsett, astro-ph:1011.4291

no signature for mass cut off

$M = (2.4 \pm 0.12) M_s ?$
van Kerkwijk et al., ApJ 728, 95 (2011)

current benchmark for NS models

$M = (1.97 \pm .04) M_0$

Demorest et al. Nature 467, 1081 (2010)

well established
heavy neutron stars

Lattimer, Prakash, astro-ph:1012.3208

hadronic model based on non-linear realization of chiral symmetry

degrees of freedom **SU(3) multiplets:**

baryons (n, Λ, Σ, Ξ) scalars (σ, ζ, δ^0) vectors (ω, ρ, φ) , pseudoscalars, glueball field χ

A) **SU(3) interaction**

$$\sim \text{Tr} [\bar{B}, M]_{\pm} B \quad , \quad (\text{Tr} \bar{B} B) \text{Tr} M$$

B) meson interactions

$$\begin{aligned} \sigma &\sim \langle u \bar{u} + d \bar{d} \rangle & \zeta &\sim \langle s \bar{s} \rangle & \delta^0 &\sim \langle u \bar{u} - d \bar{d} \rangle \\ \sim V(M) \quad \langle \sigma \rangle &= \sigma_0 \neq 0 & \langle \zeta \rangle &= \zeta_0 \neq 0 \end{aligned}$$

C) chiral symmetry $m_{\pi} = m_K = 0$

$$\text{explicit breaking} \quad \sim \text{Tr} [c \sigma] \quad (\sim m_q \bar{q} q)$$

fix scalar parameters to baryon masses, decay constants, meson masses

degrees of freedom

$$B = \begin{pmatrix} \Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -2\Lambda/\sqrt{6} \end{pmatrix} \quad \text{baryons}$$

$$\text{diag}(V) = \{ (\omega + \rho) / \sqrt{2}, (\omega - \rho) / \sqrt{2}, \phi \} \quad \text{vector mesons}$$

$$\text{diag}(X) = \{ (\sigma + \delta) / \sqrt{2}, (\sigma - \delta) / \sqrt{2}, \zeta \} \quad \text{scalar mesons}$$

$$\text{Scalar self interaction } L_0 = -\frac{1}{2} k_0 I_2 + k_1 (I_2)^2 + k_2 I_4 + 2 k_3 I_3 + L_{\text{ESB}}$$

invariants

$$I_1 = \text{Tr}(X)$$

$$I_2 = \text{Tr}(X)^2$$

$$I_3 = \det(X)$$

$$+ \text{dilaton field } L_\chi = -k_4 \chi^4 - \frac{1}{4} \chi^4 \ln(\chi^4/\chi_0^4) + \delta/3 \chi^4 \ln(I_3/\langle X \rangle)$$

fix scalar parameters to baryon masses, decay constants, scalar meson masses

+ term prop $(X - X_0)$, breaks SU(3)

Nuclear Matter and Nuclei

binding energy $E/A \sim -15.2 \text{ MeV}$ saturation $(\rho_B)_0 \sim .16/\text{fm}^3$

compressibility $\sim 223 \text{ MeV}$ asymmetry energy $\sim 31.9 \text{ MeV}$

parameter fit to known nuclear binding energies and hadron masses

2d calculation of all measured (~ 800) even-even nuclei

good charge radii $\delta r_{\text{ch}} \sim 0.5 \%$ (+ LS splittings)

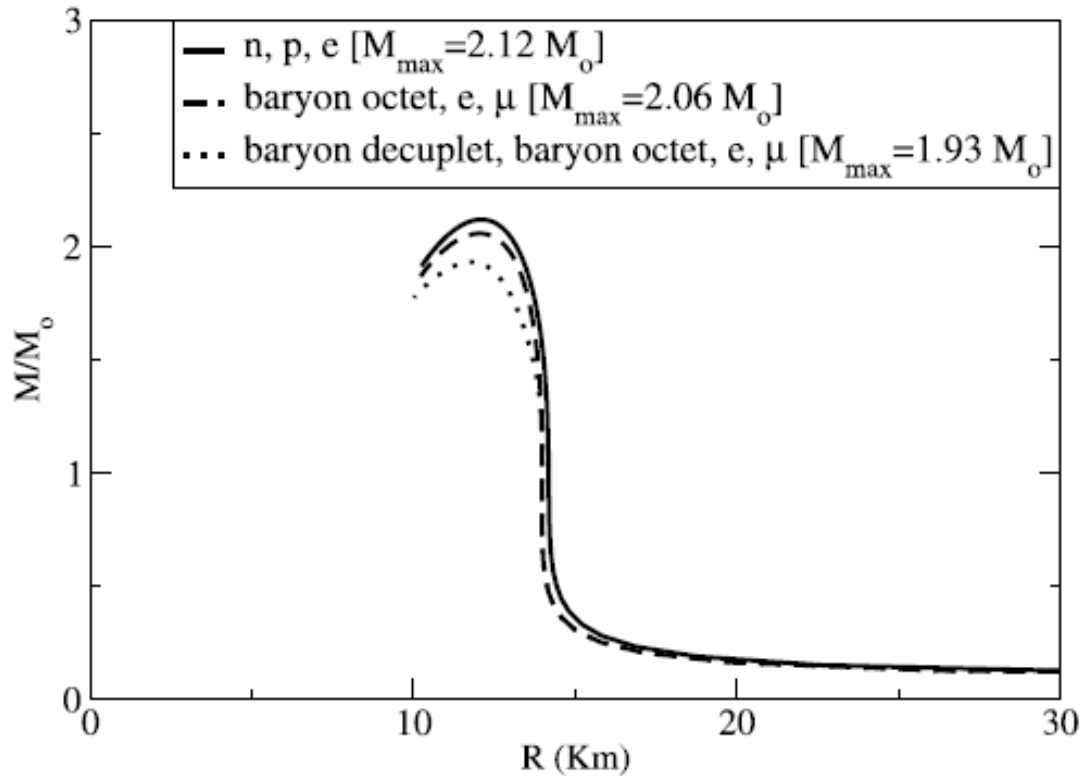
relativistic nuclear
structure models

error in energy $\varepsilon (A > 50) \sim 0.21 \%$ (NL3: 0.25 %)

$\varepsilon (A > 100) \sim 0.14 \%$ (NL3: 0.16 %)

+ correct binding energies of hypernuclei

Neutron star masses including different sets of particles



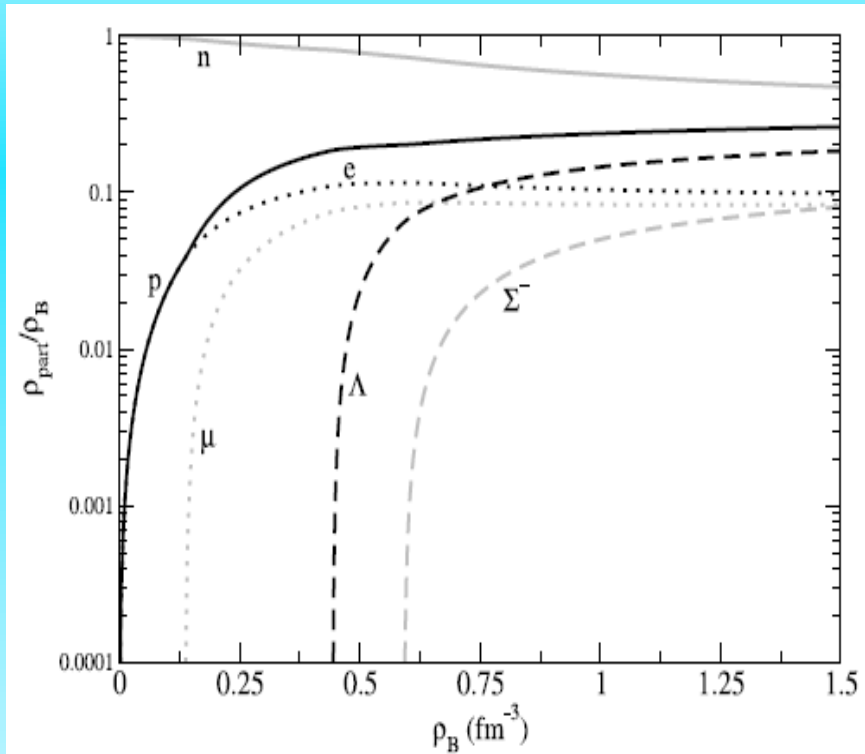
Tolman-Oppenheimer-Volkov equations, static spherical star

changing masses with degrees of freedom

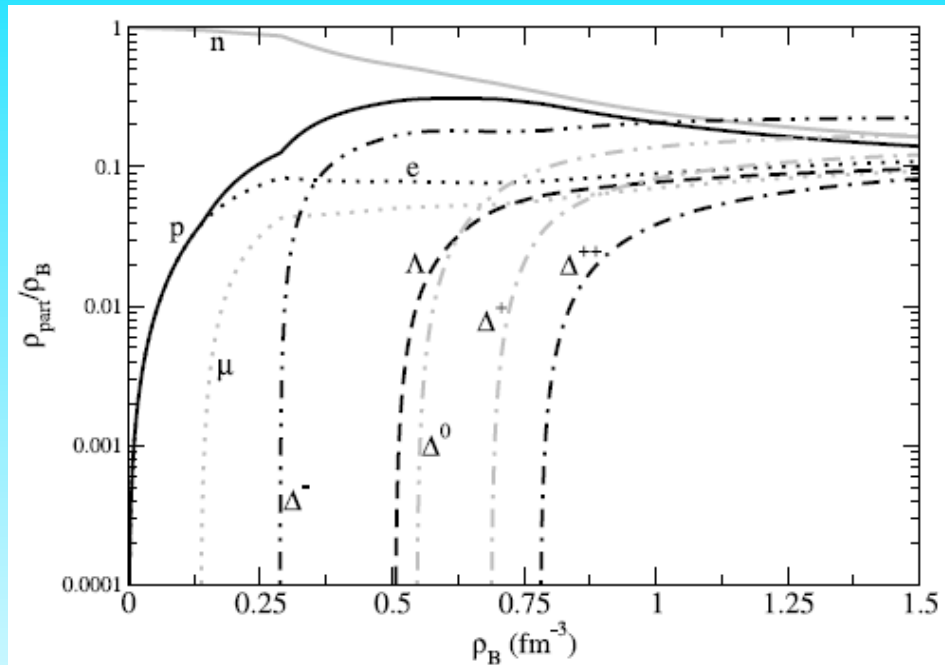
large star masses even with spin 3/2 resonances

tuned parameters

particle densities inside of the star



particle abundancies – no decuplet

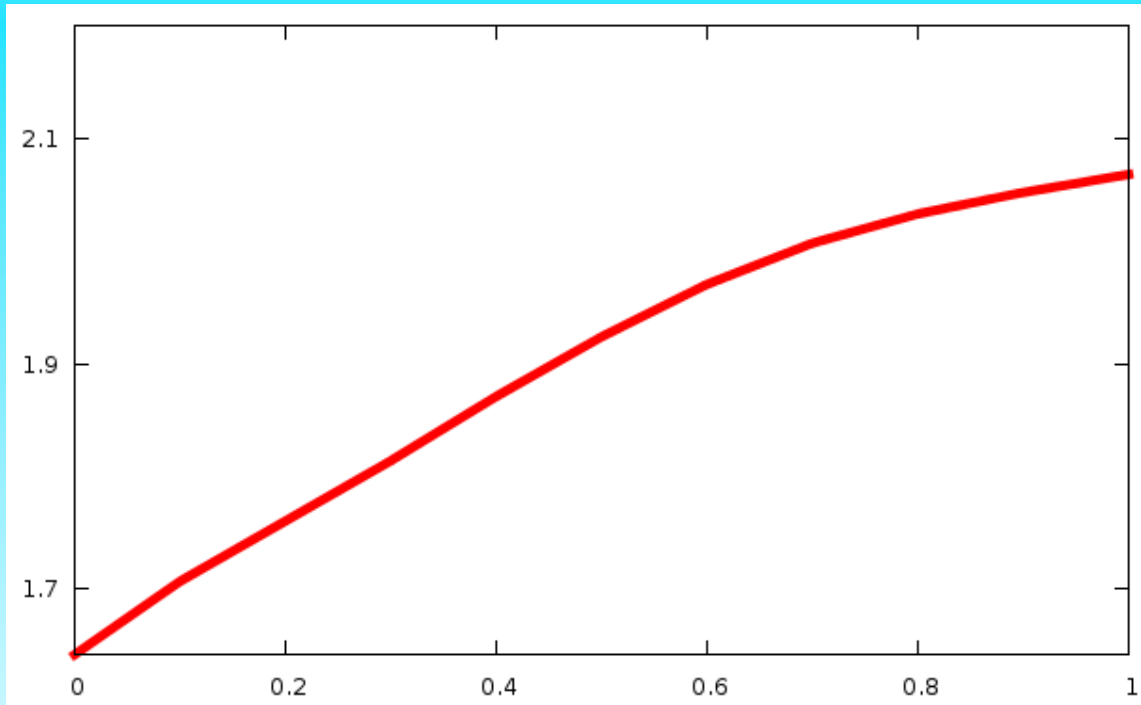


particle numbers as function of density
uncertainties from $g_{3/2}$ coupling

Impact of Φ field

rescale $g_{B\Phi}$ coupling parameters

$M_{\max} [M_{\odot}]$

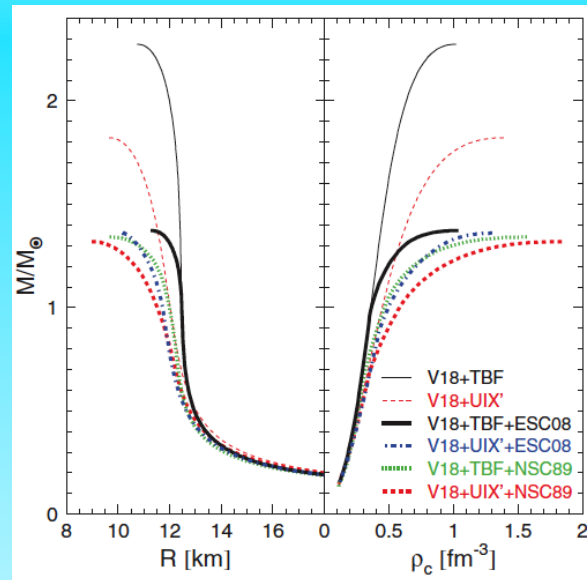


no coupling

standard fit

rather generic observation - no YY interaction \rightarrow very small star masses

Use Nijmegen potential
+ Argonne/Urbana / TBF



Schulze, Rijken, PRC 84, 035801 (2011)

Remark about SU(6) flavor-spin symmetry

SU(3) three couplings for baryon-meson interaction, SU(6) one constant

$$L_{BW} = -\sqrt{2} g_8^W (\alpha_W [\bar{B}OBW]_F + (1 - \alpha_W) [\bar{B}OBW]_D) \\ - g_1^W / \sqrt{3} \text{Tr}(\bar{B}OB) \text{Tr}(W)$$

$$[\bar{B}OBW]_F = \text{Tr}(\bar{B}OBW - \bar{B}OWB) \quad [\bar{B}OBW]_D = \text{Tr}(\bar{B}OBW + \bar{B}OWB) - 2/3 \text{Tr}(\bar{B}OB) \text{Tr}(W)$$

usually only applied for vector meson interaction

2 M_S possible

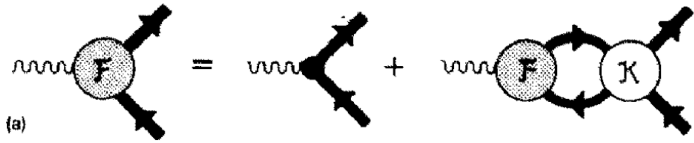
$$\text{SU}(6) \quad g_{N\omega} : g_{\Lambda\omega} : g_{\Xi\omega} = 3 : 2 : 1 \quad g_{N\phi} : g_{\Lambda\phi} : g_{\Xi\phi} = 0 : 1 : 2$$

However – deviating from SU(6) deviation, keeping SU(3) leads to $g_{N\phi} \neq 0$

$$g_{N\phi} = \sqrt{2} g_8 - 1/\sqrt{3} g_1 \quad g_1/g_8 (\text{SU6}) = \sqrt{6}$$

Experiments/Lattice show very small strangeness vector form factor

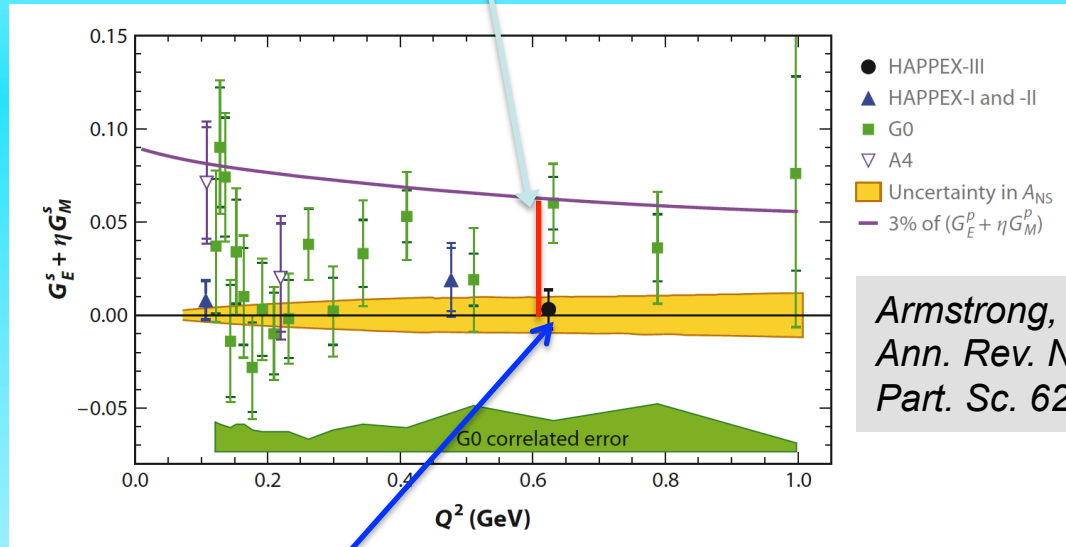
strange vector form factor of nucleon



band of possible values from calculation

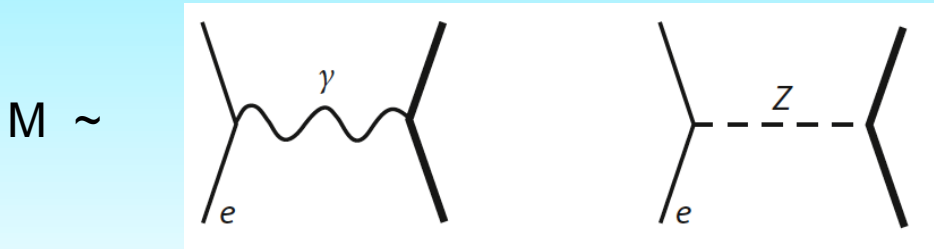
SWS, MPLA 10 1201 (1995)

Summary of PV polarized eA scattering experiments



Armstrong, McKeown, *Ann. Rev. Nucl. & Part. Sc.* 62, 337 (2012)

most recent experiment - strangeness contribution consistent with 0



interference term $\sigma_{PV} \sim M_Y M_Z^*$

asymmetry $A \sim G_F Q^2 / \alpha \sim 10^{-5}$

χ QCD collaboration small values for μ_s and r_s ,

PRD80 094503 (2009)

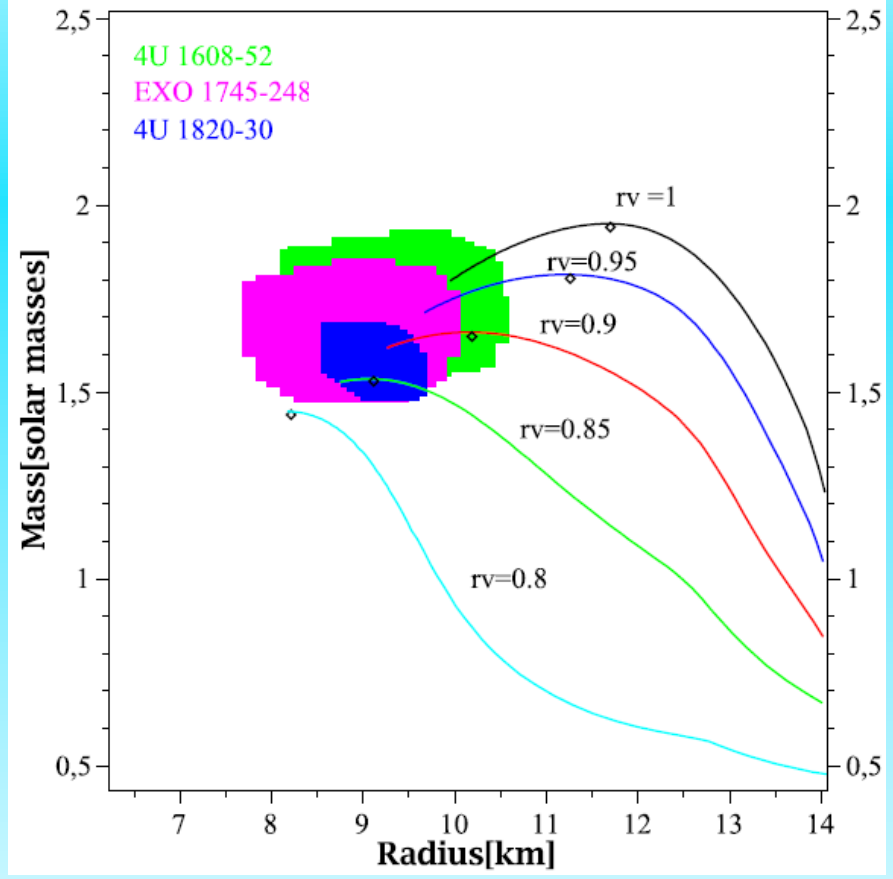
playing around with the Δ baryon

Δ resonances
scalar couplings \rightarrow vacuum masses

vector couplings unclear
moderate changes $r_V = g_{\Delta\omega} / g_{n\omega}$

not far from SU(6)

same from quasielastic eA scattering



Data from Özel et al, astro-ph:1002.3153

see however, Steiner et al, astro-ph:1005.0811

connect hadronic and quark degrees of freedom

$$\Phi = \frac{1}{N_c} \text{Tr}_c L$$

$$L(\vec{x}) = \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$$

order parameter of the phase transition

$$\langle \Phi \rangle = 0 \quad \text{confined phase}$$

$$\langle \Phi \rangle \neq 0 \quad \text{deconfined phase}$$

effective potential for Polyakov loop, fit to lattice data

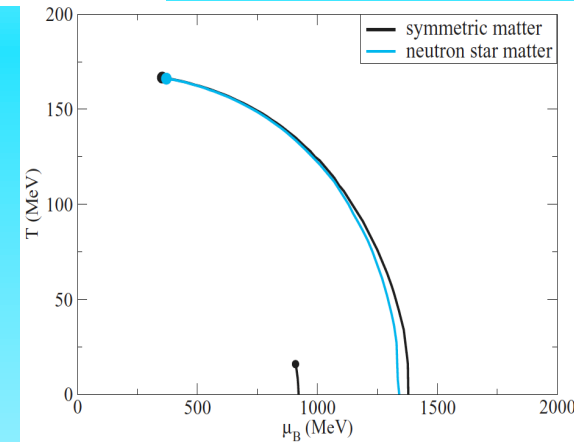
$$U = -\frac{1}{2} a(T) \Phi \Phi^* + b(T) \ln[1 - 6 \Phi \Phi^* + 4 (\Phi \Phi^*)^3 - 3 (\Phi \Phi^*)^2]$$

$$a(T) = a_0 T^4 + a_1 \mu^4 + a_2 \mu^2 T^2$$

baryonic and quark mass shift $\delta m_B \sim f(\Phi)$ $\delta m_q \sim f(1-\Phi)$

quarks couple to mean fields via g_σ^q , g_ω^q

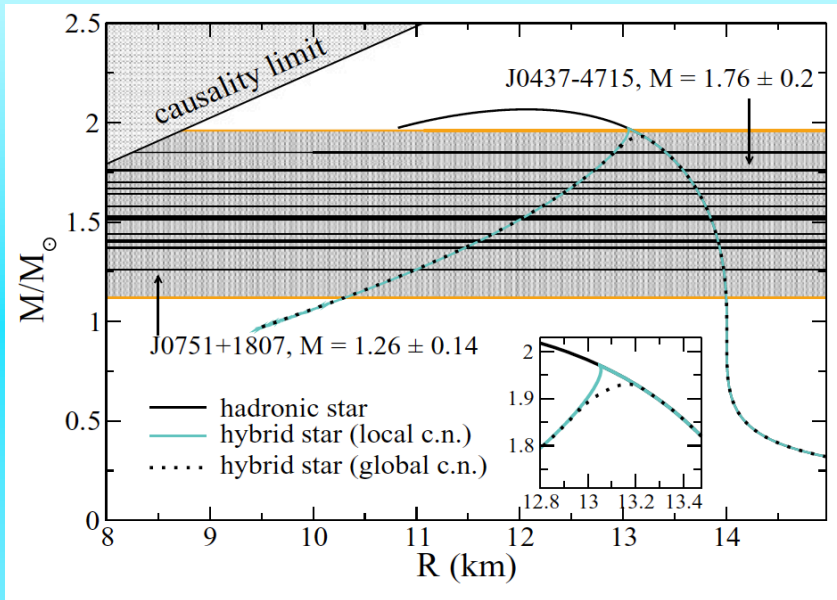
minimize grand canonical potential



V. Dexheimer, SWS, PRC 81 045201 (2010)

Ratti et al. PRD 73 014019 (2006)

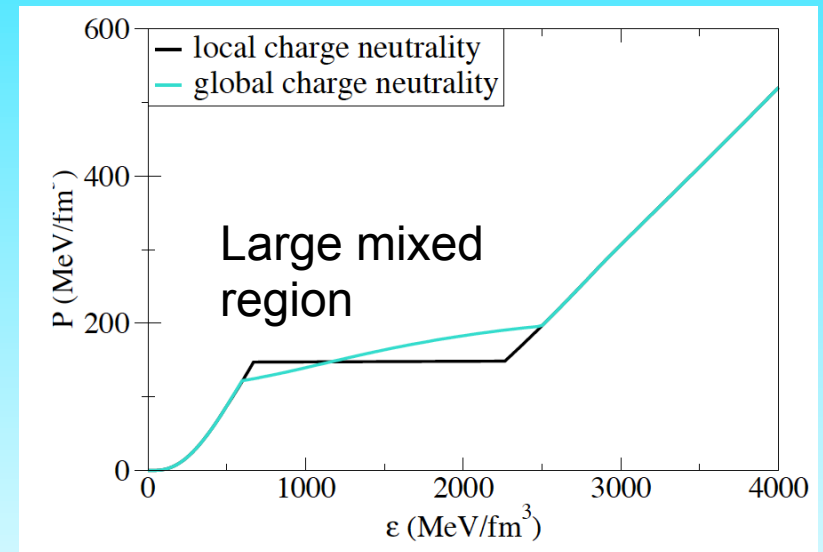
Fukushima, PLB 591, 277 (2004)



Maxwell / Gibbs construction for
local / global charge neutrality

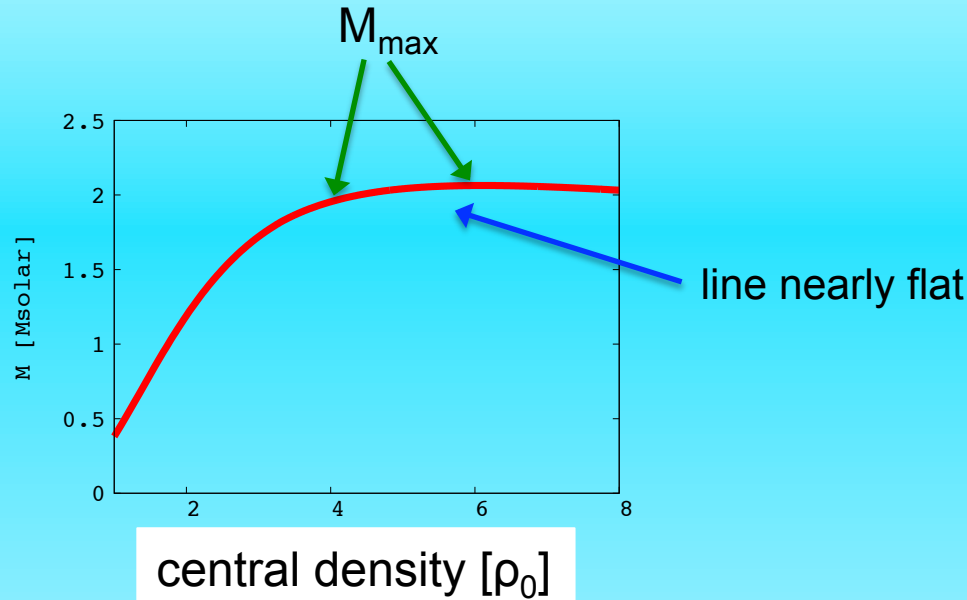
M-R diagram in QH model

baryonic star with a 2km core of quarks



Dependence of M_{\max} on quark core

SWS et al., arXiv:1112.1853 (2011)
SQM proceedings

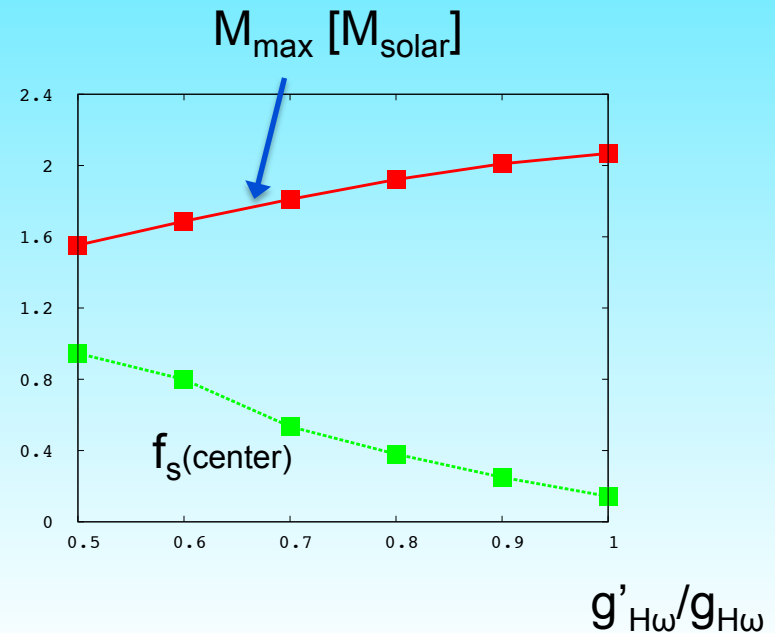


strangeness content

rather slow decay of strange condensate

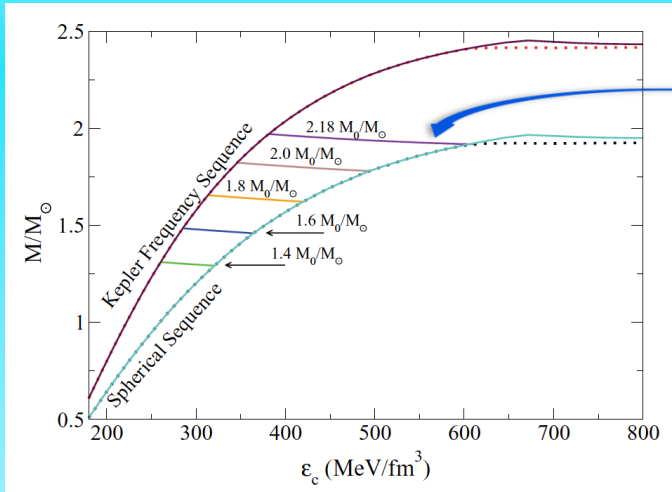
decrease (artificially)
vector potential for hyperons

preserve “canonical” values
 $U_{\Lambda} \sim -29$ MeV, $U_{\Xi} \sim -19$ MeV, $U_{\Sigma} \gtrsim 0$ MeV



Effect of rotation

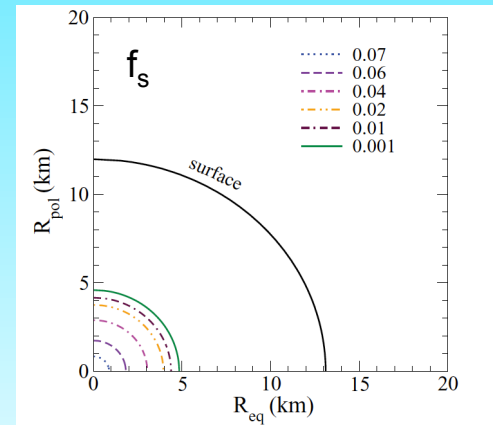
relatively small changes in M , M_{max}
substantial decrease in central density



PSR J1748-2446 $\Omega = 716 \text{ Hz}$

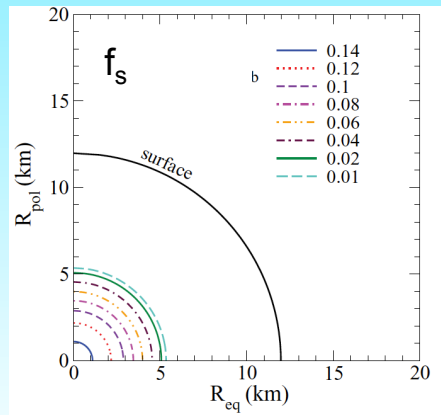
fixed baryon number ($\Omega_K \sim 990 \text{ Hz}$)

$\Omega = 600 \text{ Hz}$



f_s doubled

$\Omega = 0$



hadrons, quarks, Polyakov loop and excluded volume

Include modified distribution functions for quarks/antiquarks

$$\Omega_q = -T \sum_{j \in Q} \frac{\gamma_j}{(2\pi)^3} \int d^3k \ln \left(1 + \Phi \exp \frac{E_j^* - \mu_j}{T} \right)^*$$

Φ confinement order parameter*

Following the parametrization used in PNJL calculations

$$U = -\frac{1}{2} a(T) \Phi \Phi^* + b(T) \ln[1 - 6 \Phi \Phi^* + 4 (\Phi \Phi^*)^3 - 3 (\Phi \Phi^*)^2]$$

$$a(T) = a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2 \quad , \quad b(T) = b_3 T_0^3 T$$

The switch between the degrees of freedom is triggered by excluded volume corrections

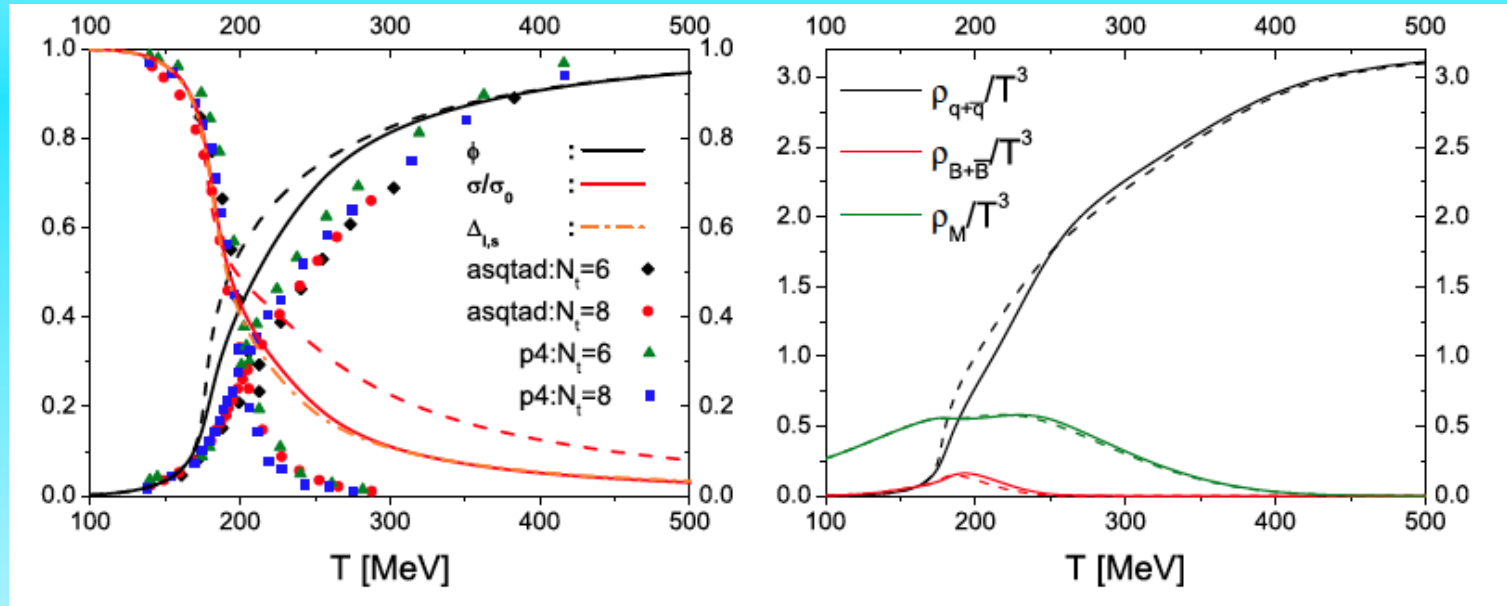
thermodynamically consistent -

no reconfinement!

$$\begin{aligned} V_q &= 0 \\ V_h &= v \\ V_m &= v / 8 \end{aligned} \quad \tilde{\mu}_i = \mu_i - v_i P \quad e = \tilde{e} / (1 + \sum v_i \tilde{\rho}_i)$$

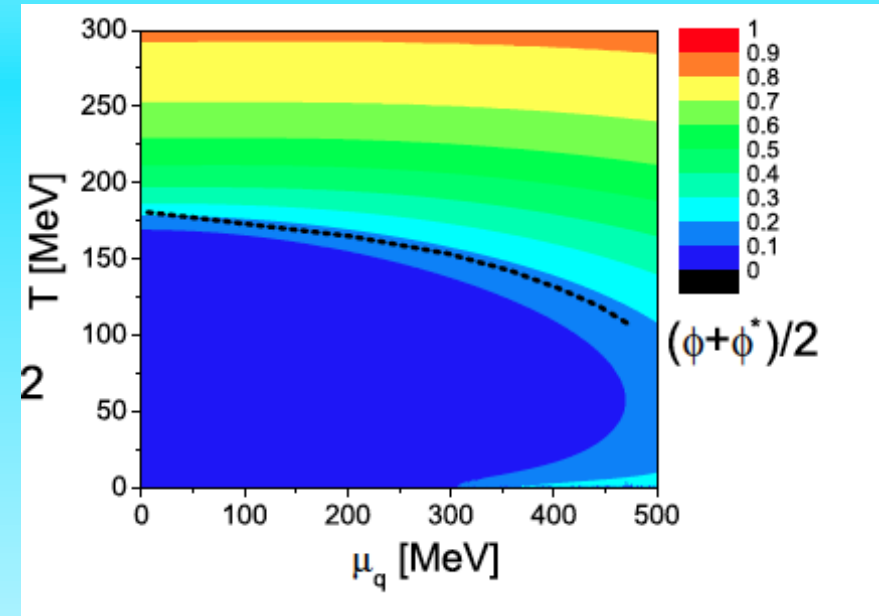
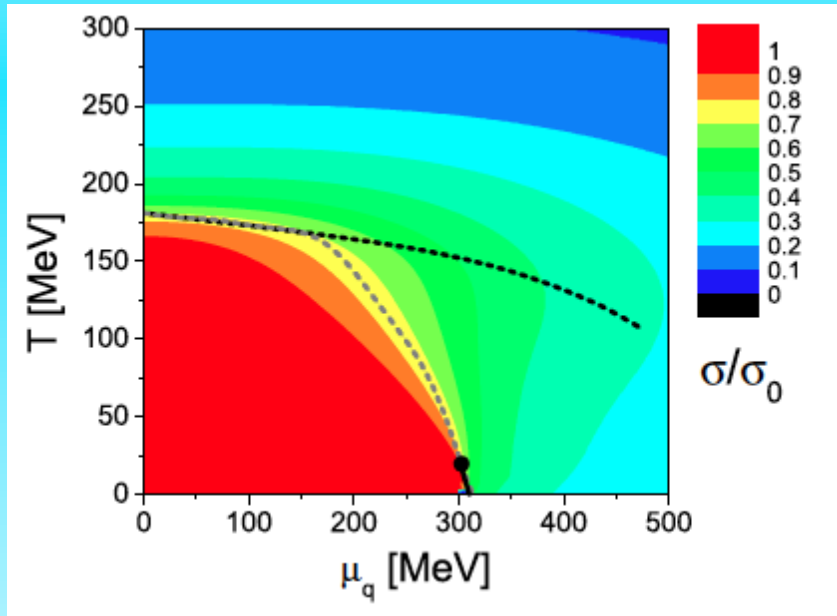
equation of state stays causal!

Results of the model at zero density



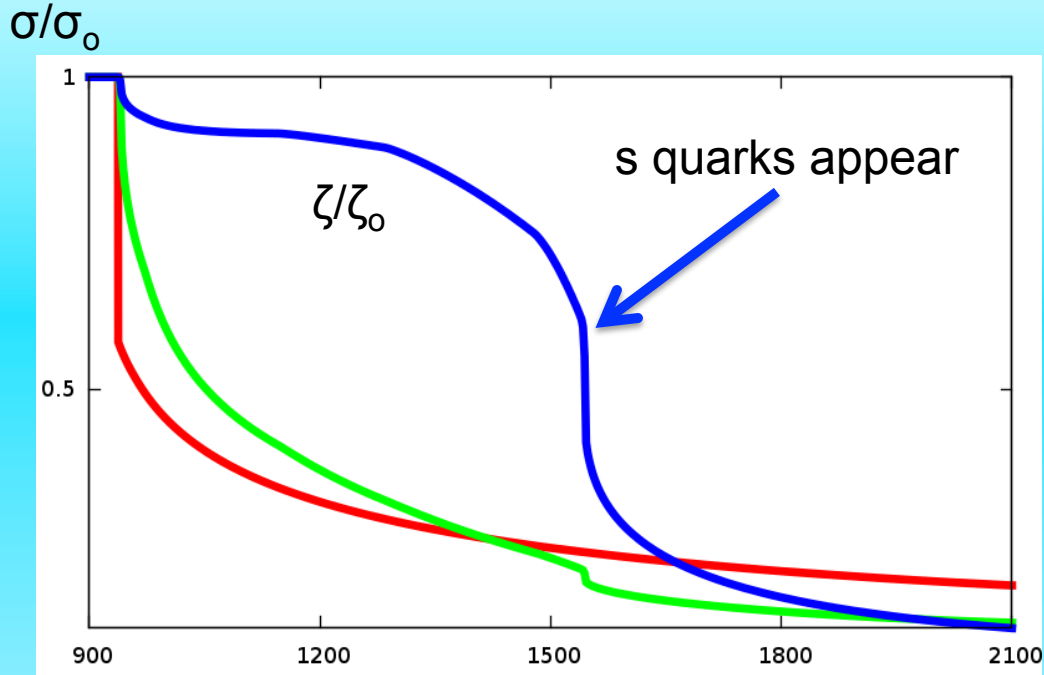
Reasonable agreement with lattice QCD
natural mixed phase of hadrons and quarks
no first-order transition

Order parameters for chiral symmetry and confinement in μ and T



except for liquid-gas no first-order transition

star matter in beta equilibrium

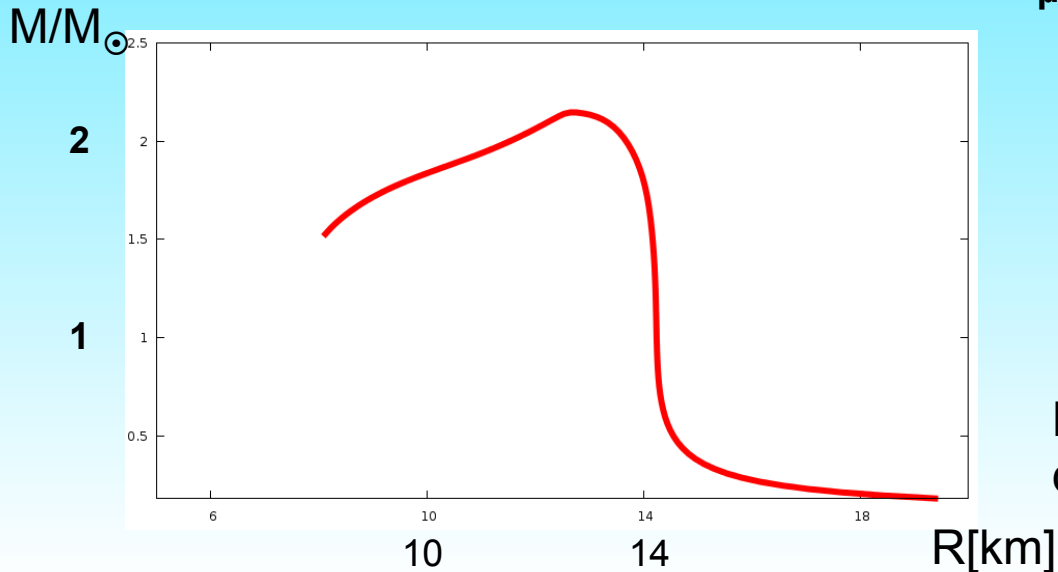


1st order phase transition
in star matter possible

cross over in symmetric matter

$f_s(\text{core}) = 0.6$ jumps to ~ 1

μ_B



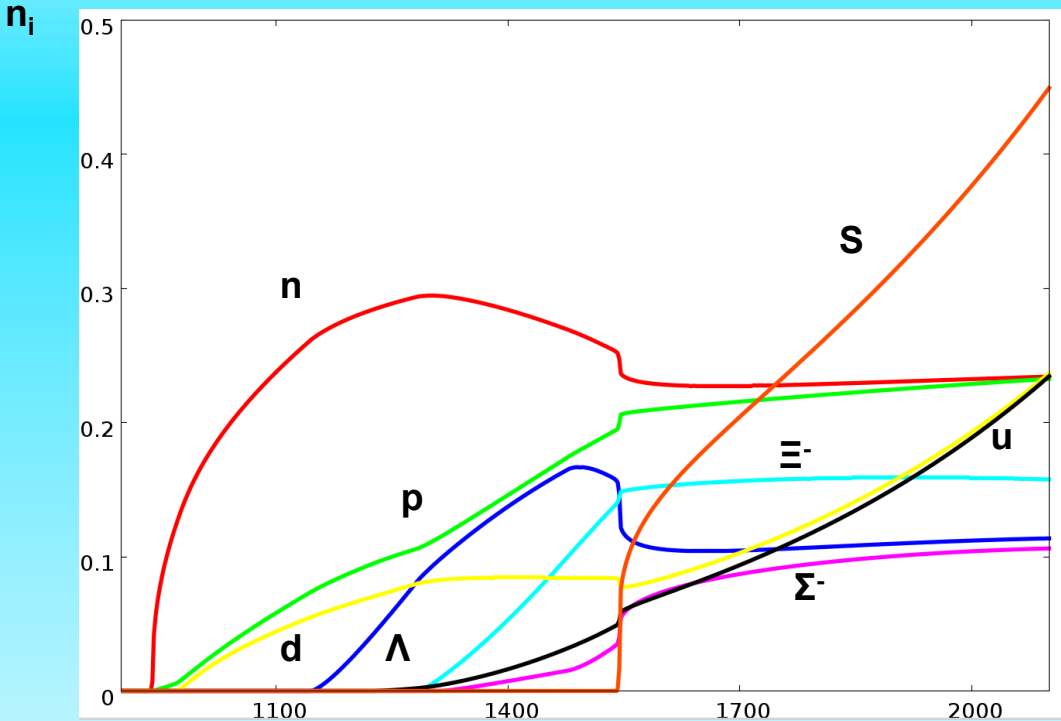
Star masses $M(R)$

Mass $\sim 2.2 M_{\odot}$ Radius ~ 13 km
depends on quark vector interaction

particle cocktail

$\kappa = 305 \text{ MeV}$
 $E_{\text{asym}} = 34.5 \text{ MeV}$
 $L = 105 \text{ MeV}$

reasonable values for
compressibility and asymmetry



large mixed phase (no Gibbs)

dense matter and stars in a parity doublet model

- treat N, N^* as positive/negative parity doublet

transformation:

$$\begin{aligned}\psi_{1R} &\longrightarrow R\psi_{1R}, & \psi_{1L} &\longrightarrow L\psi_{1L}, \\ \psi_{2R} &\longrightarrow L\psi_{2R}, & \psi_{2L} &\longrightarrow R\psi_{2L}.\end{aligned}$$

chirally invariant mass term

$$\begin{aligned}m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2) \\ = m_0(\bar{\psi}_{2L}\psi_{1R} - \bar{\psi}_{2R}\psi_{1L} - \bar{\psi}_{1L}\psi_{2R} + \bar{\psi}_{1R}\psi_{2L})\end{aligned}$$

standard σ model + vector mesons (ω, ρ)
diagonalize mass matrix ψ_1, ψ_2 to N, N^* with

$$M_{N^\pm}^* = \sqrt{\left[\frac{(M_{N_+} + M_{N_-})^2}{4} - m_0^2\right] \frac{\sigma^2}{\sigma_0^2} + m_0^2} \pm \frac{M_{N_+} - M_{N_-}}{2} \frac{\sigma}{\sigma_0}$$

degenerate in the
chiral limit

Dexheimer et al., PRC 77, 025803 (2008); EPJA 38, 105 (2008)
Zschesche et al., PRC 75, 055202 (2007)
DeTar and Kunihiro, PRD 39, 2805 (1989)
Hatsuda and Prakash, PLB 224, 11 (1989)

scan of possible parameters

SU(2)

constraint – reproduce basic n.m. saturation properties

$$E/A - m_N = -16 \pm 0.5 \text{ MeV} \quad \rho_0 = 0.15 \pm 0.015 \text{ fm}^{-3}$$

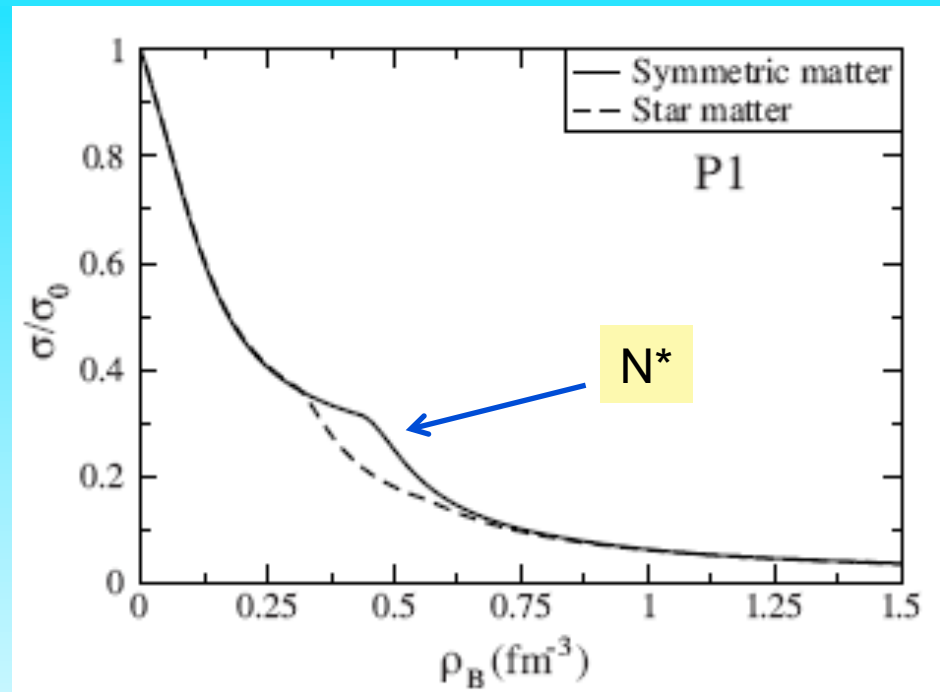
mass of the parity partner

possibility $N^*(1535)$ - *unclear*

keep mass as parameter

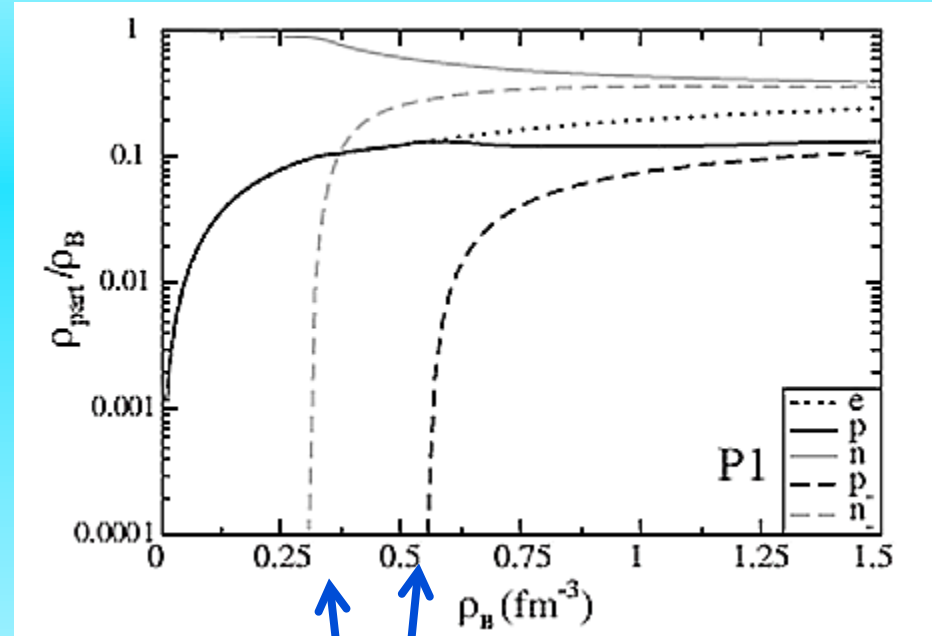
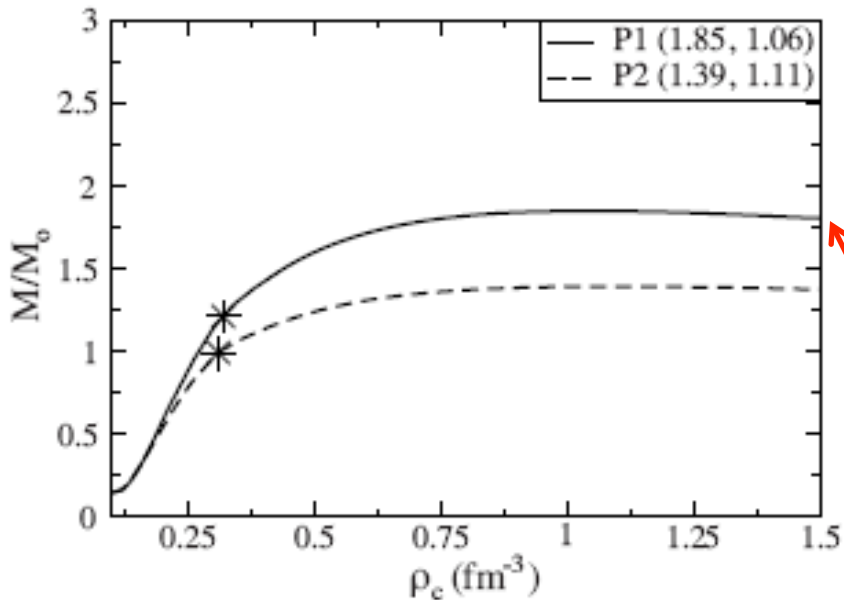
$(m_\sigma \quad g_\omega \quad m_{N^*} \quad m_0)$

$$m_{N^*} = 1200 \text{ MeV}$$



Star Masses and Particles

$m_{N^*} = 1200 \text{ MeV}$



parity partners

small vector self-interaction!

Extension of the parity model to SU(3)

Baryon SU(3) multiplet + parity doublets

Similar approach, SU(3)-invariant potential for scalar fields

single particle energies
$$E_{\pm} = \sqrt{(g_1\sigma + g_2\zeta)^2 + m_0^2} \pm (g'_1\sigma + g'_2\zeta)$$

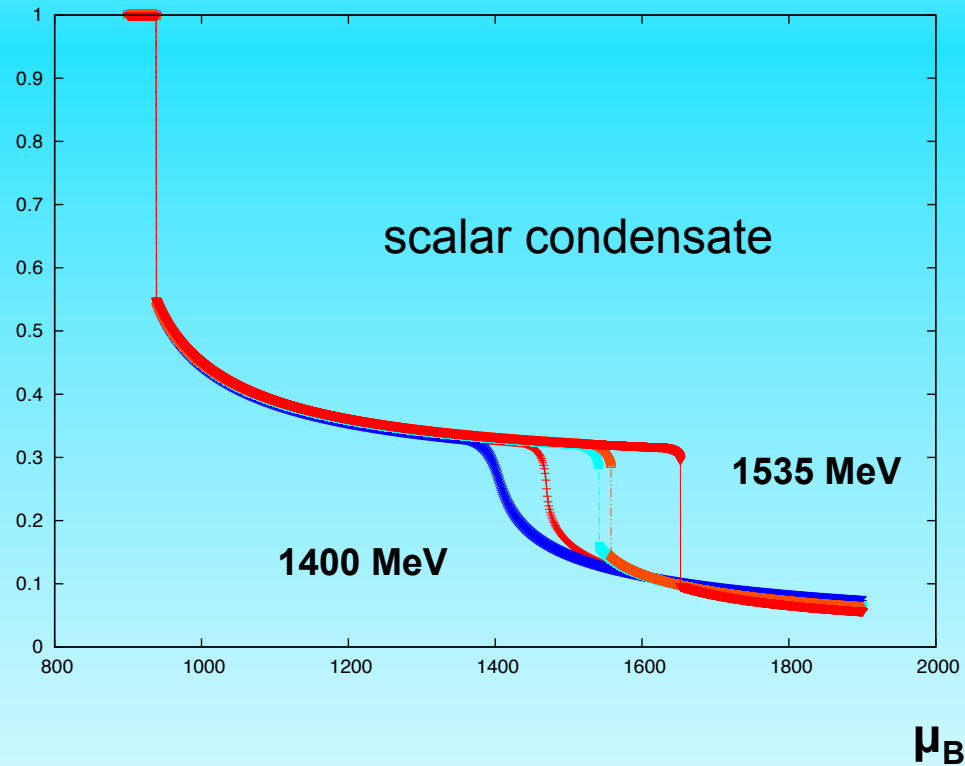
simplify investigation – same mass shift for whole octet

Candidates – $\Lambda(1670)$, $\Sigma(1750)$, Ξ (?) overall unclear

Steinheimer, SWS, Stöcker, JPhysG 38, 035001 (2011)

first study - Nemoto et al. PRD 57, 4124 (1998)

scalar condensate for different masses m_{N^*}

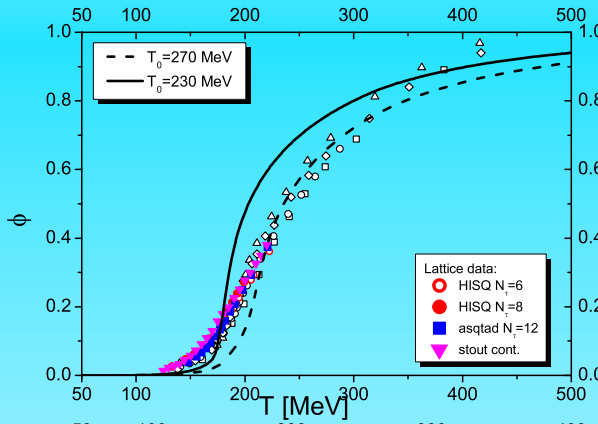


First order transition for masses ≥ 1470 MeV, below crossover

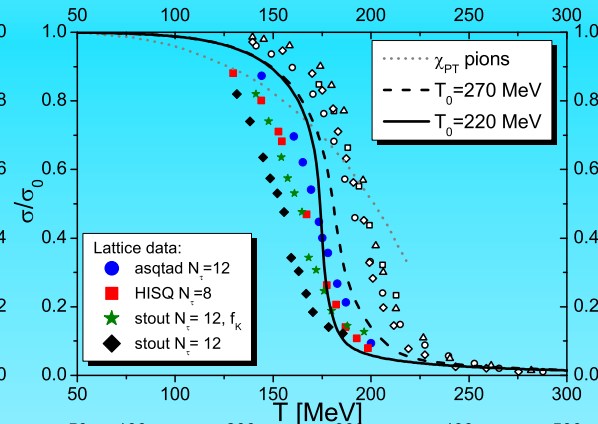
results for hot matter at vanishing chemical potential

points are various lattice results

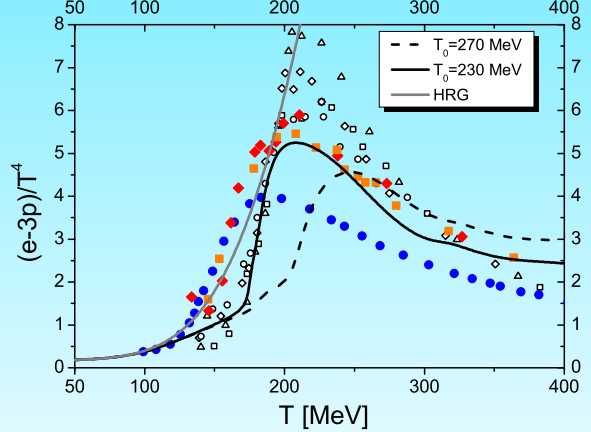
Polyakov loop



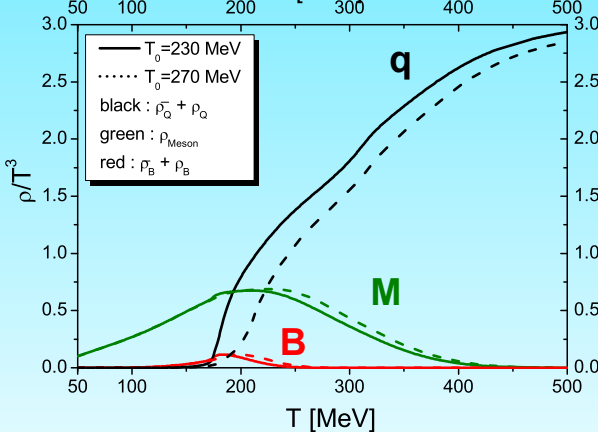
scalar condensate



Interaction measure



densities

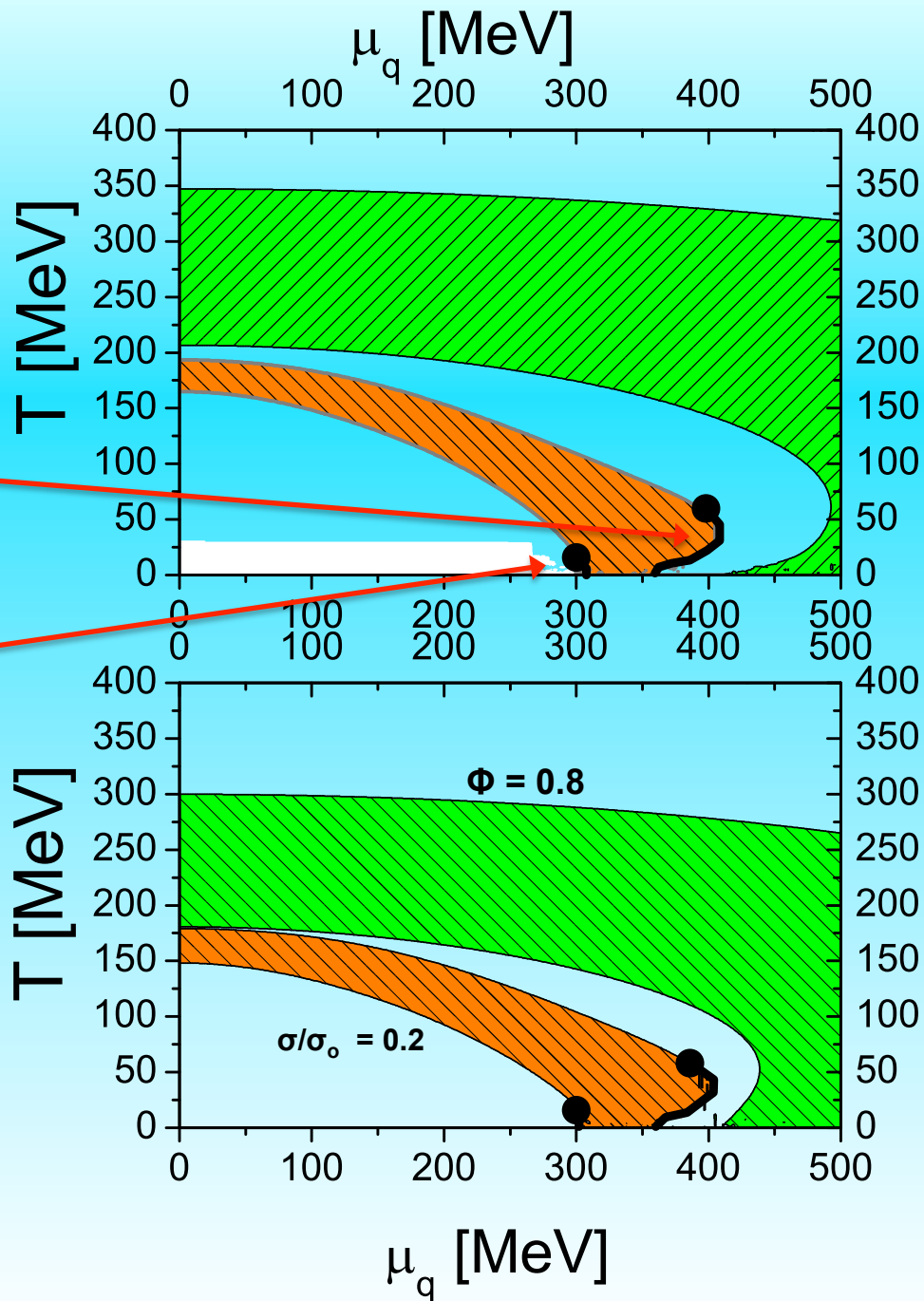


Excited quark-hadron matter in the parity-doublet approach

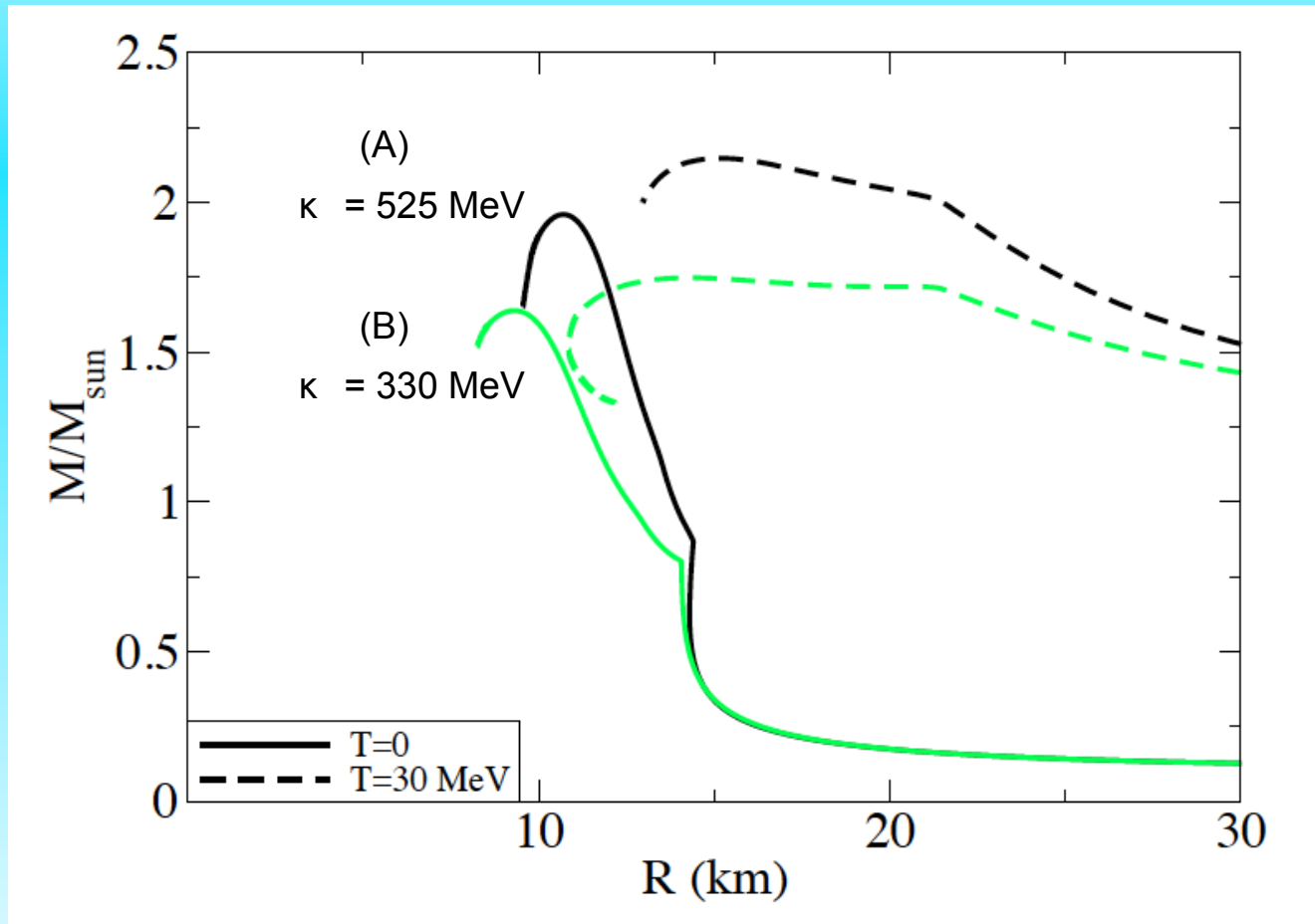
Chiral transition

Liquid-gas phase transition

2 different values for T_0

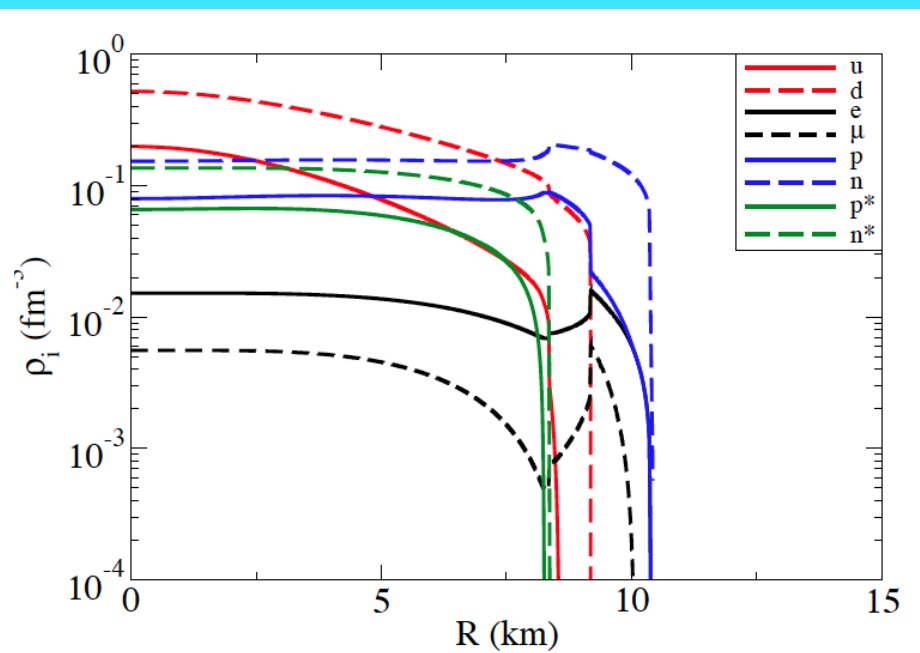


Neutron stars including quarks in the SU(3) parity doublet model

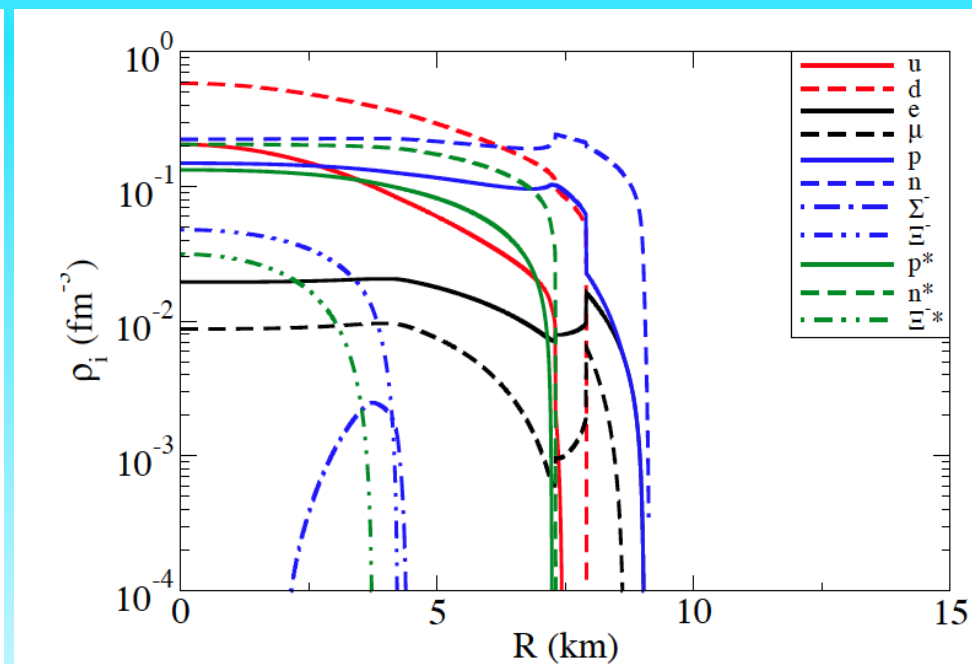


Improve on excluded volume treatment - not that trivial!

particle densities inside of the star for the parity doublet model

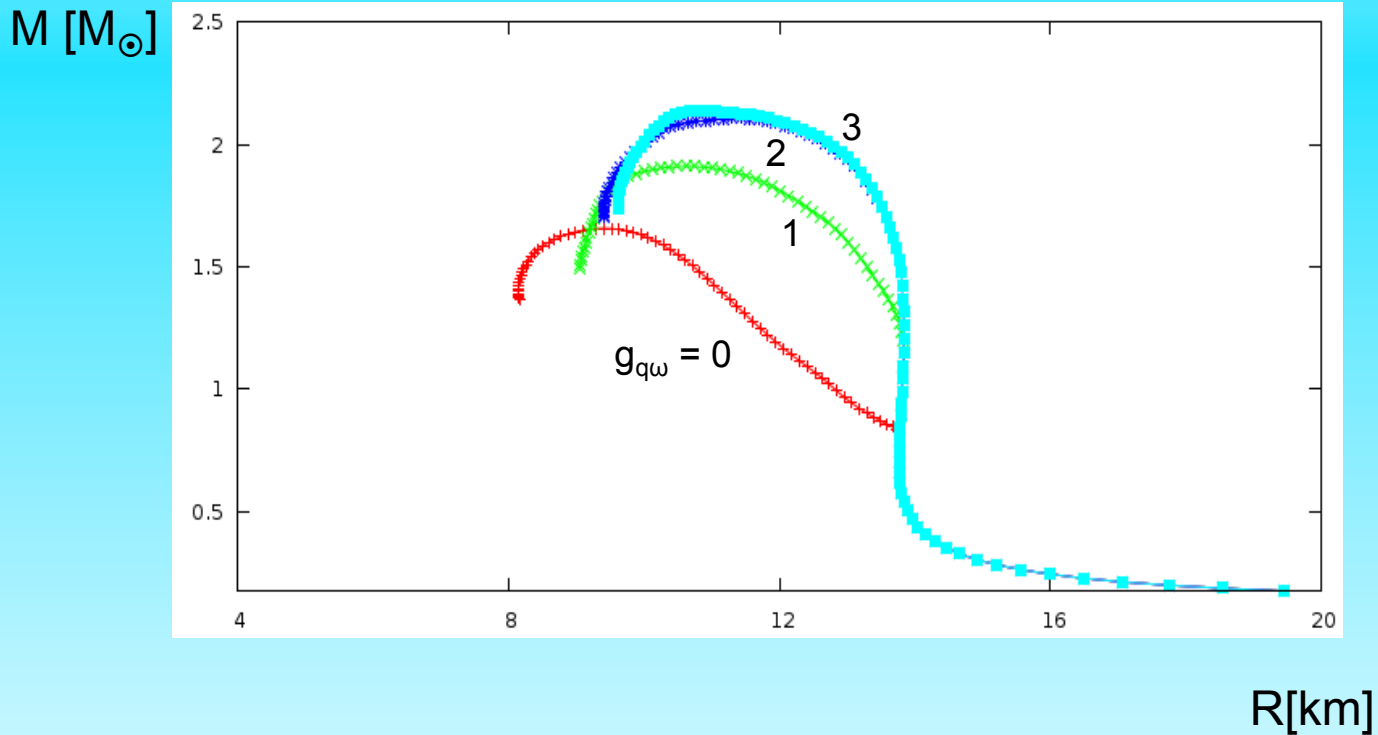


parameters (A)



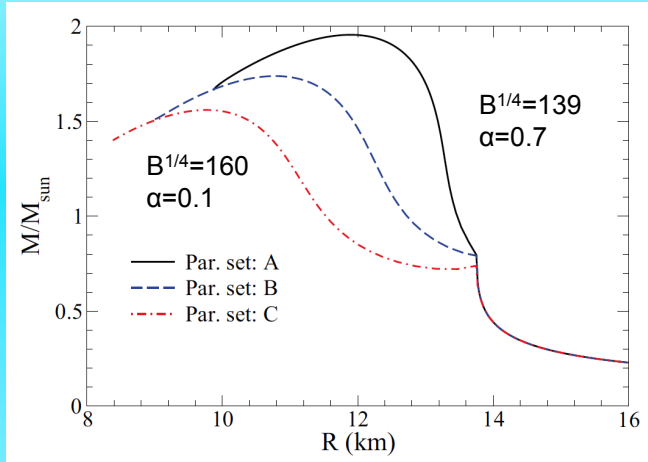
(B)

Including vector interaction for quarks



increase M / R , potential problems at $\mu = 0$

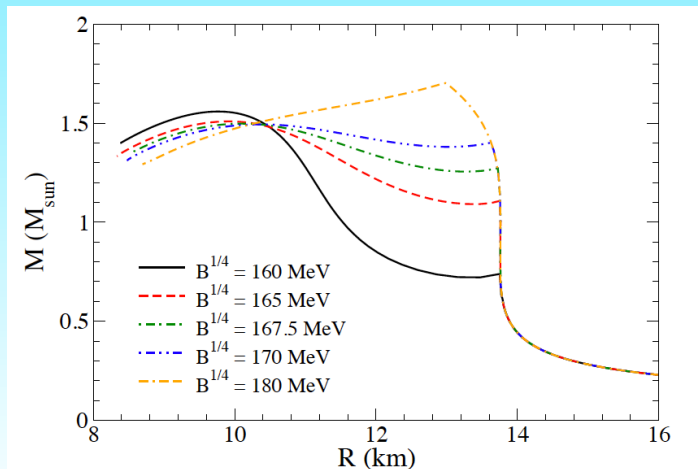
Hybrid Stars, Quark Interactions



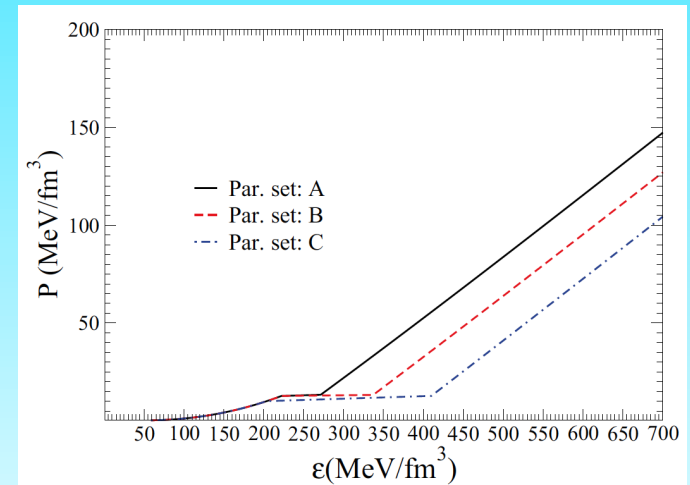
baryons alone $M_{\max} \sim 1.8 M_{\text{solar}}$

ingredients –
 Standard baryonic EOS (G300)
 plus MIT bag model + α_s corrections

Fast cooling in the quark core
 need gaps in the quark phase

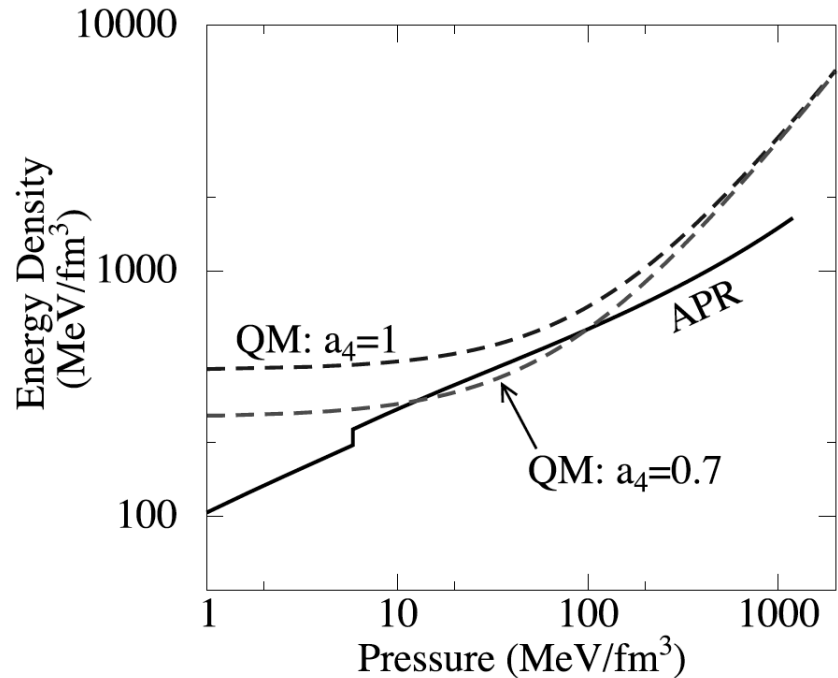
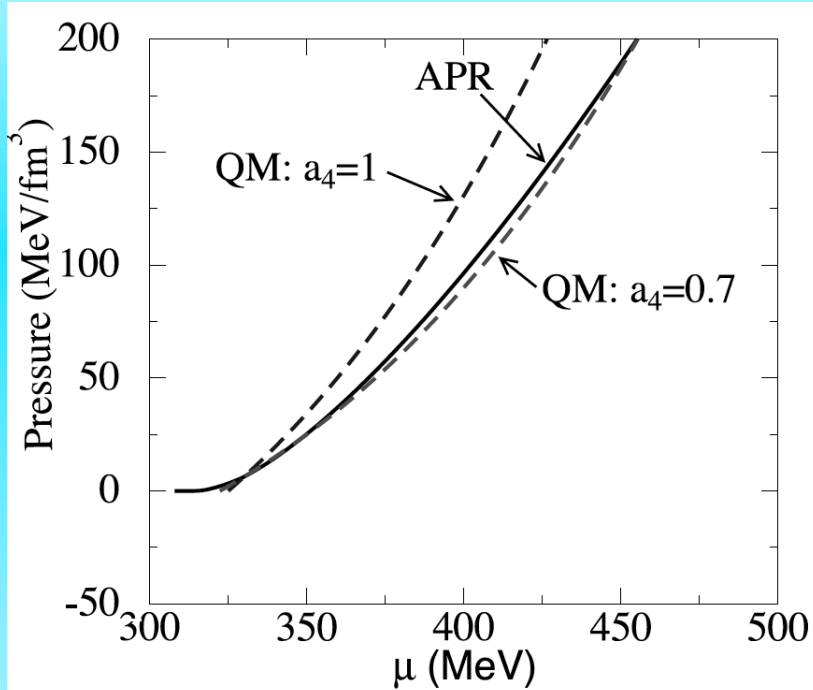


no α_s



Negreiros, Dexheimer, SWS, PRC 035805 (2012)

Quarks as Hadron Look-Alikes



$$\Omega_q \sim c a_4 \mu^4 + b a_2 \mu^2 + B_{\text{eff}}$$

$a_4 < 1$: perturbative QCD corrections

$$a_2 \sim m_s^2 - 4 \Delta^2$$

$a_4 \sim 0.7$ Fraga et al. PRD63 121702 (2001)

$$a_2 = (150 \text{ MeV})^2$$

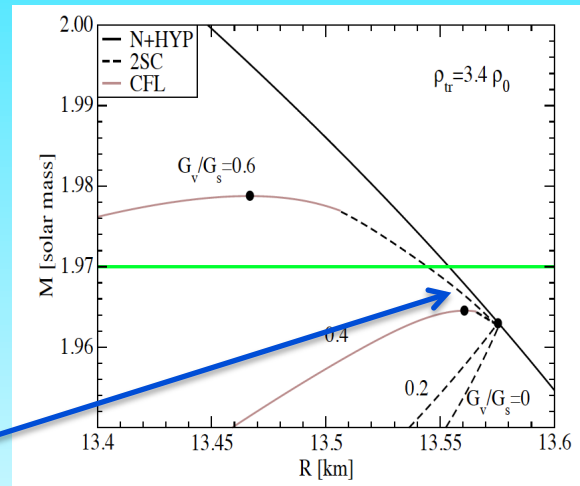
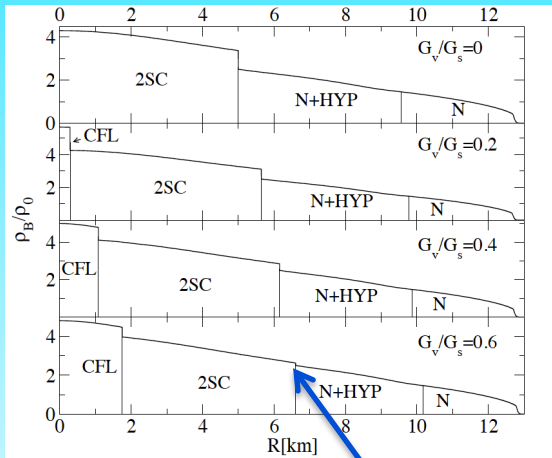
Similar EOS \rightarrow similar M(R) diagram

Hybrid star: $M_{\text{max}} \sim 2 M_{\text{solar}}$

nucleons plus hyperons
 NJL with 2SC/CFL + vector repulsion

$M_{\text{max}} \sim 1.98 M_{\text{solar}}$

Include various phases
 heavy star



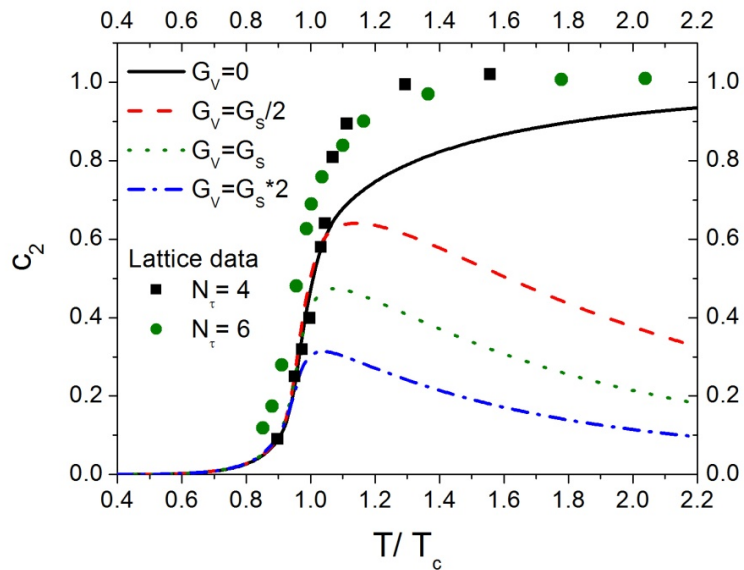
quark – hadron phases become similar
 again “look-alikes”

Susceptibility c_2 in PNJL and QH model for different quark vector interactions

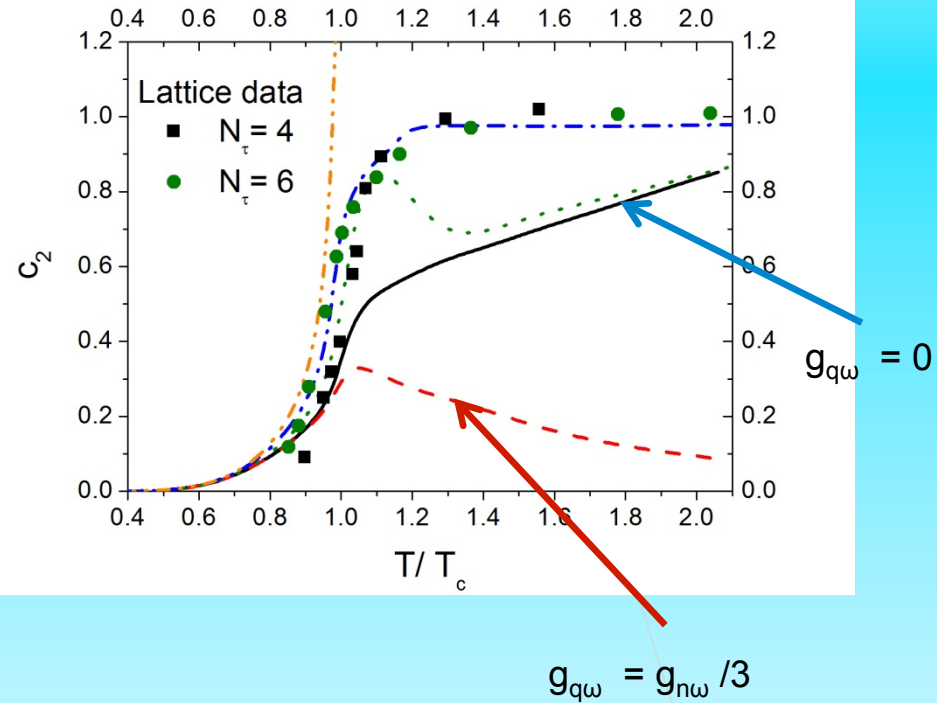
$$P(T, \mu) = P(T) + c_2(T) \mu^2 T^2 + \dots$$

small quark vector repulsion !!

PNJL

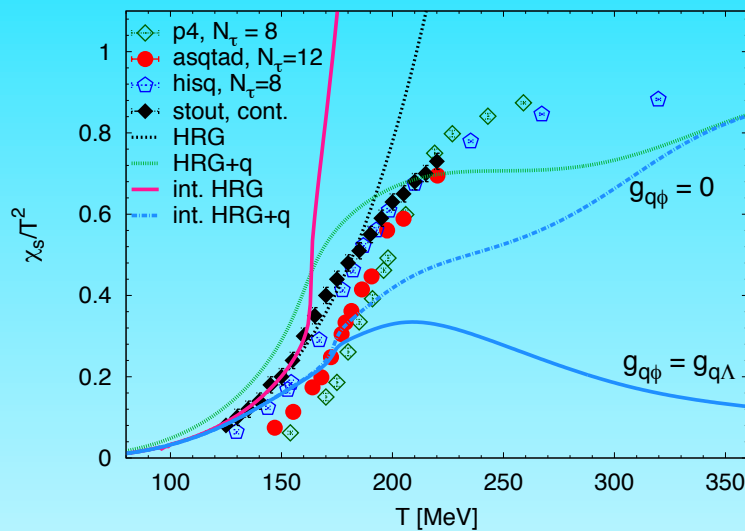


QH



analogous behaviour of strange susceptibility

$$\chi_s = d^2(P) / (d\mu_s)^2 |_{\mu_B, \mu_s = 0}$$



calc. by Philip Rau

Conclusions

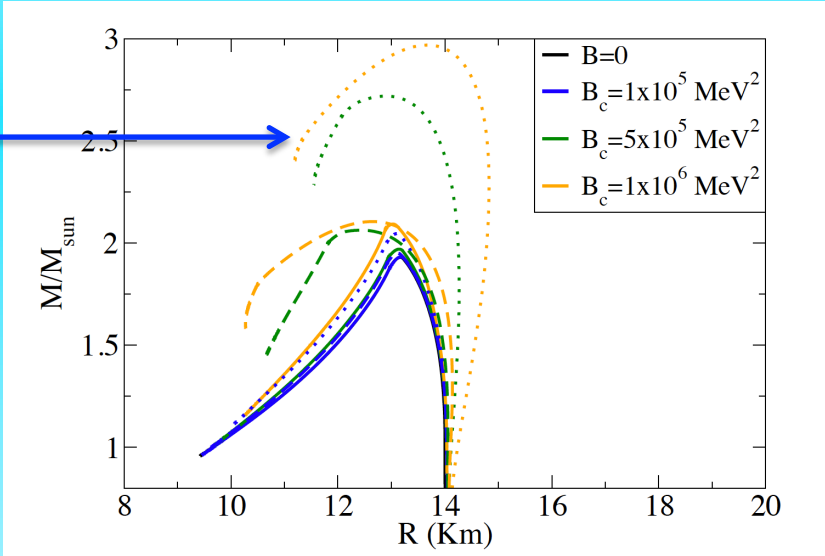
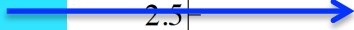
- $2 M_{\text{S}}$ does not preclude hyper-/hybrid stars
- SU(3) approach with dynamical baryon masses produces still reasonable star masses, need Φ
- Excluded volume effects interesting phase structure and heavy masses
- In general, similar results with parity doublet model
- Hybrid star approach tends to generate problems with small μ_{B} physics

Include magnetic fields

magnetars - up to 10^{15} Gauss surface field

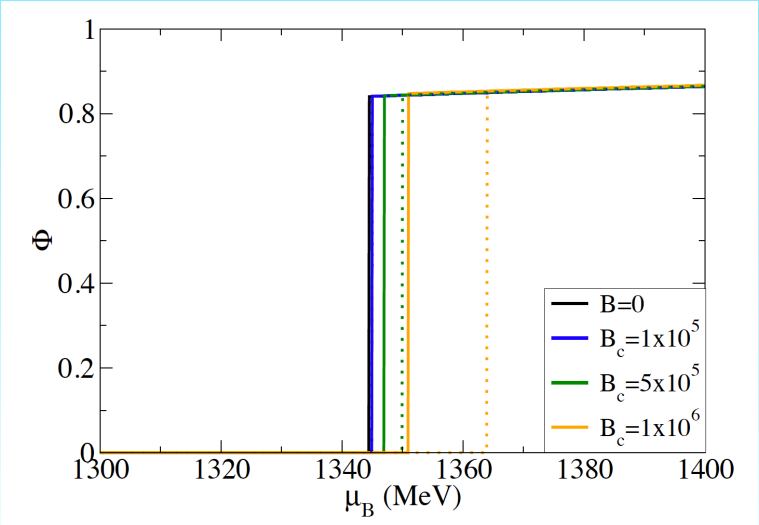
$T_{\mu\nu}^F = \text{diag}(\frac{1}{2}B^2, \frac{1}{2}B^2, \frac{1}{2}B^2, -\frac{1}{2}B^2)$
 energy-momentum tensor not isotropic
 consistent modeling of star needed

2d calculations:
 Bocquet et al. A&A 301, 757 (1995)
 Cardall et al. APJ 554, 322 (2001)

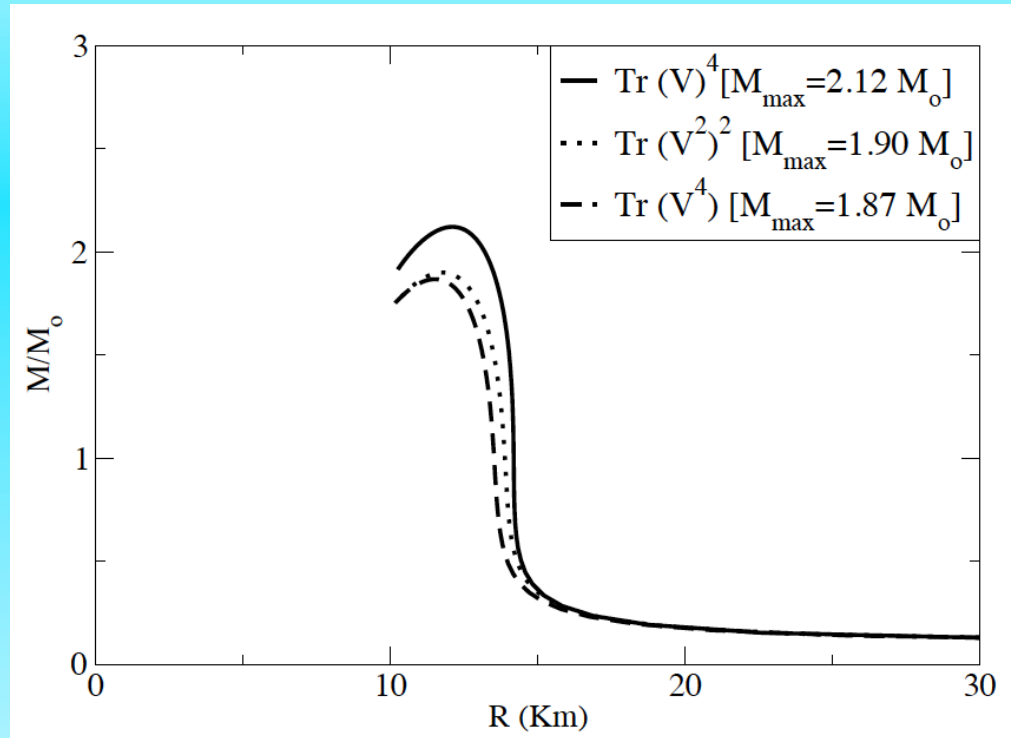


Small changes for B_c up to 10^{19} G

deconfinement transition
 magnetic field stiffens hadronic EOS
 moves transition to somewhat higher μ_B
 Cooling behavior is slightly modified

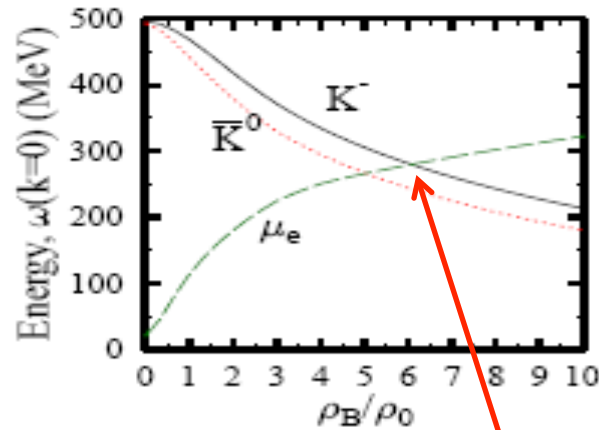
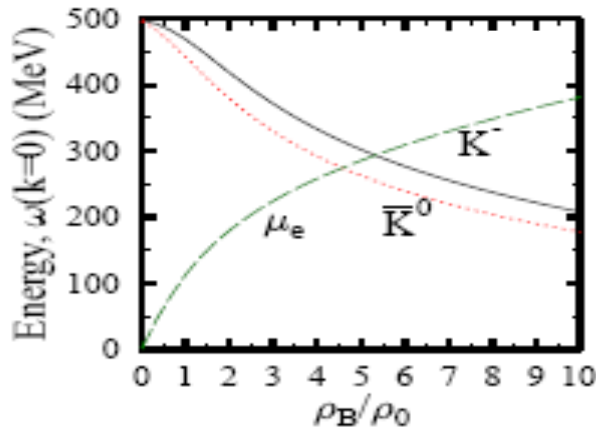
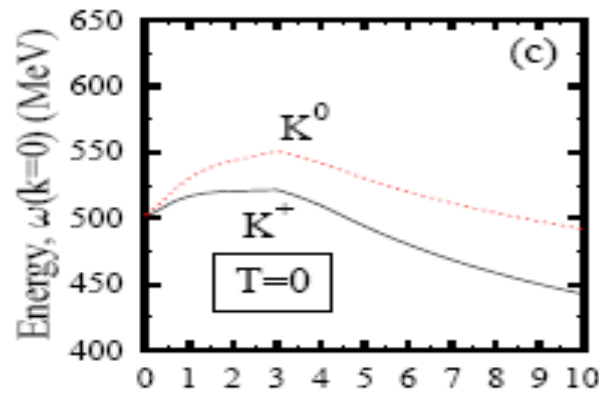
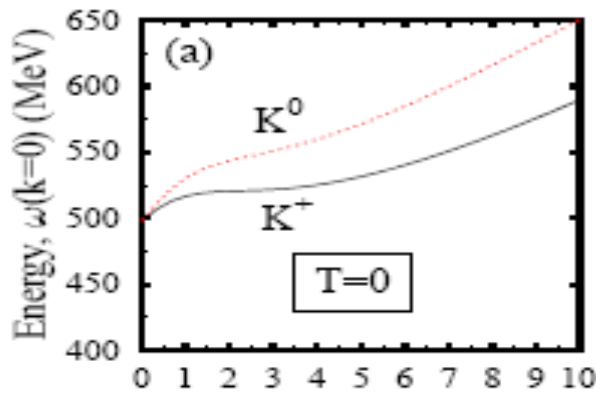


Different schemes for non-linear vector interactions



$$\text{diag}(V_{\mu}) = \{ (\omega + \rho)/\sqrt{2}, (\omega - \rho)/\sqrt{2}, \phi \}$$

kaon energies as function of density for neutron star at $T = 0$



nucleons

nucleons + hyperons

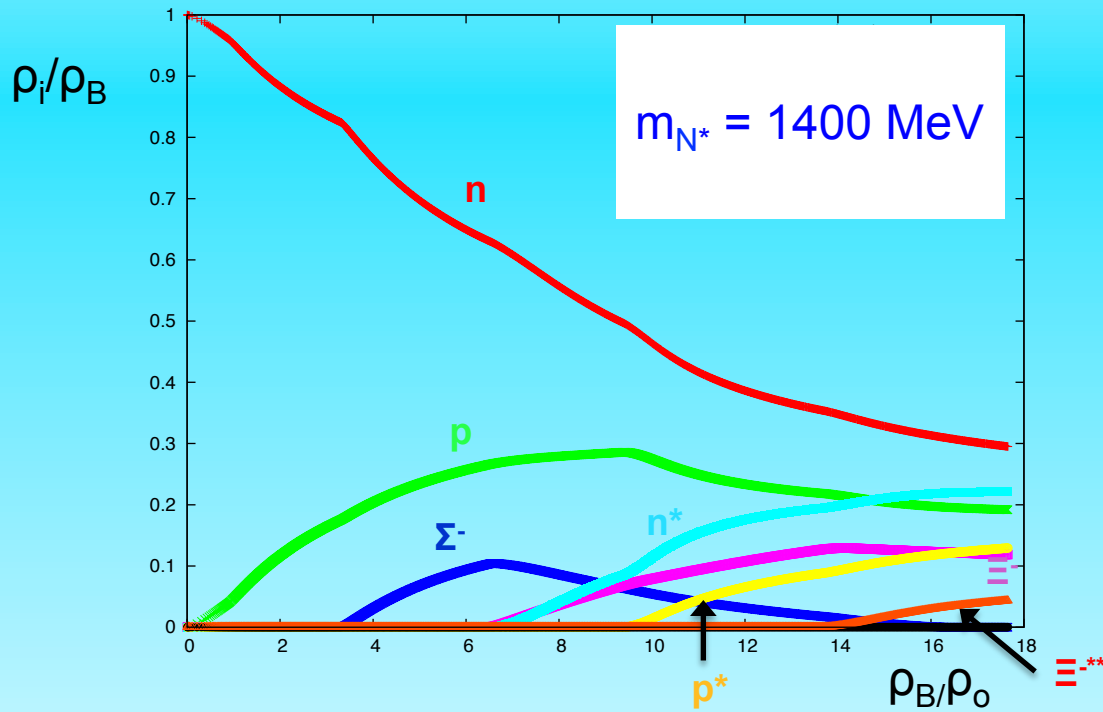
$U_{K^-}(\rho_0) \sim -50$ MeV

correct a_{KN} values

Kaon condensation sets in at around $5.5 \rho_0$
no significant change of star properties

Hyperons shift ρ_c to higher values

Particle population in nuclear matter in beta equilibrium



particle mix not too exotic for reasonably large densities