Vector-like Contributions in the NJL model for Magnetized Quark Matter

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Introduction

In the last few years, theoretical comprehension of the processes governing the nature of the hadronic matter phase transitions were improved by the observation that giant magnetic background fields would be present at the environments where these phenomena occur.

The effective field theory approach for the behavior of an infinite volume of fermionic charged matter which is subject to intense magnetic fields and extreme thermodynamical conditions can provide us a better understanding about the systems where these features become important. From condensed matter physics one can find the best examples of the relevance in considering such effects in low energy fermionic systems. The alignment and orbiting of the fermionic particles in an intense magnetic field could induce and alter properties like condensation and superconductivity of quantum states as well modifying bulk properties, such as the particle number density and viscosity.

On the other side, the consideration of a vector interaction channel in the framework of the theory becomes important in order to bring an saturation mechanism, since its repulsive character reflects the sustaining pressure that avoids matter from collapsing to higher densities as one can observe by the application of the Walecka model to the nucleus hard core potential. Study the counterbalance between magnetic background field and a vector-like contribution can imply new features that are not observed by the assumption of these effects separately.

At finite temperatures and nonzero chemical potential we observe the Inverse Magnetic Catalysis [1] which decreases the coexistence chemical potential and expands the first order region in the QCD phase diagram. The consideration of a repulsive vector interaction induces an opposite effect by shrinking the first order transition line and setting the coexistence chemical potential to higher values. In this way, the chiral transition can take place at density values which are more closely related to the case where the magnetic field is absent. In the following, the main aspects of the inclusion of a vector coupling channel to the standard Nambu–Jona-Lasinio model are discussed when quark matter is subject to external magnetic fields.

Effective Langrangian in SU(2)

We start with the simple SU(2) NJL lagrangian containing just the light quarks up and down, adding the corresponding term of the vector coupling Gv. The kinematical effect of a constant background magnetic field B will be given by the shift in the momentum four-vector of charged q_f (q_u = +2/3e e q_d = −1/3e) particles

$$\mathscr{L}_f = \bar{\psi}_f(i\gamma_\mu\partial^\mu + q_f\gamma_\mu A^\mu - m)\psi_f + G_S[(\bar{\psi}_f\psi_f)^2 + (\bar{\psi}_f i\gamma_5\vec{\tau}\psi_f)^2] - G_V(\bar{\psi}_f\gamma_\mu\psi_f)^2$$

The additional consideration of a magnetic field B in the z direction will break the spherical spatial invariance of the fermion fields in the xy plane where particles and antiparticles of spin s will describe circular orbits of quantized momentum $p_x^2+p_y^2$ = (2n+1-s)qB in the n Landau levels, so the total momentum and energy can be written as:

$$\mathbf{p}^2 \rightarrow {p_z}^2 + (2n+1-s)|q_f|B \qquad E_{p,k}(B) = \sqrt{p_z^2 + 2k|q_f|B + M^2}$$

Where the degeneracy of the lowest Landau level (LLL) can be accounted if we replace the running letter n by k with the inclusion of a degeneracy factor $\alpha_k = 2 - \delta_{k0}$. The substitution of the four-momentum phase space integration by the specific Matsubara frequencies ν and the magnetically allowed quantized n Landau levels can be summarized with the substitution:

$$\int \frac{d^4p}{(2\pi)^4} \rightarrow i\frac{T|q_f|B}{2\pi}\sum_{\nu=-\infty}^{\infty}\sum_{n=0}^{\infty}\int\frac{dp_z}{(2\pi)}$$

Applying the last results in the effective potential, one finds:

$$\Omega = \frac{(M-m)^2}{4G_s} - \frac{(\mu-\tilde{\mu})^2}{4G_V} - \frac{N_c}{2\pi}\sum_{f=u}^{d}\sum_{k=0}^{\infty}\alpha_k(|q_f|B)\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}E_{p,k} + \\ -\frac{N_c}{2\pi}\sum_{f=u}^{d}\sum_{k=0}^{\infty}\alpha_k(|q_f|B)\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\left\{T\ln\left[1+e^{-(E_{p,k}+\tilde{\mu})/T}\right] + T\ln\left[1+e^{-(E_{p,k}-\tilde{\mu})/T}\right]\right\}$$

This expression is more easily to treat if calling every contribution in the following terms:

$$\Omega_f = \frac{(M-m)^2}{4G_s} - \frac{(\mu-\tilde{\mu})^2}{4G_V} + \Omega_f^{vac} + \Omega_f^{mag} + \Omega_f^{med}$$

Where the vacuum contribution can be calculated as:

$$\Omega_f^{vac} = \frac{N_cN_f}{8\pi^2}M^4\left\{\ln\left[\frac{(\Lambda+\epsilon_\Lambda)}{M}\right] - \epsilon_\Lambda\Lambda[\Lambda^2+{\epsilon_\Lambda}^2]\right\}$$

Here the symbol ε_Λ represents the energy $(\Lambda^2+M)^{1/2}$ at the cutoff momentum value Λ. The magnetic part of the effective potential can be calculated defining a new variable x_f and and using the properties of the Riemann-Hurwitz zeta function

$$x_f = \frac{M^2}{(2|q_f|B)} \qquad \zeta'(-1,x_f) = \left.\frac{d\zeta(z,x_f)}{dz}\right|_{z=-1}$$

with these assumptions, we may found the magnetic contribution:

$$\Omega_f^{mag} = -\sum_{f=u}^{d}\frac{N_c(|q_f|B)^2}{2\pi^2}\left\{\zeta'[-1,x_f] - \frac{1}{2}(x_f^2-x_f)\ln x_f + \frac{x_f^2}{4}\right\}$$

The medium contribution at finite temperature and chemical potential also reads:

$$\Omega_f^{med} = -\frac{N_c}{2\pi}\sum_{f=u}^{d}\sum_{k=0}^{\infty}\alpha_k(|q_f|B)\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\left\{T\ln\left[1+e^{-(E_{p,k}+\tilde{\mu})/T}\right] + T\ln\left[1+e^{-(E_{p,k}-\tilde{\mu})/T}\right]\right\}$$

Applying the corresponding minimization procedure to the constituent mass and the effective chemical potential:

$$\frac{\delta\Omega}{\delta M} = \frac{(M-m)}{2G_s} - \frac{N_c}{2\pi}\sum_{f=u}^{d}\sum_{k=0}^{\infty}\alpha_k(|q_f|B)\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\frac{M}{E_{p,B}}\Big(1 - n_{p,B}(\tilde{\mu},T) - \bar{n}_{p,B}(\tilde{\mu},T)\Big) = 0$$

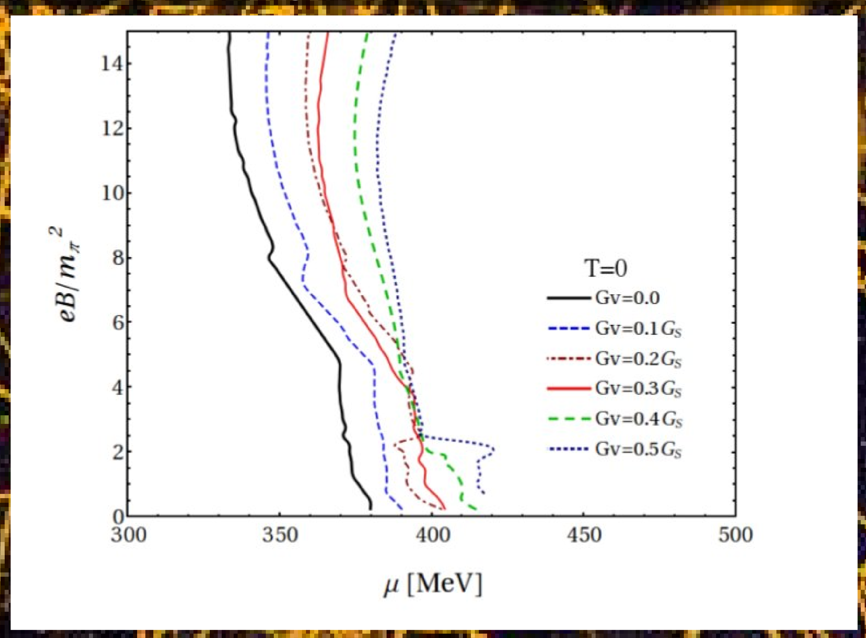
$$\frac{\delta\Omega}{\delta\tilde{\mu}} = \frac{(\mu-\tilde{\mu})}{2G_V} - \frac{N_c}{2\pi}\sum_{f=u}^{d}\sum_{k=0}^{\infty}\alpha_k(|q_f|B)\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\Big(n_{p,B}(\tilde{\mu},T) - \bar{n}_{p,B}(\tilde{\mu},T)\Big) = 0$$

Solving consistently this coupled set of equations will supply us with the stationary values of constituent mass and shifted chemical potential at a given state (μ,T) and parameter space of Gv and B values. These solutions can yield all the thermodynamical quantities of interest.

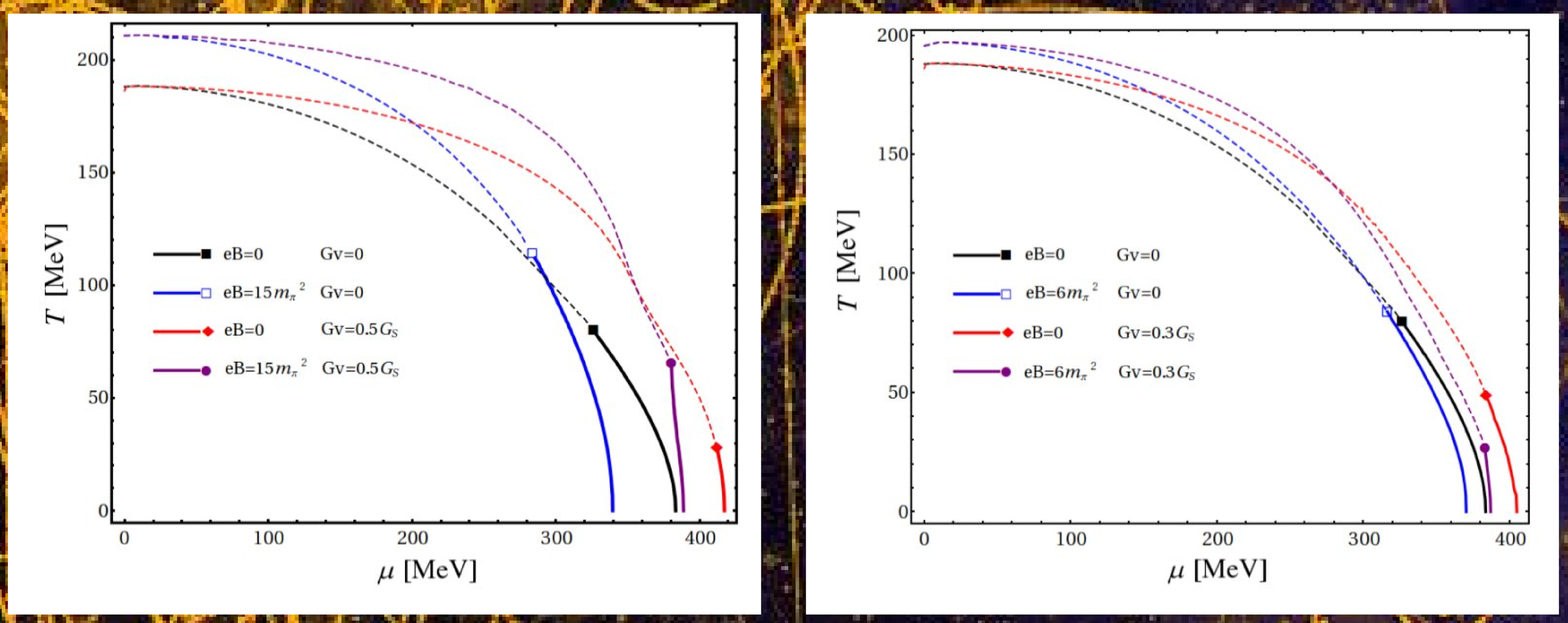
Phase Diagram Dependence Analysis

The repulsive character of the vector contribution will play a central role because its balancing with the attractive scalar coupling. The magnetic field interplay with the dense medium energy cost actually decreases the coexistence chemical potential value. This effect was named Inverse Magnetic Catalysis [1][2], because the existence of the background field reduces the μ coexistence values in the chiral symmetry phase restoration.

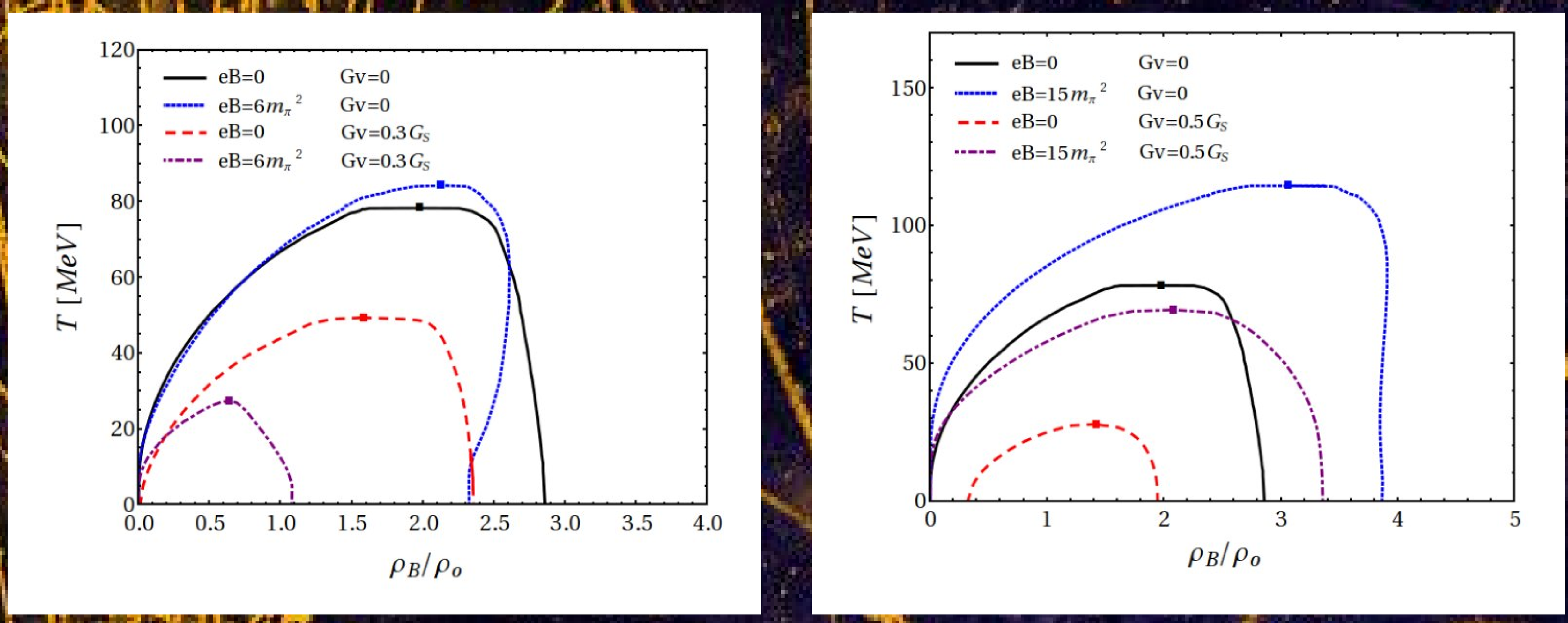
In the next figure are shown the curves corresponding the dependence of the coexistence chemical potential at zero temperature with the magnitude of the external magnetic field and vector interaction coupling.



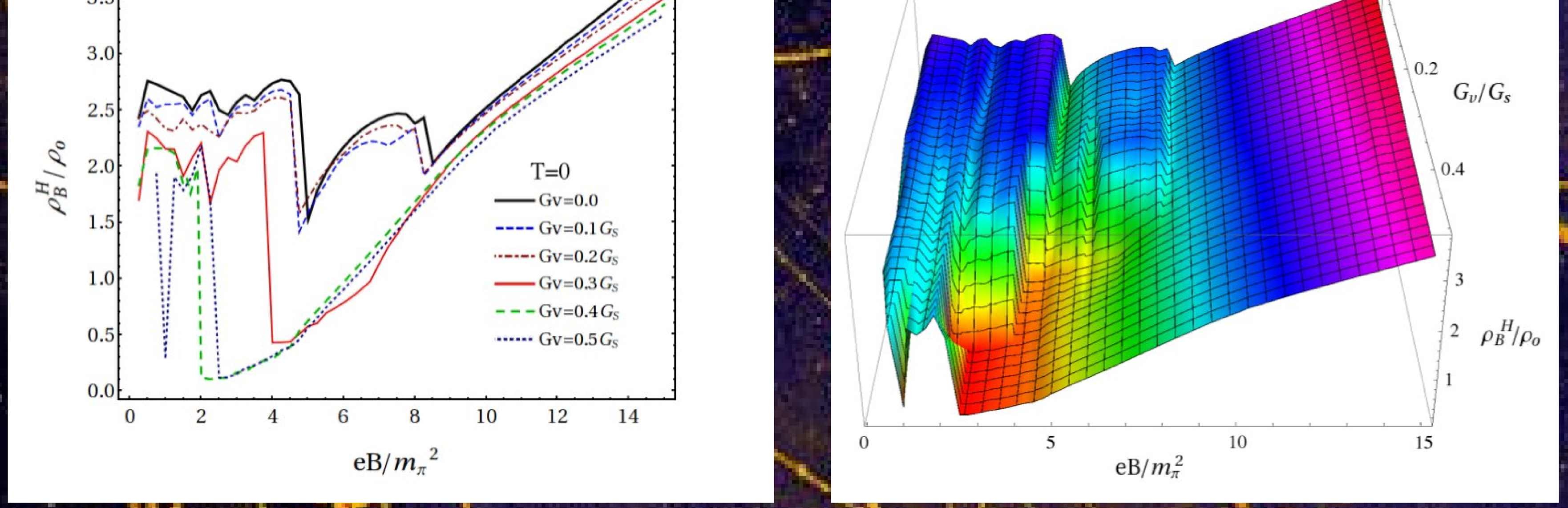
Setting just the vector coupling effect, assuming a starting value of Gv=0.5Gs, one observes the shrinkage of the first order region and the rising of the magnitude of chemical potential values. Inserting together both contributions (B=15 $m\pi^2$/e, Gv=0.5Gs), we reproduce an intermediary situation, almost comparable with the case without any effects (B=0, Gv=0) at small values of temperature. Beyond T=0, though, the curvature of these phase boundaries are clearly different, thus emphasizing that the unperturbed behavior cannot be restored within these two influences. We find that this result can be important only at T=0, when the presence of a background magnetic field could reduce or even cancel the vector coupling interaction. Analyzing the phase diagram features at a weaker magnetic field B=6 $m\pi^2$/e, one encounters almost the same pattern as before, where the first difference to note is the displacement between curves.



The first order phase coexistence is best regarded when we look to the ρ-T plane. In the first order transition, the two coexisting densities can be calculated determining the constituent mass of the equal pressure effective potential minima. At a given temperature value is possible to coexist two different phases, with the consequent bubbles and droplets formation. The application of an external magnetic field of B=15 $m\pi^2$/e enlarges this coexistence region when all the states are in the LLL. Otherwise, in a weak field strength B=6 $m\pi^2$/e, this region is reduced. The vector coupling channel alone has the same shrinking effect in both coupling values.



Finally, are shown the corresponding magnetic oscillations in the quark number density at zero temperature. The result at Gv=0 agree with other evaluations [1] employing holographic methods like the Sakai-Sugimoto model [3]. Furthermore, we found that as the vector coupling Gv is increased, these oscillations are suppressed until disappear. Such oscillatory behavior ensures a clear manifestation of the De Haas-van Alphen effect to the fermionic matter subject to an external magnetic field. In fact, one can assign its quantum character because the evident disagreement with the Bohr-van Leeuwen Theorem; this theorem states that classically, any property of a given system in thermal equilibrium must depend on the magnetic field strength.



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