Density dependent magnetic field and the equation of state of baryonic matter.

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Outlook

- Used model.
- Equations.
- Chosen parameterization.

Lagrangian:

$$\mathcal{L}_b = \sum_{b=n,p,hyp} \overline{\psi}_b[i\gamma_\mu\partial^\mu + q_b\gamma_\mu A^\mu - g_{\rho b}\tau_{3b}\gamma_\mu\vec{\rho}^\mu - m_b^* \quad (1)$$

$$-g_{\omega b}\gamma_\mu\omega^\mu - \frac{1}{2}\mu_N k_b \sigma_{\mu\nu}F^{\mu\nu}]\psi_b$$

$$\mathcal{L}_m = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{3}bm_n(g_{\sigma N}\sigma)^3 - \frac{1}{4}c(g_{\sigma N}\sigma)^4 \quad (2)$$

$$+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$-\frac{1}{4}P^{\mu\nu}P_{\mu\nu} + \frac{1}{4!}\xi g_\omega^4(\omega_\mu\omega^\mu)^2 + \Lambda_\omega(g_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu)(g_\omega^2\omega_\mu\omega^\mu).$$

$$\mathcal{L}_l = \sum_{l=e,\mu} \overline{\psi}_l[i\gamma_\mu\partial^\mu + q_l\gamma_\mu A^\mu - m_l]\psi_l \quad (3)$$

$$\mathcal{L} = \mathcal{L}_b + \mathcal{L}_m + \mathcal{L}_l \quad (4)$$

Lagrangian.

- (ψ_b) - Baryon field
- (ψ_l) - Lepton field
- (σ) - Scalar meson
- (ω^μ) - Vectorial meson
- (ρ^μ) - Isovetorial meson
- g_σ, g_ω, g_ρ - Meson-baryon coupling constants
- γ_μ - Dirac Matrices
- $A^\mu = (V, \vec{A})$ - Lorentz tensor
- $k_p = 1.79285$, $k_n = -1.91315$ - Anomalous magnetic moment (AMM)
- τ_3 - Isospin projection
- q_b - Electrical charge of the particle b
- μ_N - Nuclear magneton.

Lagrangian.

- $m_b^* = (m_b - g_{\sigma b}\sigma)$ - Baryon effective mass
- $\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$
- $P_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$
- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ - Electromagnétic tensor
- ξ - Omega meson self-interaction parameter
- Λ_ω - Parameter that changes the density dependence of the symmetry energy
- b and c - Parameters related with the weights of the non-linear scalar terms

Magnetic field.

$$B(n) = B_{surf} + B_0 \left[1 - \exp\left(-\beta \left(\frac{n}{n_0} \right) \right)^{\gamma} \right] \quad (5)$$

- $B_{surf} = 10^{15}\ G,\ n_0 = 0.153\ fm^{-3}$
- Fast: $\gamma = 4$, $\beta = 1.83$ Slow: $\gamma = 1$, $\beta = 1.92$
- The unit of the magnetic field is the critical field $B_e^c = 4.414x10^{13}\ G$, so that $B = B_0/B_e^c$.

Magnetic field.

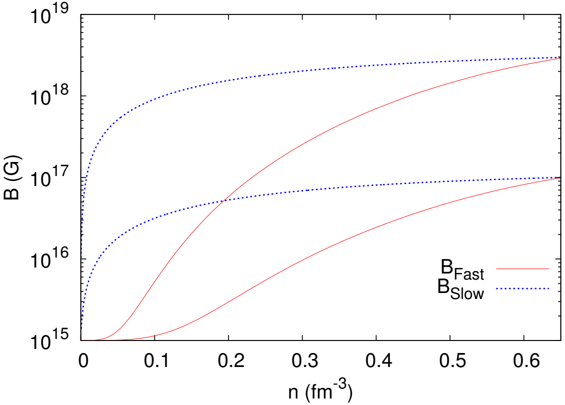

Figura: Variable density dependent magnetic field.

Magnetic field.

Dividing the work in two types:

- Fixed magnetic field (FIX): The magnetic field is considerate constant on the equations of state.
- Variable density dependent magnetic field (VAR): The magnetic field varies on the equations of state, see eq. (**??**).
- Both cases consider a density dependent variable magnetic field B term on the total energy density and pressure, eq. (**??**).

Mesonic field equations:

Utilizing the mean field approximation and the Euler-Lagrange equations.:

$$\begin{aligned} m_\sigma^2 \sigma_0 &= -c g_\sigma^4 \sigma_0^3 - b m_n g_\sigma^3 \sigma_0^2 + g_\sigma n^s \\ m_\omega^{*2} \omega_0 &= -\frac{\xi}{6} g_\omega^4 \omega_0^3 + g_\omega n_b \\ m_\rho^{*2} \rho_{03} &= \tau_{3b} g_\rho n_3, \end{aligned} \tag{6}$$

- $m_\rho^{*2} = m_\rho^2 + 2\Lambda_\omega g_\rho^2 g_\omega^2 \omega_0^2$
- $m_\omega^{*2} = m_\omega^2 + 2\Lambda_\omega g_\rho^2 g_\omega^2 \rho_{03}^2$
- $g_{iH} = X_{iH} g_{iN}$
- $X_{\sigma H} = 0.700, X_{\omega H} = 0.738,$ $X_{\rho H} = 0.600$
- $n^s = n_p^s + n_n^s$
- $n_b = n_p + n_n$
- $n_3 = \frac{(n_p - n_n)}{2}$

Densities:

Scalar[1]:

$$n_b^s = \frac{q_p B m_p^*}{2\pi^2} \sum_{\nu}^{\nu_{max}} \sum_s \frac{\sqrt{m_p^{*2} + 2\nu q_p B} - s\mu_N k_p B}{\sqrt{m_p^{*2} + 2\nu q_p B}} \tag{7}$$

$$\times \ln \left| \frac{p_{F,\nu,s}^p + E_F^p}{\sqrt{m_p^{*2} + 2\nu q_p B} - s\mu_N k_p B} \right|$$

$$n_b^s = \frac{m_n^*}{4\pi^2} \sum_s \left[E_F^n p_{F,s}^n - \overline{m}_n^2 \ln \left| \frac{p_{F,s}^n + E_F^n}{\overline{m}_n} \right| \right]$$

[1] J.Phys.G: Nucl. Part. Phys. **35** (2008) 125201 (17pp)

Densities:

Vectorial:

$$n_b = \frac{q_p B}{2\pi^2} \sum_{\nu}^{\nu_{max}} \sum_{s} p^p_{F,\nu,s} \tag{8}$$

$$n_b = \frac{1}{2\pi^2} \sum_{s} \left[\frac{1}{3}(p^n_{F,s})^3 - \frac{1}{2} s\mu_N k_n B \right.$$

$$\left. \times \left(\overline{m}_n p^n_{F,s} + E^{n2}_F \left(\arcsin\left(\frac{\overline{m}_n}{E^n_F} \right) - \frac{\pi}{2} \right) \right) \right]$$

$$n_l = \frac{|q_l| B}{2\pi^2} \sum_{\nu}^{\nu_{max}} \sum_{s} p^l_{F,\nu,s}$$

$$n_B = \sum_{b=n,p,hyp} n_b + n_l \tag{9}$$

Baryons:

b	M_b (MeV)	q_b	μ_b/μ_N	k_b
p	938.3	1	2.79	1.79
n	939.6	0	-1.91	-1.91
Λ^0	1115.7	0	-0.61	-0.61
Σ^+	1189.4	1	2.46	1.67
Σ^0	1189.4	0	-1.61	-1.61
Σ^-	1189.4	-1	-1.16	-0.38
Ξ^0	1314.9	0	-1.25	-1.25
Ξ^-	1314.9	-1	-0.65	0.06

Tabela: Masses, charges, magnetic moments e anomalous magnetic moments for baryons.

Where $k_b = \frac{\mu_b}{\mu_N} - q_b \frac{m_p}{m_b}$.

Parametrizations:

Modelo	m_σ	m_ω	m_ρ	g_σ	g_ω	g_ρ
GM1	512.0	783.0	770.0	8.910	10.610	8.196

Tabela: Masses and coupling constants for mesons.

Modelo	c	b	ξ	Λ_ω
GM1	-0.001070	0.002947	0.00	0.00

Tabela: Constants.

Energy spectra:

$$E^b_{\nu,s} = \sqrt{p_z^2 + (\sqrt{m_p^* + 2\nu q_p B} - s\mu_N k_p B)^2} + g_\omega \omega^0 + \frac{1}{2} g_\rho \rho^0 \quad (10)$$

$$E^b_s = \sqrt{p_z^2 + (\sqrt{m_n^* + p_\perp^2} - s\mu_N k_n B)^2} + g_\omega \omega^0 - \frac{1}{2} g_\rho \rho^0,$$

- $\nu = n + \frac{1}{2} - \text{sgn}(q_b)\frac{s}{2} = 0, 1, 2, ...$ - landau levels for fermions with electrical charge q
- s is the spin, which assumes values $+1$ for spin *up* and -1 for spin *down*
- $p_\perp^2 = p_x^2 + p_y^2$

Momenta:

$$
\begin{aligned}
p^b_{F,\nu,s} &= E^{b2}_F - [\sqrt{m^{*2}_b + 2\nu q_b B} - s\mu_N k_b B]^2 \\
p^b_{F,s} &= E^b_F - \overline{m}^2_b \\
p^l_{F,\nu,s} &= E^l_F - \overline{m}^2_l
\end{aligned}
\qquad (11)
$$

- Where $p^b_{F,\nu,s}$ is the Fermi momentum for the charged baryons, $p^b_{F,s}$ for the neutron and $p^l_{F,\nu,s}$ for the leptons.
- $\overline{m}_b = m^* - s\mu_N k_b B$, $\overline{m}_l = m^2_l - 2\nu|q_l B|$.

Landau levels:

$$\nu_{max} = \left[\frac{(E_F^b + s\mu_N k_b B)^2 - m_b^*}{2|q_b|B} \right] \quad (12)$$
$$\nu_{max} = \left[\frac{(E_F^{l2}) - m_l^2}{2|q_l|B} \right]$$

- Landau levels are summed up to its maximum value, for which the square of the Fermi moment of the particle is still positive.

Chemical Potencials:

$$
\begin{aligned}
\mu_p &= E_F^p + g_\omega \omega^0 + \frac{1}{2} g_\rho \rho^0 \qquad (13)\\
\mu_n &= E_F^n + g_\omega \omega^0 - \frac{1}{2} g_\rho \rho^0 \\
\mu_p &= \mu_{\Sigma^+} = \mu_n - \mu_e \\
\mu_\Lambda &= \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n \\
\mu_{\Sigma^-} &= \mu_{\Xi^-} = \mu_n + \mu_e \\
\mu_\mu &= \mu_e \\
\sum_{b=n,p,hyp} q_b n_b &+ \sum_l q_l n_l = 0
\end{aligned}
$$

Total energy density:

$$\varepsilon_m = \sum_{b=p,n,hyp} \varepsilon_b + \sum_{l=e,\mu} \varepsilon_l + \frac{1}{2} m_\sigma \sigma_0^2 + \frac{1}{2} m_\omega \omega_0^2 + \frac{1}{2} m_\rho \rho_0^2 \quad (14)$$

$$+\frac{1}{8}\xi (g_\omega \omega_0)^4 + \frac{1}{3} b m_n (g_\sigma \sigma_0)^3 + \frac{1}{4} c (g_\sigma \sigma_0)^4$$

$$+3\Lambda_\omega (g_\omega \omega_0)^2 (g_\rho \rho_0)^2,$$

Energy density:

$$\varepsilon_b = \sum_{n=0}^{n_{max}} \sum_s \frac{|q_b| B}{4\pi^2} \left[p^b_{F,n,s} E^b_F + (\sqrt{m_b^2 + 2q_b B n} - s\mu_N k_b B)^2 \right. \tag{15}$$

$$\left. \times ln \left| \frac{p^b_{F,n,s} + E^b_F}{(\sqrt{m_b^2 + 2q_b B n} - s\mu_N k_b B)} \right| \right],$$

$$\varepsilon_b = \frac{1}{4\pi^2} \sum_s \frac{1}{2} E_F^{b3} p^b_{F,n,s} - \frac{2}{3} s\mu_N k_b B E_F^{b3} \left(arc\sin \frac{\overline{m}}{E^b_F} - \frac{\pi}{2} \right)$$

$$-\left(\frac{1}{3} s\mu_N k_b B + \frac{1}{4} \overline{m} \right) \left[\overline{m} p^b_{F,n,s} E^b_F + \overline{m}^3 ln \left(\left| \frac{E^b_F + p^b_{F,n,s}}{\overline{m}} \right| \right) \right]$$

$$\varepsilon_l = \sum_{n=0}^{n_{max}} \sum_s \frac{|q_l| B}{4\pi^2} \left[p^l_{F,n,s} E^l_F + \overline{m}_l^2 \times ln \left| \frac{p^l_{F,n,s} + E^l_F}{\overline{m}_l} \right| \right]$$

Pressure and total energy densities:

$$P_m = \sum_{i=e,\mu,n,p,hyp} \mu_i n_i - \varepsilon_m \tag{16}$$

Attributing the contributions of magnetic fields, we find the total energy density and pressure:

$$\varepsilon_T = \varepsilon_m + \frac{B^2}{2} \qquad P_T = P_m + \frac{B^2}{2} \tag{17}$$

Symmetry energy and slope:

$$
\begin{aligned}
a_{sym} &= \frac{1}{2n_B}\frac{\partial^2 E}{\partial t^2} \\
L &= 3n_0 \frac{\partial a_{sym}}{\partial n_B}\Big|_{n_B \to n_0}
\end{aligned}
\tag{18}
$$

- $n_0 = 0.153\ fm^{-3}$ - Nuclear saturation density
- $t = \frac{n_n - n_p}{n_p + n_n}$ - Asymmetry parameter.

- THANK YOU!