The neutrino escape and gravitational wave generation in protoneutron star cooling

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Introduction

The nascent Neutron stars are fast rotating systems with periods of revolution of few milliseconds. Deformed rotating massive system, according to the General Theory of Relativity, can convert part of their rotational energy into gravitational waves .

We describe the dynamical evolution of a rotating proto- neutron star approximately as a rotating compressible homogeneous triaxial ellipsoid. The dynamical evolution of the system is described by an effective Lagrangean ,

$$L = K - W - U_{int} - U_{rot}$$

with K being the translational kinetic energy, W the gravitational potential energy, U_{int} the internal energy, and U_{rot} the rotational kinetic energy. All quantities are written in terms of the semi-axes of the ellipsoid, a_1, a_2, and a_3 and their time derivatives. With the kinetic terms given by,

$$K = \frac{1}{10} M\left(\dot{a}_1^2 + \dot{a}_2^2 + \dot{a}_3^2\right) \qquad U_{rot} = \frac{5}{2}\frac{J^2}{M(a_1^2 + a_2^2)}$$

and the gravitational energy as

$$W = -\frac{3}{10} G M^2 \frac{A}{a_1 a_2 a_3} \qquad \text{with} \qquad A = \sum_{i=1}^{3} A_i a_i^2 \,,$$

where the parameters are defined as,

$$A_i = a_1 a_2 a_3 \int_0^\infty \frac{d\zeta}{\Delta\,(a_i^2 + \zeta)} \qquad \text{with}$$

$$\Delta^2 = (a_1^2 + \zeta)(a_2^2 + \zeta)(a_3^2 + \zeta)^{1/2} \,.$$

The internal energy of the gas is calculated by mean of hadronic equation of state , taking into account the pressure of the trapped neutrino gas.

Equations of motion and neutrino escape

From the effective Lagrangean we obtain the dynamical equation for ellipsoid axis evolution ,

$$\begin{aligned}
\ddot{a}_1 &= -\frac{3}{2}\frac{GM}{a_2 a_3} A_1 + \frac{25 J^2}{M^2}\frac{a_1}{(a_1^2 + a_2^2)^2} + \frac{20\pi}{3M} P a_2 a_3, \\
\ddot{a}_2 &= -\frac{3}{2}\frac{GM}{a_1 a_3} A_2 + \frac{25 J^2}{M^2}\frac{a_2}{(a_1^2 + a_2^2)^2} + \frac{20\pi}{3M} P a_1 a_3, \\
\ddot{a}_3 &= -\frac{3}{2}\frac{GM}{a_1 a_2} A_3 + \frac{20\pi}{3M} P a_1 a_2,
\end{aligned}$$

where the neutrino escape is coupled to the dynamical evolution of the system by the angular momentum variation and changes in the pressure. The number of trapped neutrinos is parameterized an exponential form ,

$$N_\nu(t) = N_\nu(0)\exp(-t/\tau) \quad ,$$

where τ represents the time scale for the neutrino escape throughout the surface of the star. The initial number of trapped neutrinos is determined by the equation of state and weak interaction equilibrium between hadronic and leptonic sectors.

When the angular velocity Ω is known the time rate of angular momentum is determined as a function of the axis values and rates by,

We solve numerically the equations of motion for the three semi-axes of the core star, for a given initial condition. The generation of gravitational waves is treated within the weak filed limit approximation, providing two additional equations: one describing the rate of the radiated energy and other one describing the rate of the angular momentum loss.

$$-\frac{dE}{dt} = \frac{32}{125}\frac{GM^2\Omega^6}{c^5}\left(a_1^2 - a_2^2\right)^2 \quad \text{(radiated power of GW)}$$

$$\frac{dJ}{dt} = -\frac{32GM^2}{125c^5}\Omega^5(a_1^2 - a_2^2)^2 + \frac{M}{5}\,\Omega(a_1\dot{a_1} + a_2\dot{a_2})$$

(rate of angular momentum loss)

To solve numerically the equation of motion we used a pseudo viscosity numerical artifact to avoid high frequency surface oscillations. This procedure consists in the addition of a velocity dependent term defined as $\gamma = -\frac{\dot{a}_i}{\alpha\tau_c}$, where the α-parameter scales the pseudo viscosity estimated in with the free collapse time scale, τ_c .

Results

Here we assume the initial equilibrium configuration of protoneutron star is an homogeneous tri-axial ellipsoid, composed of purely hadronic matter with trapped neutrinos. The mass, the eccentricity of the meridional sections and the semi-axes are determined by the total energy total minimization for a given mass *M* and total angular momentum *J*.

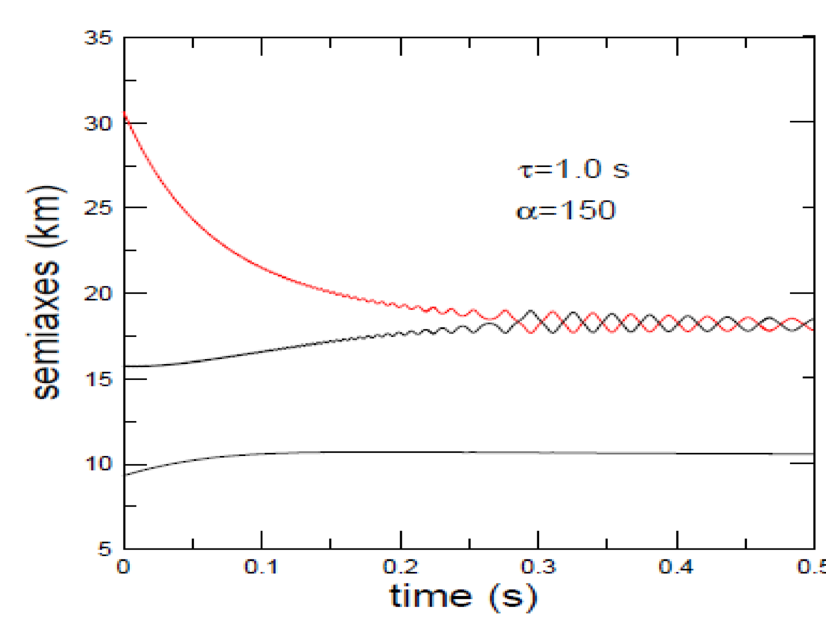

Fig. 1: Time evolution of the three semi-axes of the ellipsoidal. Due to the neutrino escape the protoneutro star ends its evolution as a oblate spheroid.

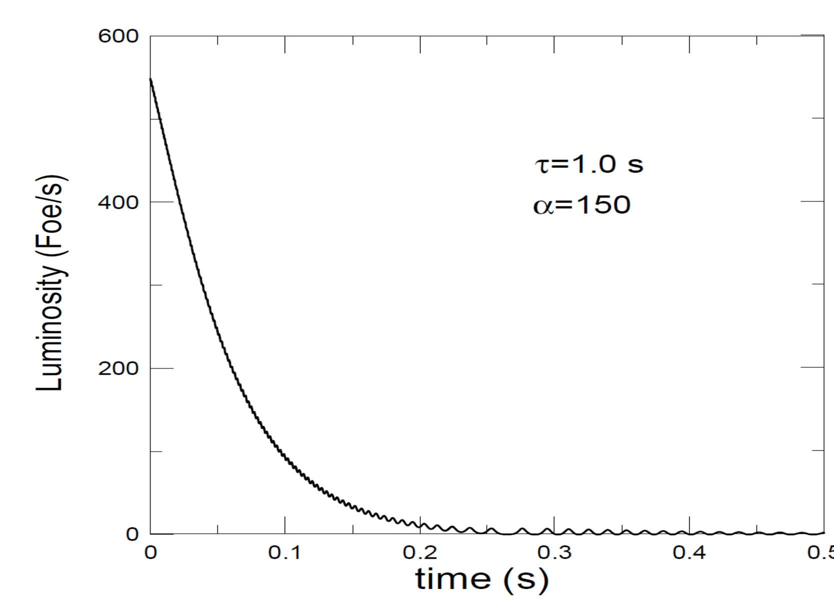

Fig.2: Gravitational wave luminosity as a function of time during the contraction of the system..

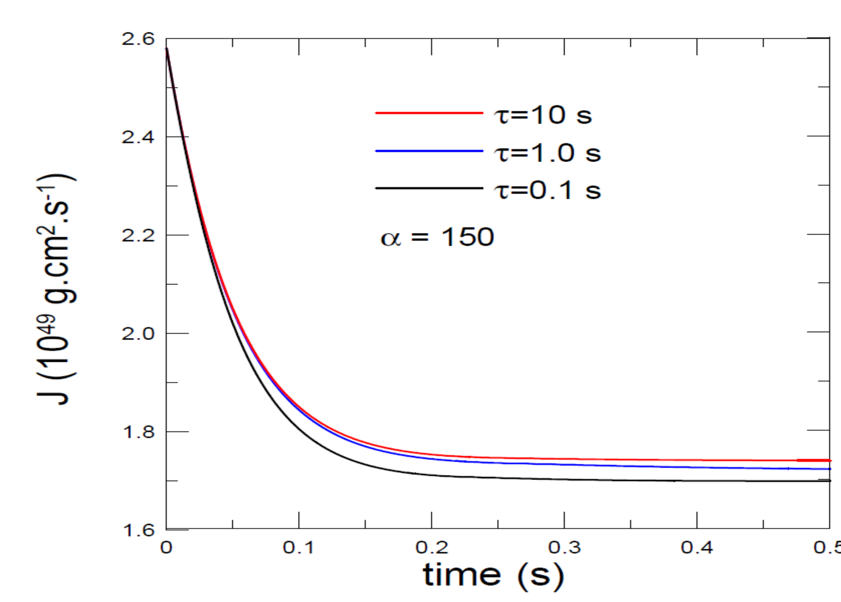

Fig. 3: Angular momentum evolution for different neutrino escape scales.

Conclusions

We presented a simplified model for generation of gravitational waves in protoneutron star evolution, starting from a tri-axial ellipsoidal shape and where neutrinos are initially trapped neutrinos. This object assumes the final shape of a fast rotating oblate spheroid due to the system contraction started by the neutrino escape. The final state of the collapsing cores are uniformly rotating system with periods, radii, densities and chemical compositions typical of neutron stars. We determine the intensity of the radiated gravitational waves for different time scales of the neutrino escape

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