Entropy, Disequilibrium and Complexity in Compact Stars: A different approach to their Composition

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Context

I am going to present you part of the research done during my Ph.D. project on neutron stars. Our goal was:

- to address the composition of neutron stars: hierarchy of EoSs through informational theoretic techniques.

Potentially: quantify the effects of interactions on the informational content of these objects.

Mass radius diagram reflects the composition

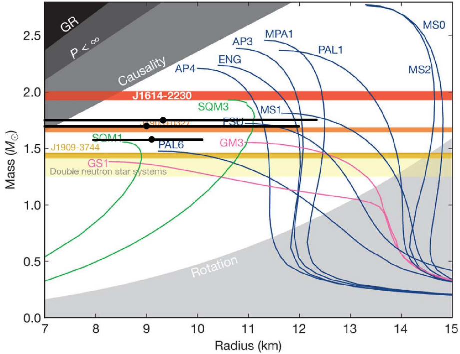

Figure: Different compositions and some constraints

How much information does the different EoSs possess? Which one, then, would be more likely to be realized in Nature?

Complexity and Information

First, concepts and definitions:

- Complexity: what does not match the requirements of being simple (tautology?). In physics, we always begin with ideal systems as the simplest systems possible;
- Information: what we can get from observing the occurrence of an event (how surprising, or unexpected or what else).

With a certain reductionism:

- definition of information in terms of the probability of an event to occur.

Information

From some desired mathematical properties of information we can derive:

$$I(p) = -log_b(p) \tag{1}$$

for some probability p and basis b (that gives the unit). $b = 2$ give us *bits*.

- flipping a fair coin once give you $-log_2(1/2) = 1$ bit of information.

Information

If a source provides n symbols $\{a_i\}$ with probability $\{p_i\}$, then the average amount of information in the stream of symbols is:

$$\frac{I}{N} = -\sum_{i=0}^{N} p_i log_b(p_i) \equiv H(P). \quad (2)$$

This quantity is defined as the *entropy* of the probability distribution $P = \{p_i\}$.

Information: property

The maximum of this quantity is achieved at equiprobability $p_i = 1/n$.
Example: a student and his grades:

- if the grades are A, B, C, D and F, with equal probabilities $\Rightarrow$ 2.32 bits of information;
- if instead the probabilities are $\{1/10, 1/5, 2/5, 1/5, 1/10\}$ $\Rightarrow$ 2.12 bits of information;
- if $\{0, 0, 0, 0, 1\}$ $\Rightarrow$ 0 bit of information.

Complexity in physical systems: ideal cases

Let us allow complexity to encode order and disorder (or the self-organization of a system): two ideal systems, extremes in all aspects and opposites as well:

- Perfect crystal: zero complexity by definition; strict symmetry rules $\Rightarrow$ probability density centered around the prevailing state of perfect symmetry $\Rightarrow$ minimal information. Completely ordered.
- Ideal gas: zero complexity by definition; accessible states are equiprobable $\Rightarrow$ maximal information. Totally disordered.

Key concept: Disequilibrium

The information alone is not enough to define complexity. We define then the *disequilibrium* as the distance to the equiprobability. Now we define complexity as:

$$C \equiv H \times D \tag{3}$$

Getting some intuition first

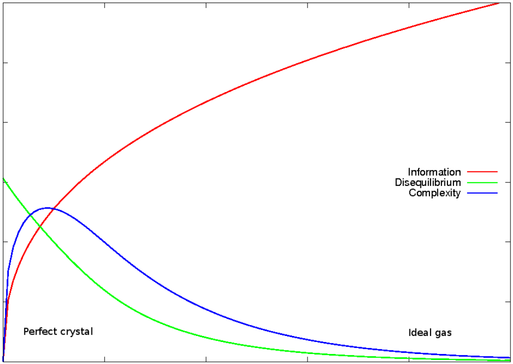

Figure: Intuitive definition of complexity

Disequilibrium

As an expression to disequilibrium the proposal is

$$D = \sum_{i=1}^{N} \left(p_i - \frac{1}{N} \right)^2 \tag{4}$$

Continuous case

In the continuous case with large N we get:

$$H = -\int p(x) log_b[p(x)]dx \tag{5}$$

$$D = \int p^2(x)dx \tag{6}$$

$$C \equiv H \times D \quad or \quad C \equiv e^H \times D. \tag{7}$$

We shall adopt the last version of complexity for convenience.

Compact stars

Application to neutron stars with two different compositions:

- Hadronic composition with SLy4 equation of state;
- Quark composition with three flavours in equal amounts or strange quark matter.

How does the composition affect the measures of these quantities?
But first, what should we adopt as $p(x)$?

Compact stars

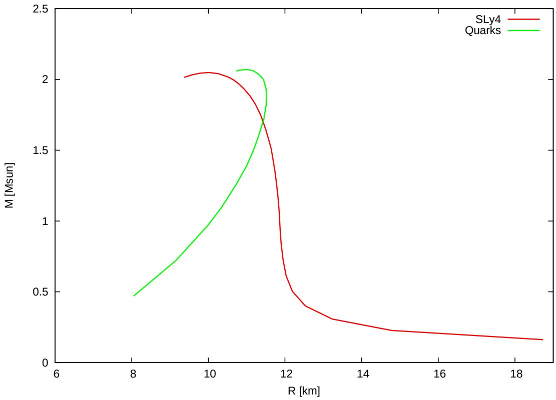

Figure: Mass-radius relation for two different EoSs

Compact stars

In order to do that we slightly modified the equations:

$$S = -b_0 \int \bar{\epsilon}(\mathbf{r}) ln[\bar{\epsilon}(\mathbf{r})] d\mathbf{r} \tag{8}$$

$$D = b_0 \int \bar{\epsilon}^2(\mathbf{r}) d\mathbf{r} \tag{9}$$

where $\bar{\epsilon}(\mathbf{r})$ is the dimensionless energy density (which is just $c^2\rho(\mathbf{r})/\epsilon_0$). The parameter b_0 makes S and D dimensionless.

Results

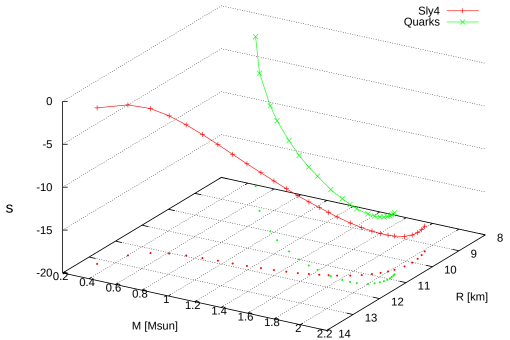

Figure: 3-D version: composite behaviour S *vs* M *vs* R

Results

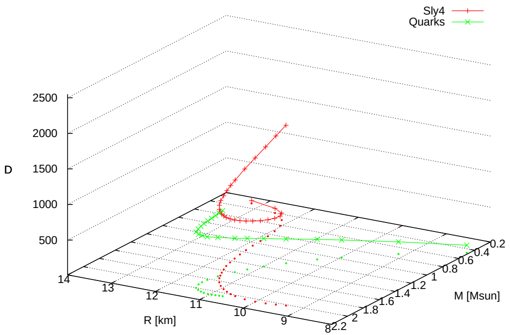

Figure: 3-D version: composite behaviour D *vs* M *vs* R

Results

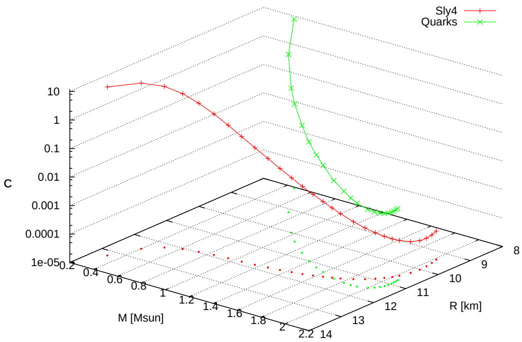

Figure: 3-D version: composite behaviour C *vs* M *vs* R

Summary

Summary of the results:

- Hadronic stars: low complexity, ordered systems, tend to the perfect crystal;
- Quark stars: low complexity, less ordered systems, more distant from perfect crystal.

The white dwarf case: complexity *grows* with increasing mass, reaching a maximum finite value at the Chandrasekhar mass. Resemblance to atomic systems.

Conclusions

Conclusions:

- If order costs energy, then nature should favour exotic strange quark stars;
- There is a trend for these stars to be at a state of minimum complexity. Calbet and López-Ruiz have shown that for a system out of equilibrium there is, in fact, a tendency of the complexity to reach an extremum. If a transition hadronic $\rightarrow$ quark occur, that would be the case.

Future

Some perspectives regarding the development of these concepts:

- Perform calculations for other realistic equations of state; calculate the amount of information encoded by the EoS as a whole, not only for each star;
- Study the case of global charge neutrality (see Souza, Manreza, de Avellar and Horvath [poster]);
- Improve the very concept of $p(x)$ used here in order to make it compatible to the analogous in information theory;
- Link to the thermodynamics and the gravitational collapse;
- ...

References

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