Superconducting phases of strange quark matter in the NJL model

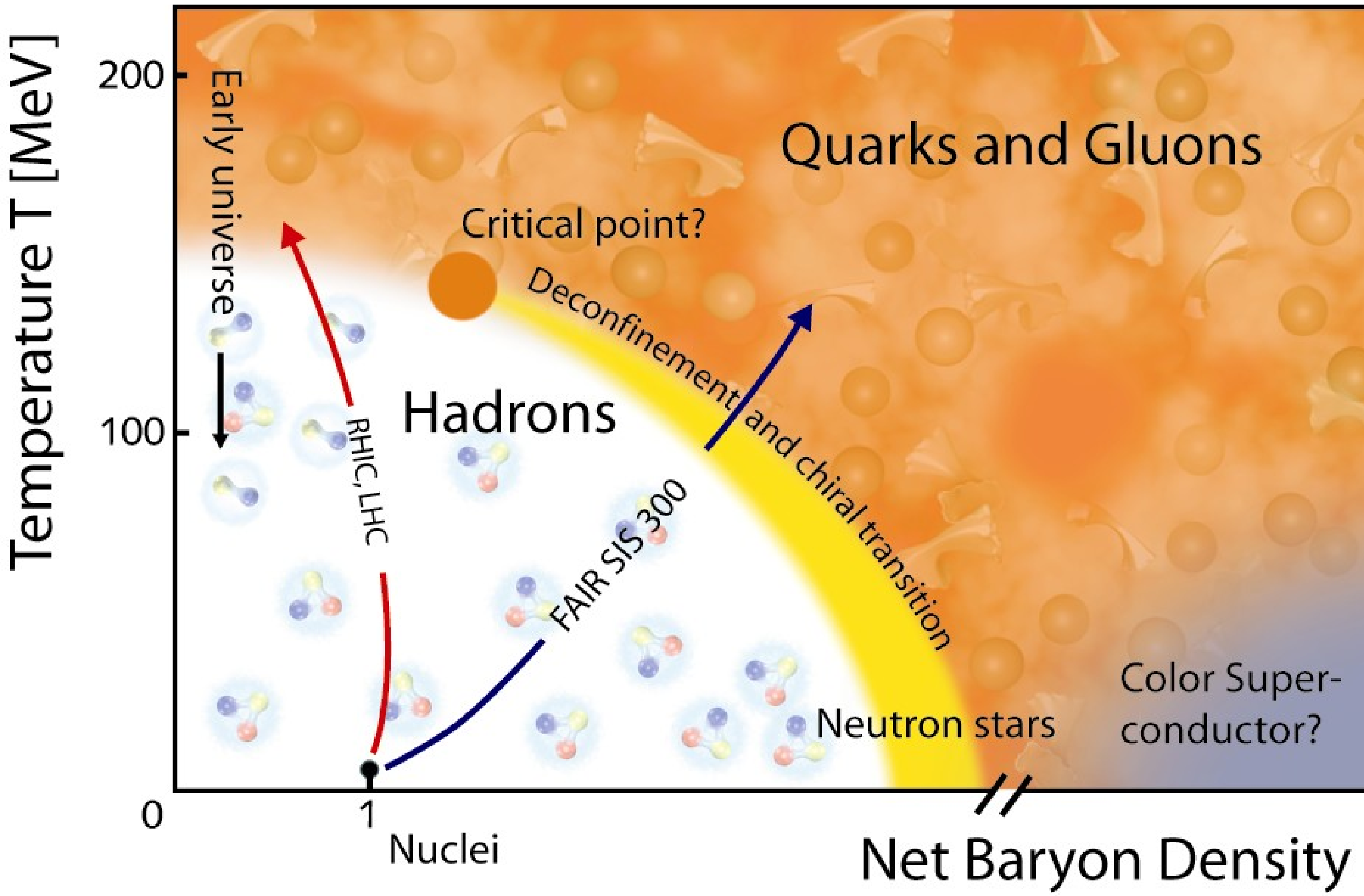
Laura Paulucci[1], Vivian de la Incera[2], Efrain J. Ferrer[2] and Jorge E. Horvath[3]

[1]Universidade Federal do ABC

[2]The University of Texas at El Paso

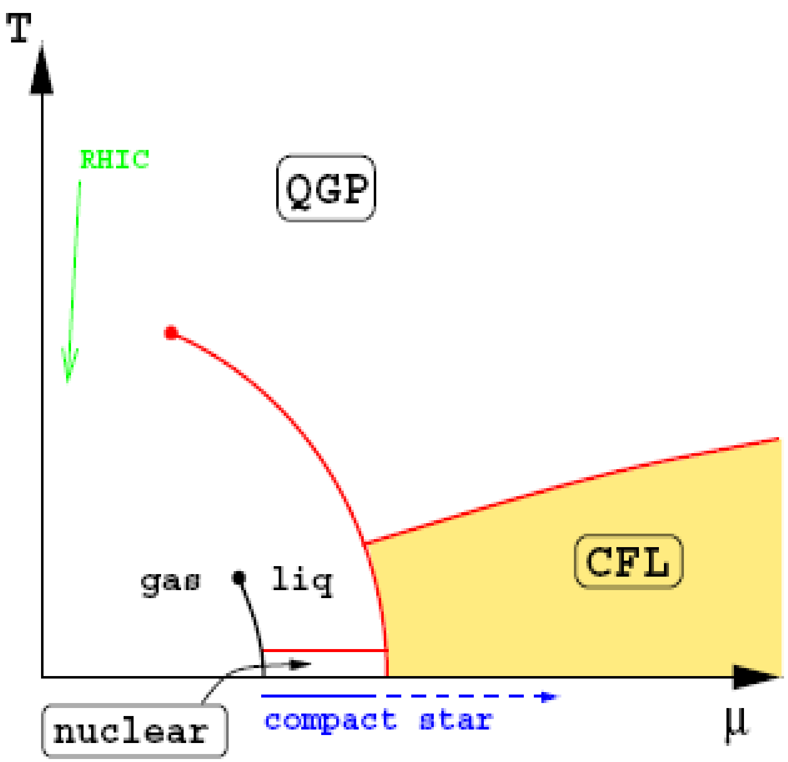
[3]IAG-USP

Phase Diagram

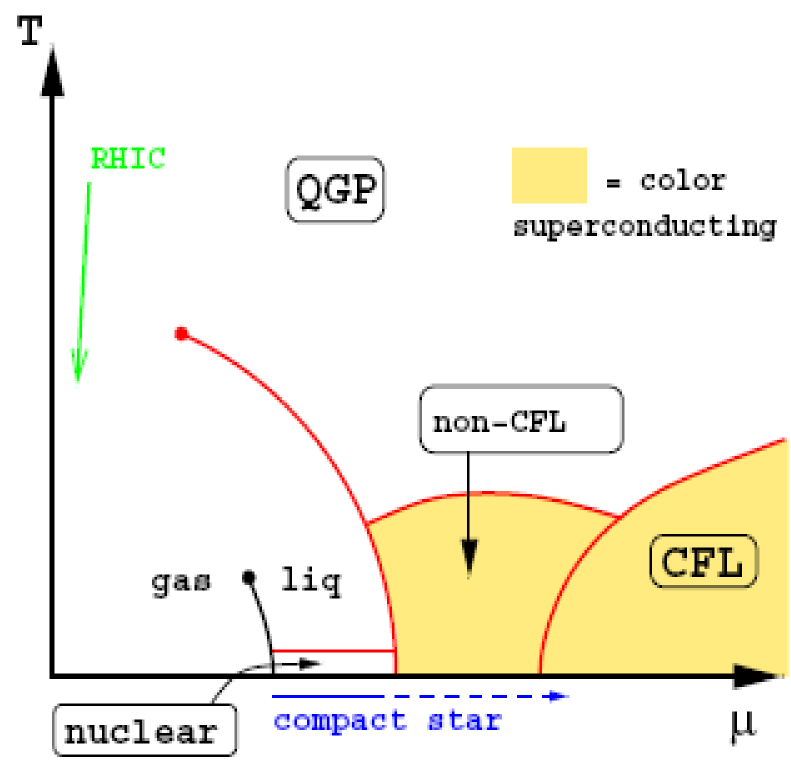


Phase Diagram

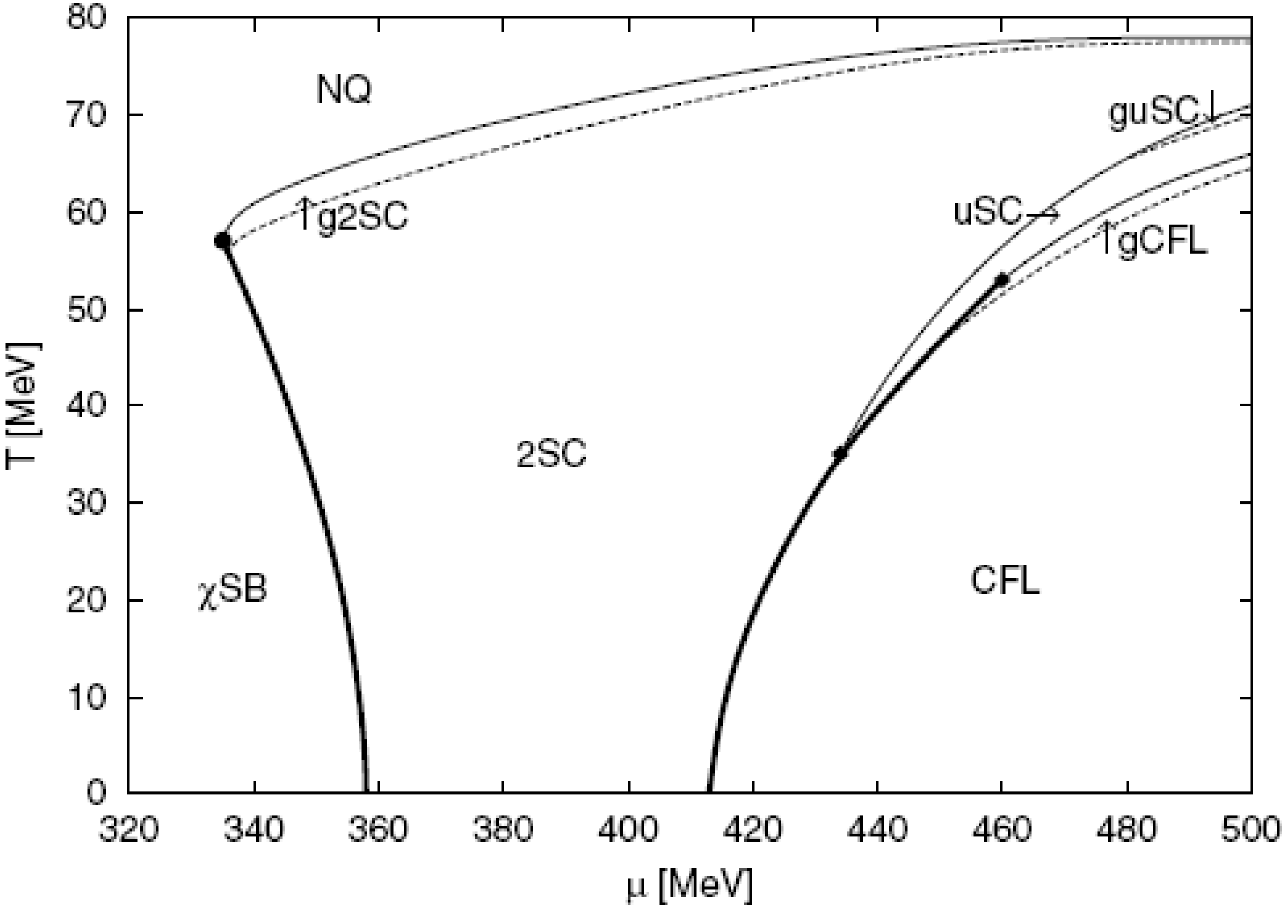
Light strange quark
or large pairing gap



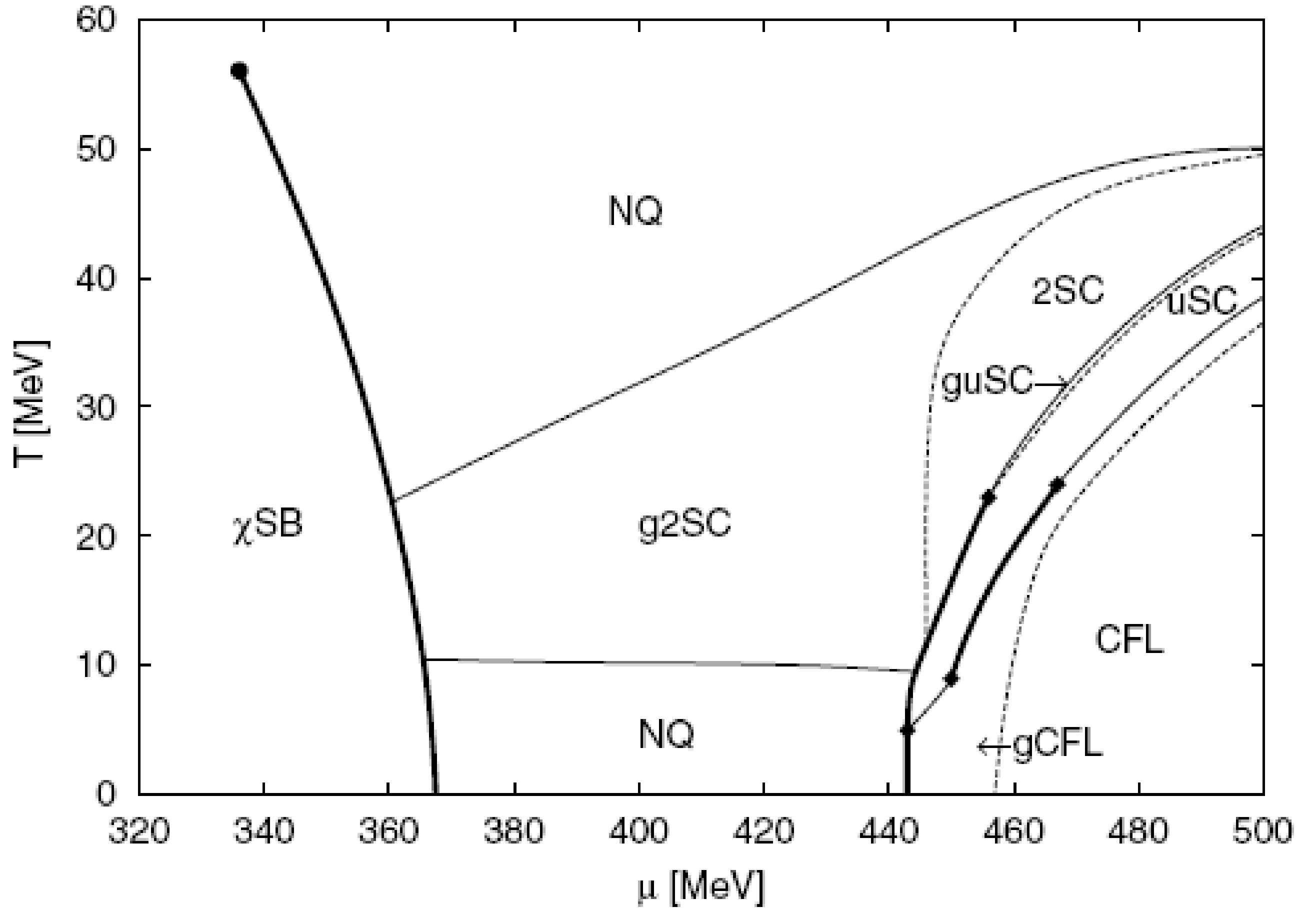
Heavy strange quark
or small pairing gap



Phase Diagram



Phase Diagram



Color-flavor-Locked Matter

- The model:
  - three-flavor Nambu-Jona-Lasinio (NJL) theory
  - we neglect all quark masses
    - color and electrical neutralities satisfied
    - only nonzero chemical potential will be the baryonic one μ
  - Locally uniform and constant magnetic field (MCFL phase)

Color-flavor-Locked Matter

- MCFL thermodynamic potential

$$\Omega_{MCFL} = \Omega_C + \Omega_N$$

$$\Omega_C = -\frac{\tilde{e}\widetilde{H}}{4\pi^2}\sum_{n=0}^{\infty}(1-\frac{\delta_{n0}}{2})\int_0^{\infty} dp_3 e^{-(p_3^2+2\tilde{e}\widetilde{H}n)/\Lambda^2}[8|\varepsilon^{(c)}| + 8|\overline{\varepsilon}^{(c)}||],$$

$$\Omega_N = -\frac{1}{4\pi^2}\int_0^{\infty} dpp^2 e^{-p^2/\Lambda^2}[6|\varepsilon^{(0)}|+6|\overline{\varepsilon}^{(0)}||]-\frac{1}{4\pi^2}\int_0^{\infty} dpp^2 e^{-p^2/\Lambda^2}\sum_{j=1}^{2}[2|\varepsilon_j^{(0)}|+2|\overline{\varepsilon}_j^{(0)}||]+\frac{\Delta^2}{G}+\frac{2\Delta_H^2}{G}$$

Color-flavor-Locked Matter

- And the dispersion relations

$$\varepsilon^{(c)} = \pm\sqrt{(\sqrt{p_3^2 + 2\tilde{e}\widetilde{H}n} - \mu)^2 + \Delta_H^2},$$
$$\overline{\varepsilon}^{(c)} = \pm\sqrt{(\sqrt{p_3^2 + 2\tilde{e}\widetilde{H}n} + \mu)^2 + \Delta_H^2},$$

$$\varepsilon^{(0)} = \pm\sqrt{(p-\mu)^2 + \Delta^2}, \qquad \overline{\varepsilon}^{(0)} = \pm\sqrt{(p+\mu)^2 + \Delta^2},$$
$$\varepsilon_1^{(0)} = \pm\sqrt{(p-\mu)^2 + \Delta_a^2}, \qquad \overline{\varepsilon}_1^{(0)} = \pm\sqrt{(p+\mu)^2 + \Delta_a^2},$$
$$\varepsilon_2^{(0)} = \pm\sqrt{(p-\mu)^2 + \Delta_b^2}, \qquad \overline{\varepsilon}_2^{(0)} = \pm\sqrt{(p+\mu)^2 + \Delta_b^2},$$

$$\Delta_{a/b}^2 = \frac{1}{4}(\Delta \pm \sqrt{\Delta^2 + 8\Delta_H^2})^2$$

Color-flavor-Locked Matter

- Pairing gaps:

$$\Delta_1 = \Delta_2 = \Delta_3$$ ← CFL

$$\Delta_1 = \Delta,$$ ← (d, s) pairing gap (neutral quarks)

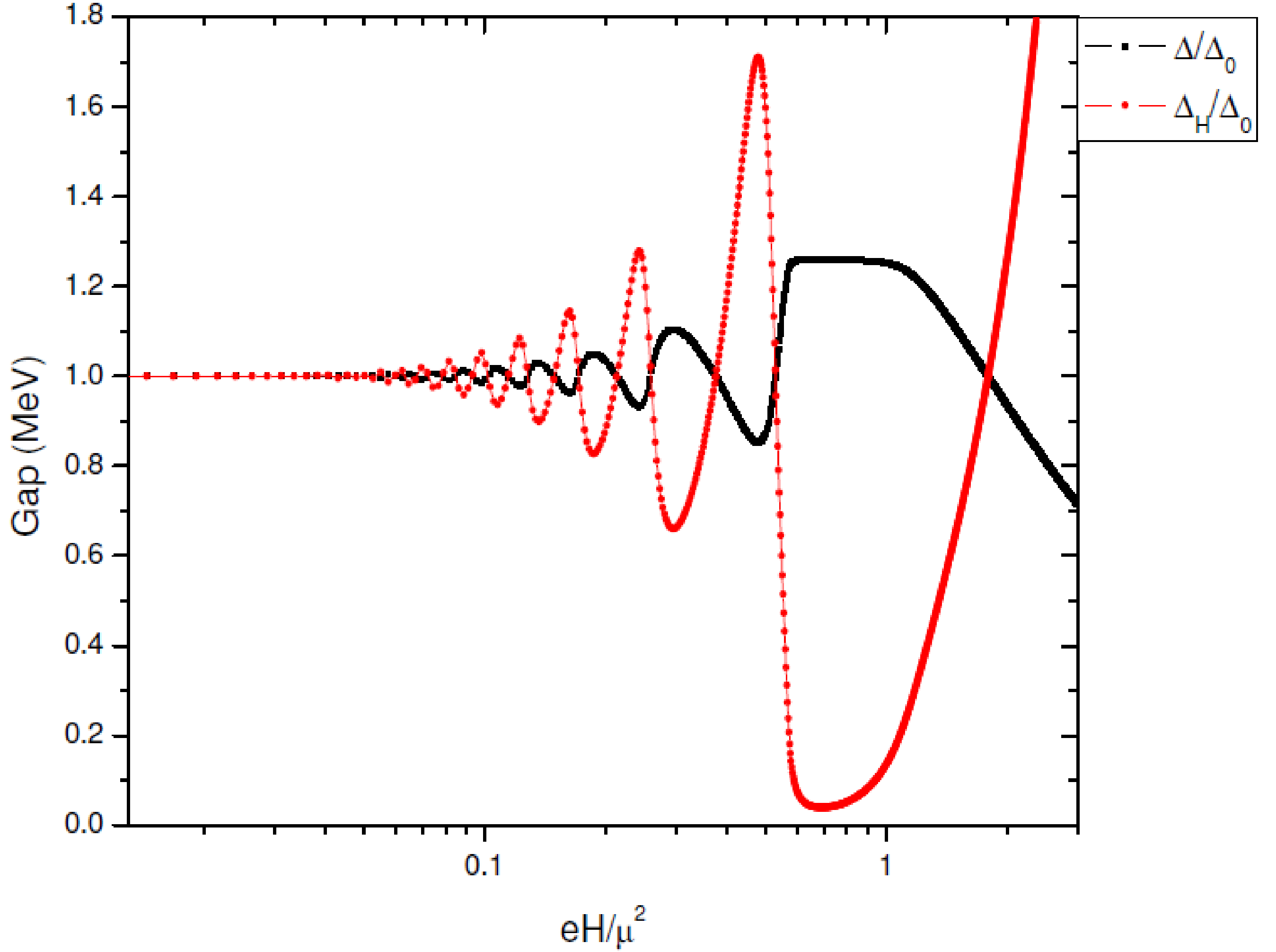
$$\Delta_2 = \Delta_3 = \Delta_H.$$ ← (u, s) and (u, d) pairing gaps (pairs of charged and neutral quarks)

$$\Omega_H = \Omega_{MCFL} + B + \frac{\widetilde{H}^2}{2},$$

- Gap equations:

$$\frac{\partial \Omega_{MCFL}}{\partial \Delta} = 0, \qquad \frac{\partial \Omega_{MCFL}}{\partial \Delta_H} = 0.$$

Color-flavor-Locked Matter



J. L. Noronha and I. A. Shovkovy, PRD **76**, 105030 (2007)

MCFL equation of state

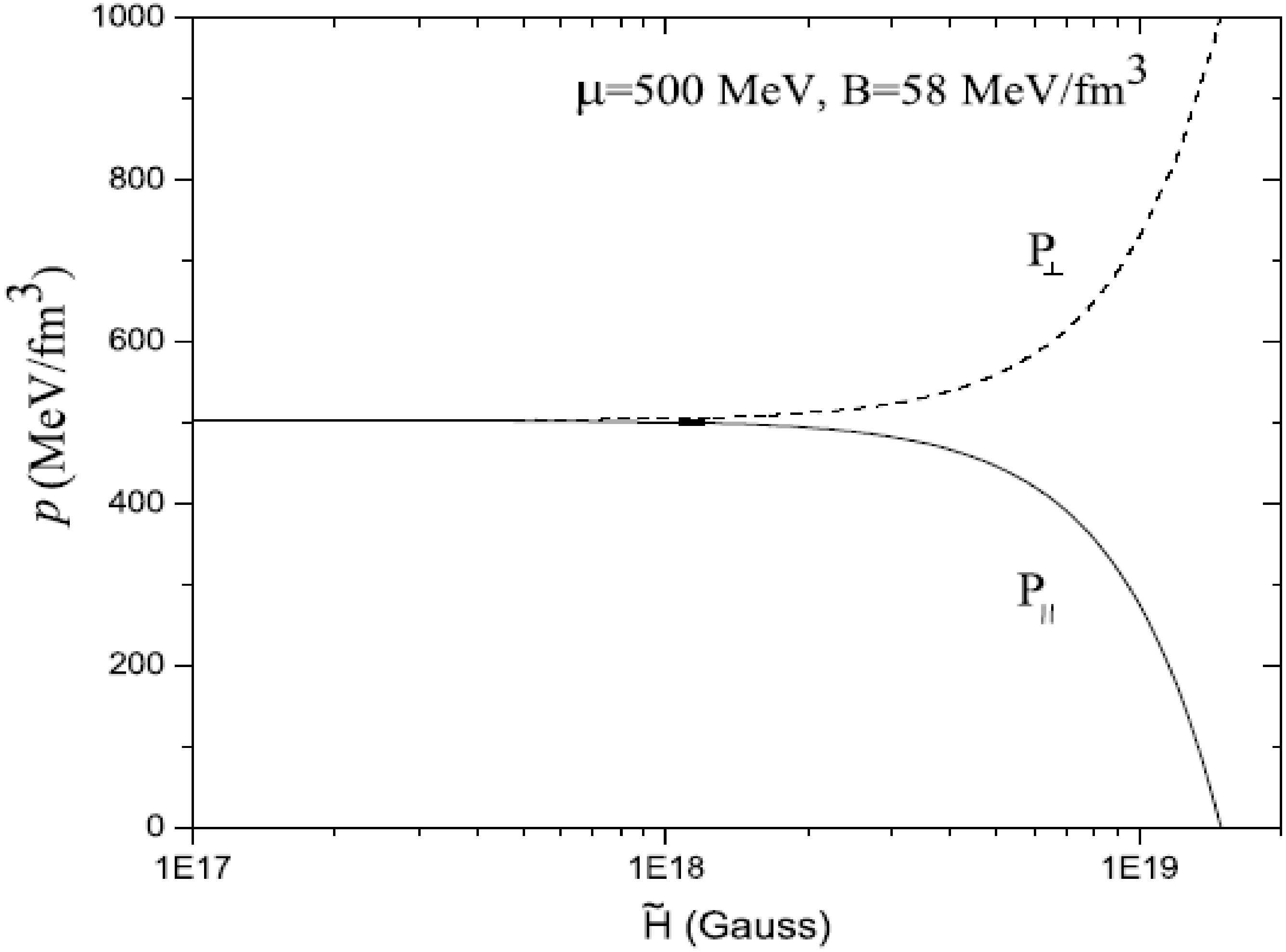
$$\epsilon_{MCFL} = \Omega_H - \mu \frac{\partial \Omega_H}{\partial \mu},$$

$$p^{\parallel}_{MCFL} = -\Omega_H,$$

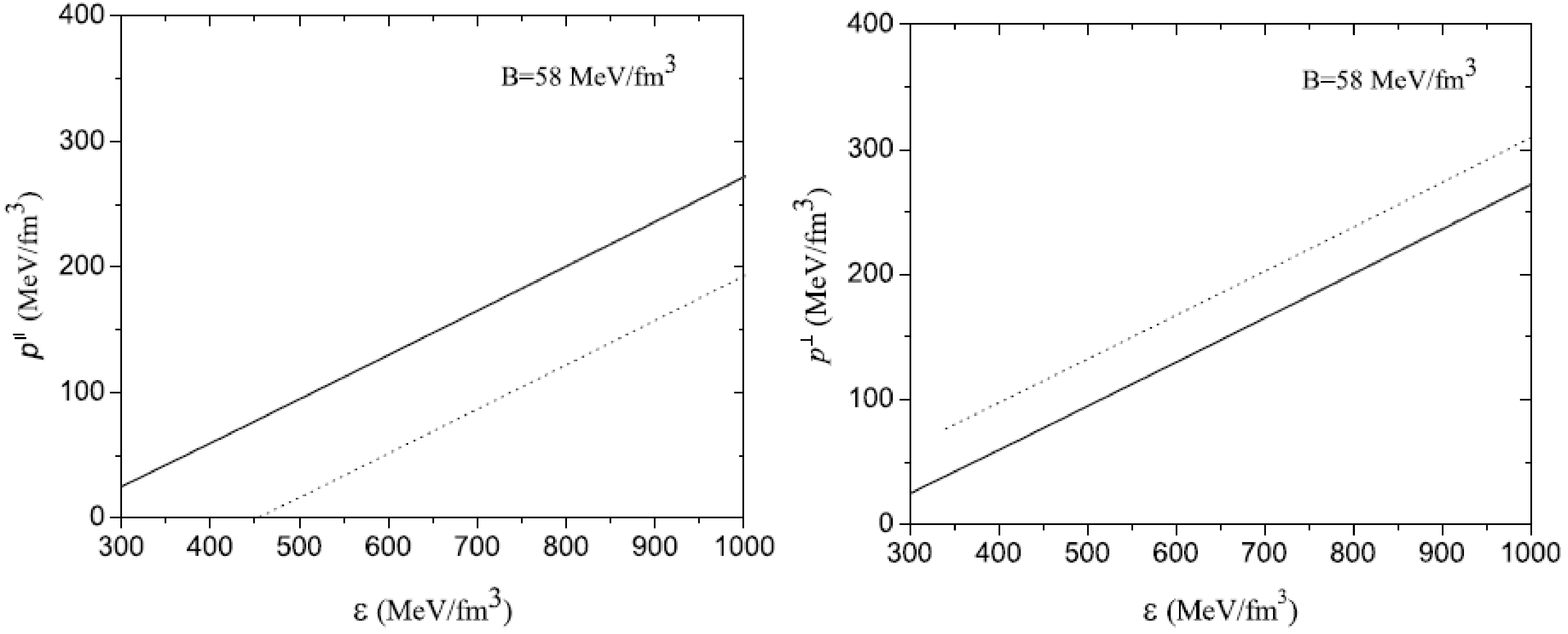
$$p^{\perp}_{MCFL} = -\Omega_H + \widetilde{H} \frac{\partial \Omega_H}{\partial \widetilde{H}}$$

Taken μ = 500 MeV, G = 4.32 GeV^{-2} (Δ_{CFL} = 10 MeV) and Λ = 1 GeV.

MCFL equation of state



MCFL equation of state



H: zero field (solid line), 10^{17} G (dashed line overlapped to the solid line), and 3 x 10^{18} G (dotted line).

MCFL stability conditions

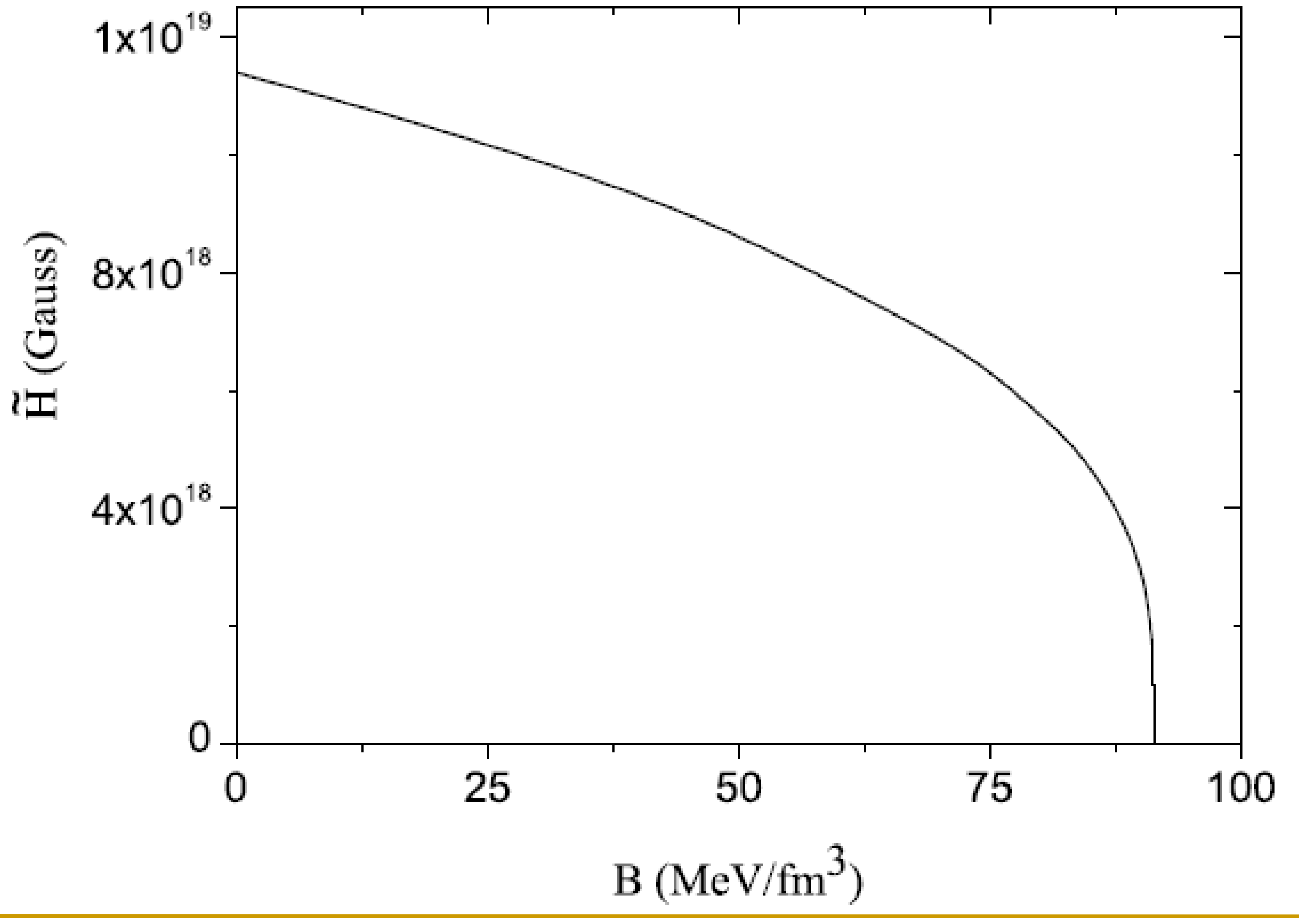
- Both the parallel and perpendicular pressures need to vanish simultaneously:

$$p^{\parallel}_{MCFL} = -\Omega_{MCFL} - B - \frac{\widetilde{H}^2}{2} = 0,$$

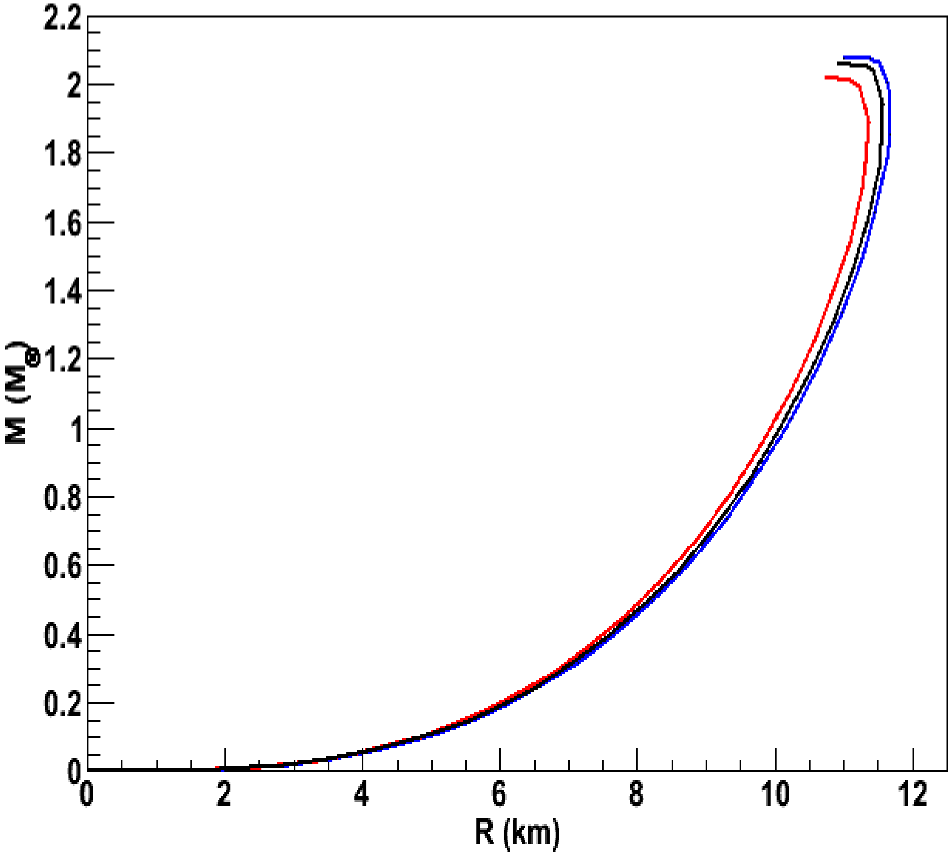
$$p^{\perp}_{MCFL} = \widetilde{H}\frac{\partial \Omega_{MCFL}}{\partial \widetilde{H}} + \widetilde{H}\frac{\partial B}{\partial \widetilde{H}} + \widetilde{H}^2 = 0$$

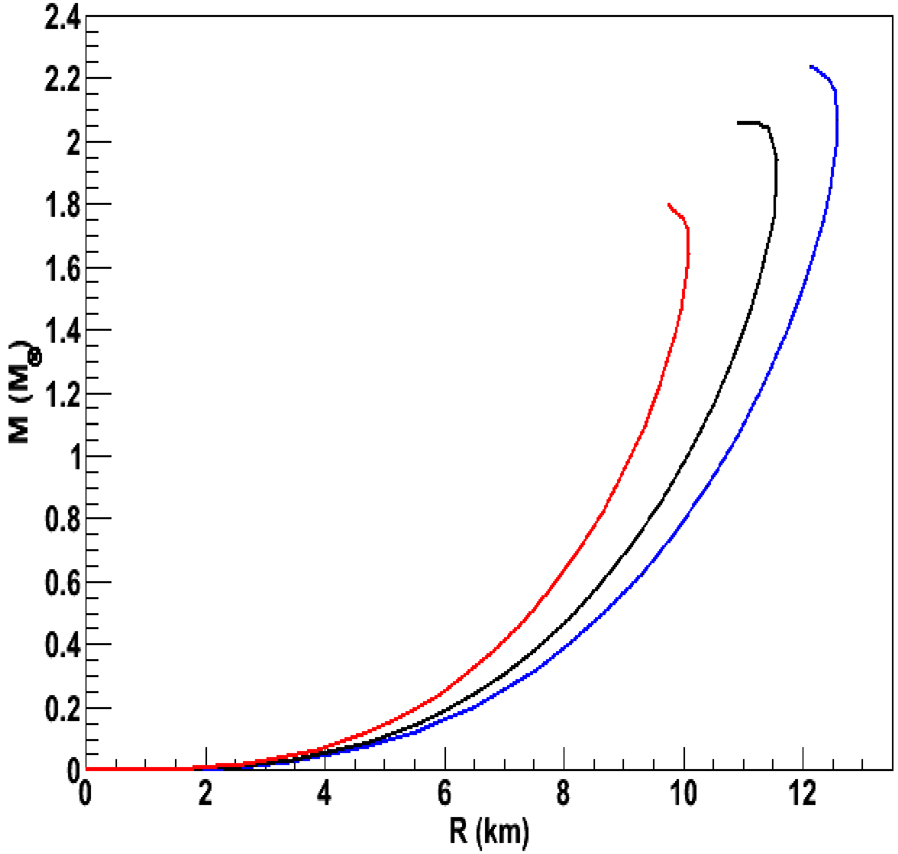
$$\widetilde{H} = M - \frac{\partial B}{\partial \widetilde{H}}$$

MCFL stability conditions



MCFL stars





Black line – P_{CFL}; red line – Ppar; blue line - Pperp

H = 1.7 x 10^{17} G

H=3 x 10^{18} G

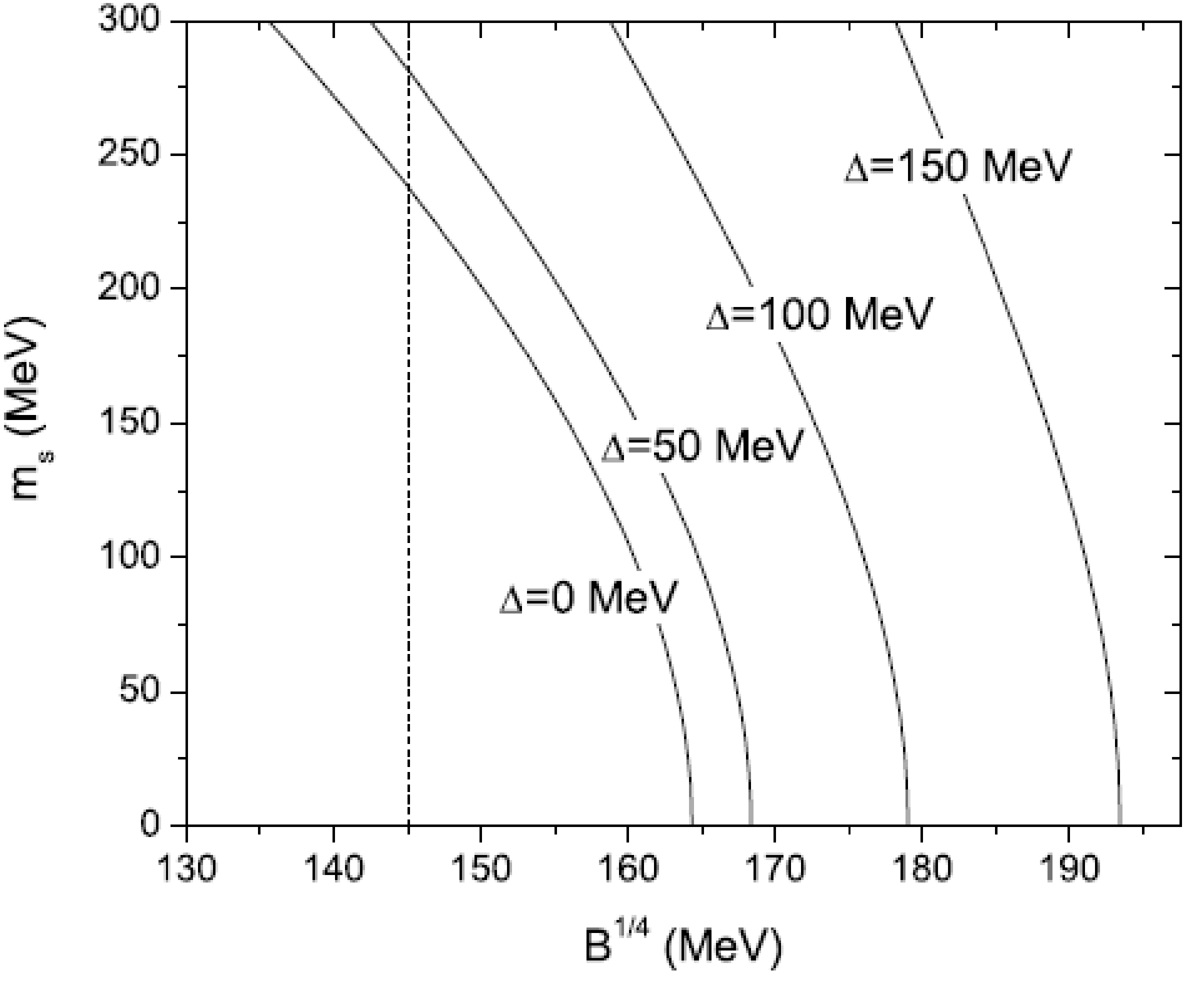
MIT bag model

$$\Omega_{CFL} = \sum_i \Omega_i - \frac{3}{\pi^2}\Delta^2\mu^2 + B$$

$$P = -\Omega_{CFL}$$

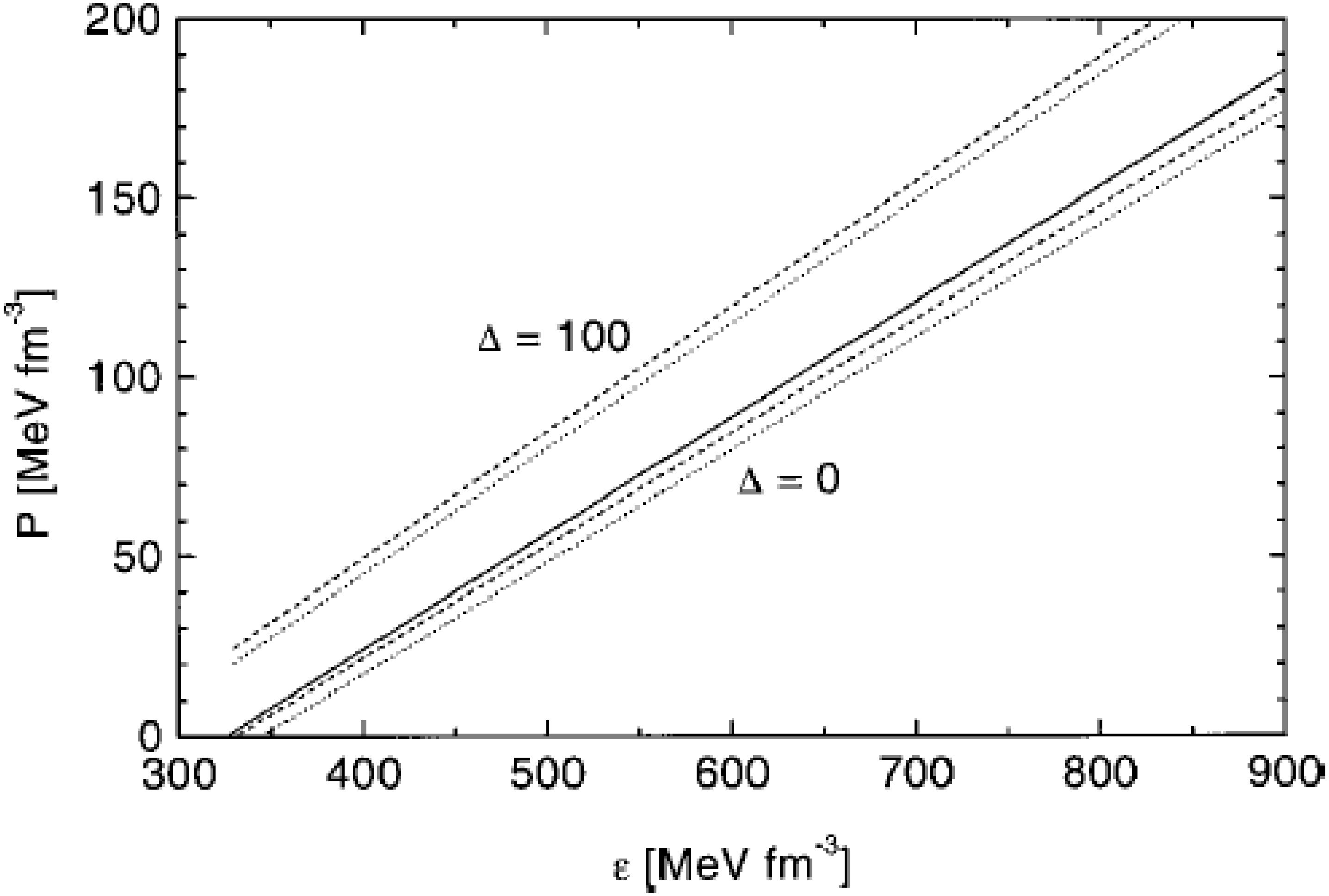
$$\varepsilon = \sum_i \mu_i n_i + \Omega_{CFL} = 3\mu n_B - P.$$

MIT bag model



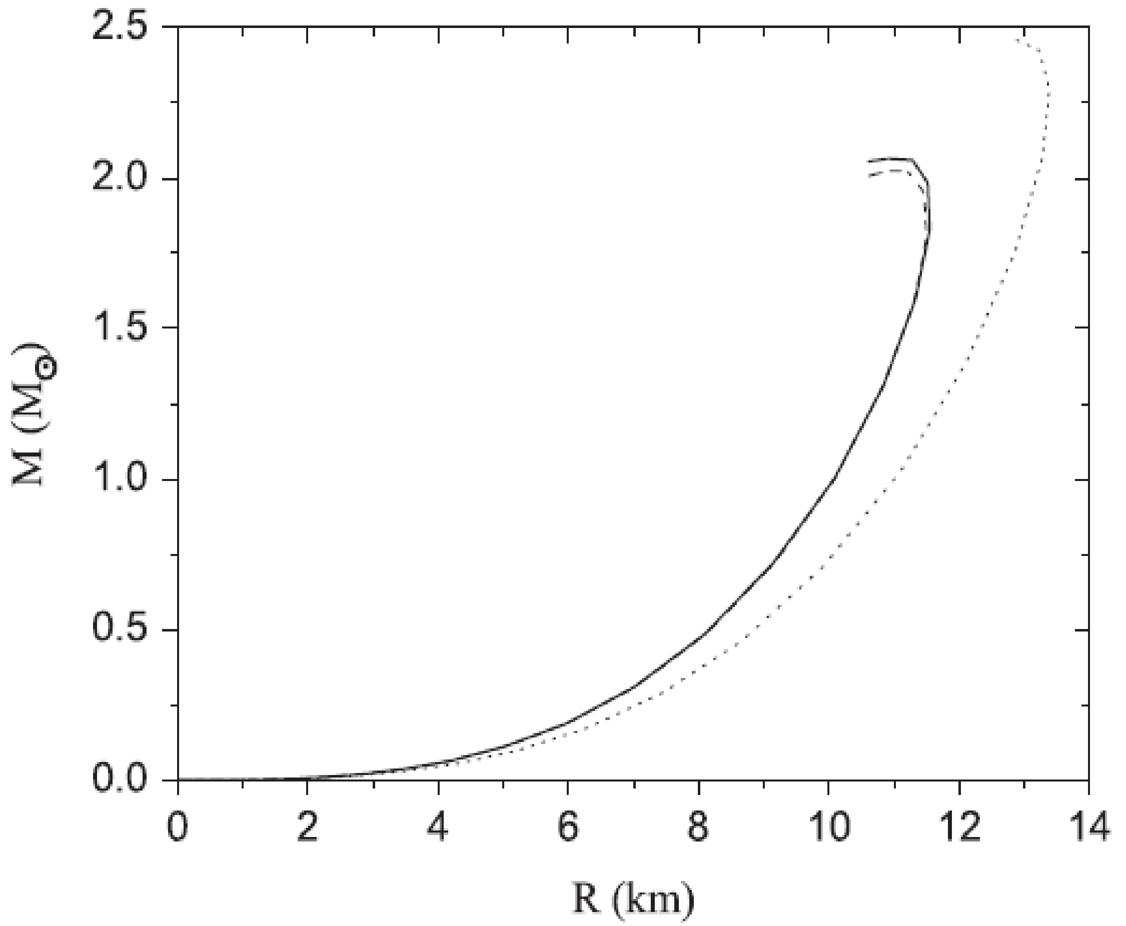
G. Lugones and J. E. Horvath, PRD **66**, 074017 (2002)

MIT bag model



G. Lugones and J. E. Horvath, PRD **66**, 074017 (2002)

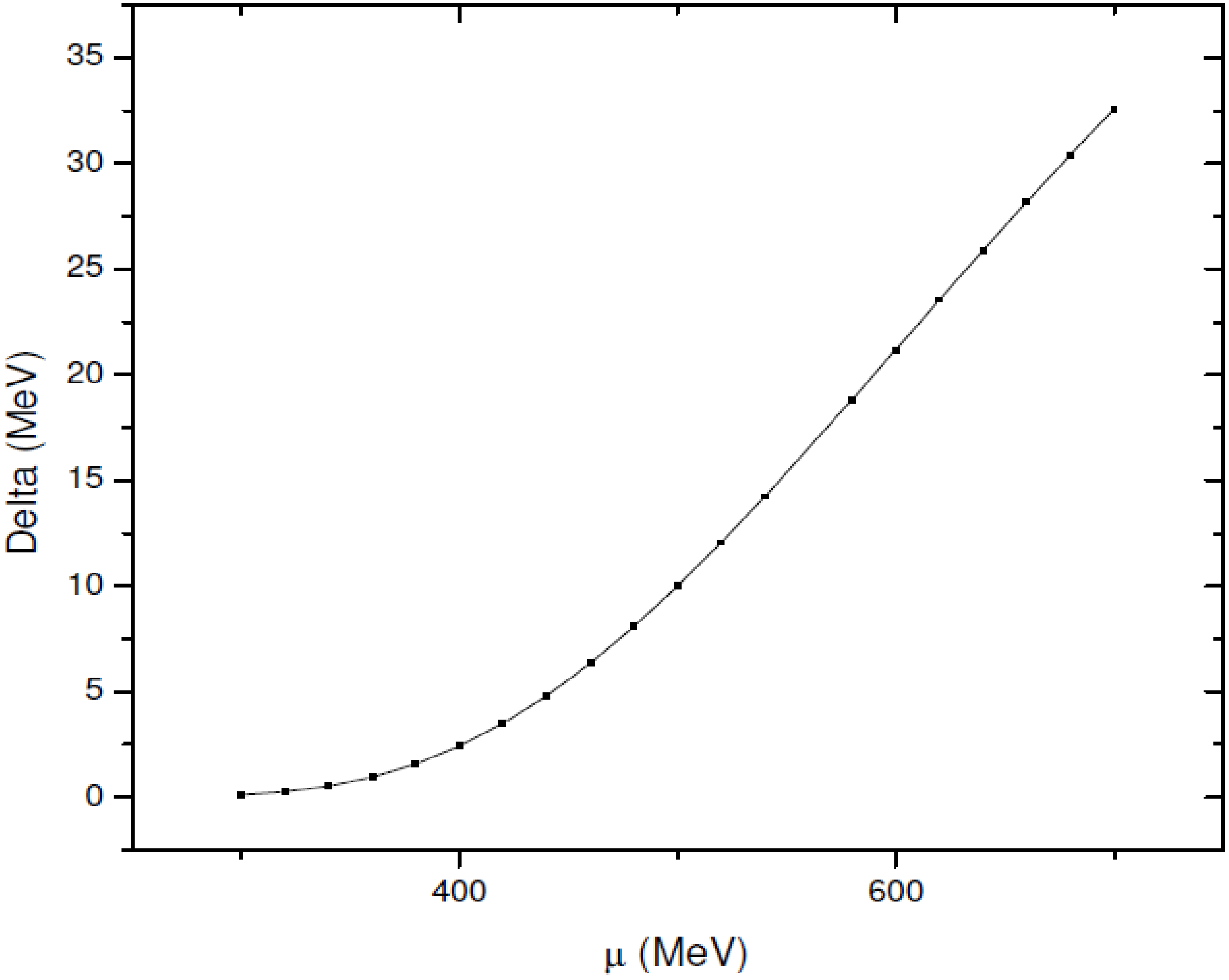
Comparing models



NJL CFL (H = 0). MIT for Δ = 10 MeV (dashed line) and Δ = 100 MeV (dotted line), and NJL (solid line).

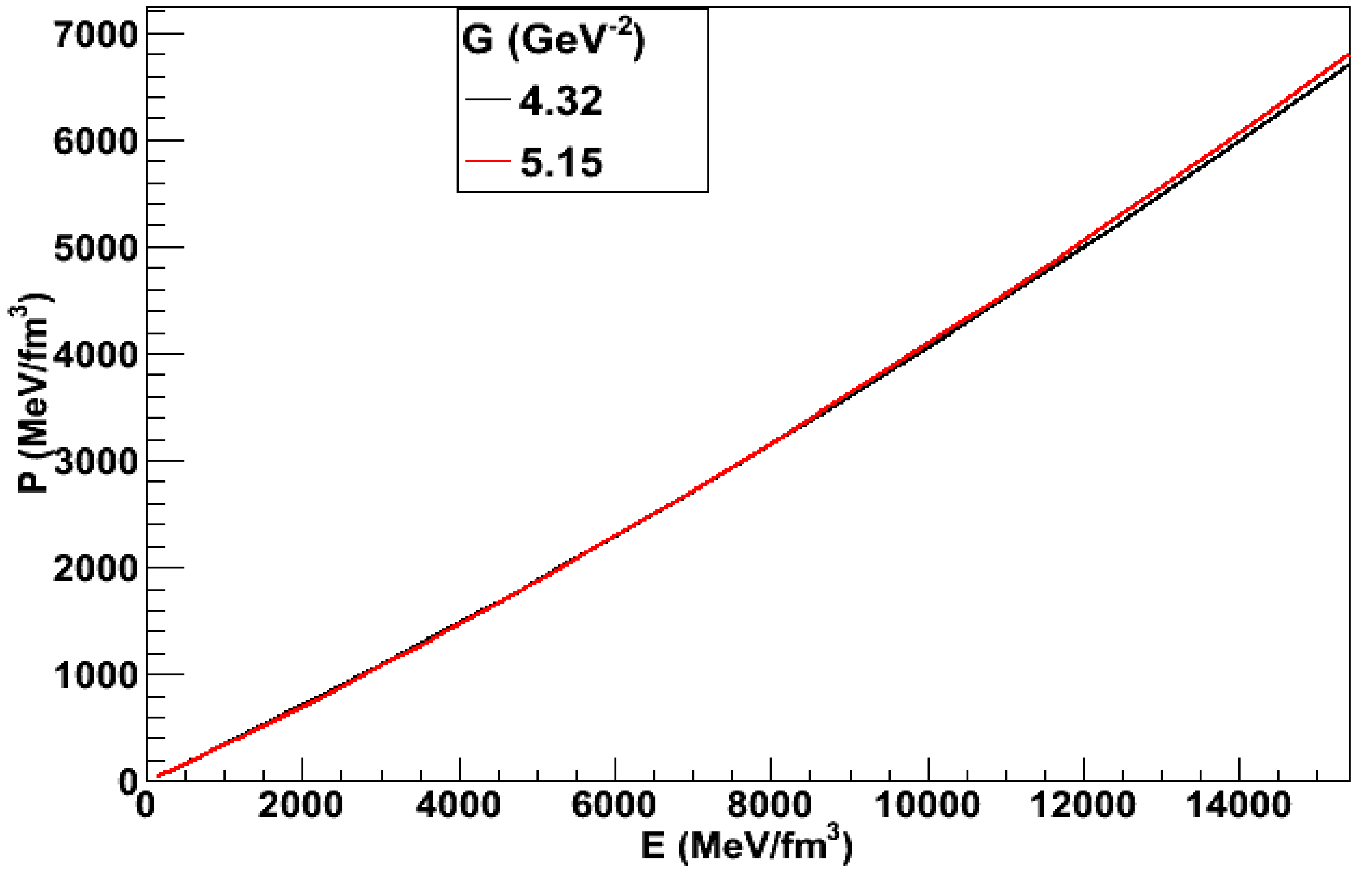
NJ L

CFL



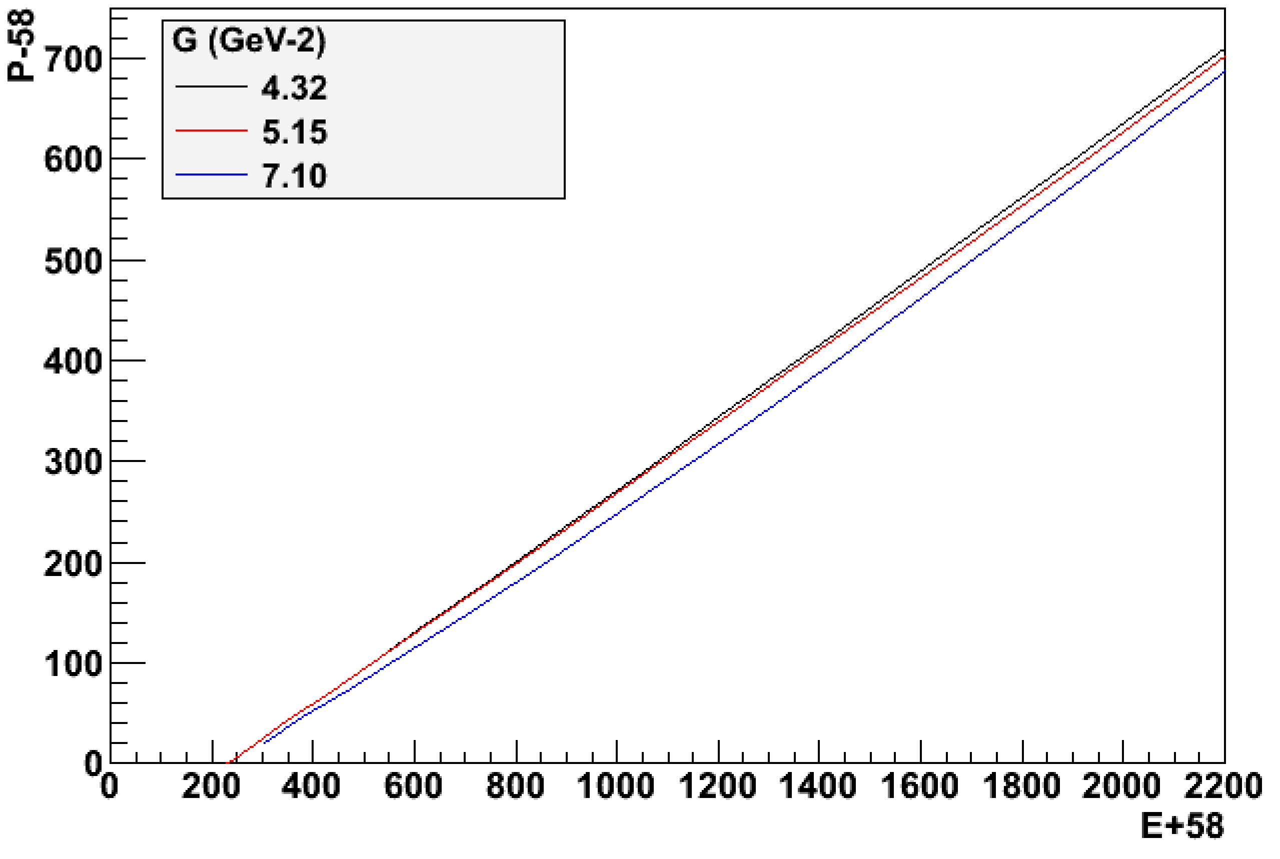
G=4.32 GeV^{-2}

NJL



G (GeV^{-2})
4.32
5.15
7000
6000
5000
4000
3000
2000
1000
0
P (MeV/fm^3)
0
2000
4000
6000
8000
10000
12000
14000
E (MeV/fm^3)

NJ L



P-58
G (GeV-2)
4.32
5.15
7.10
0
100
200
300
400
500
600
700
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
E+58

Conclusions and Perspectives

A magnetic field in CFL matter would enforce a new condition (a field dependent vacuum “bag constant”) for stability.

The EoS is largely linear and substantially modified only at sufficiently high fields.

In the anisotropic regime we need a stellar structure formalism in agreement with the system cylindrical symmetry.

Conclusions and Perspectives

The EoS is not made substantially harder within this model with the increase in the value of the gap parameter (change in G).

Calculate hybrid sequences