FERMION PAIRINGS IN B

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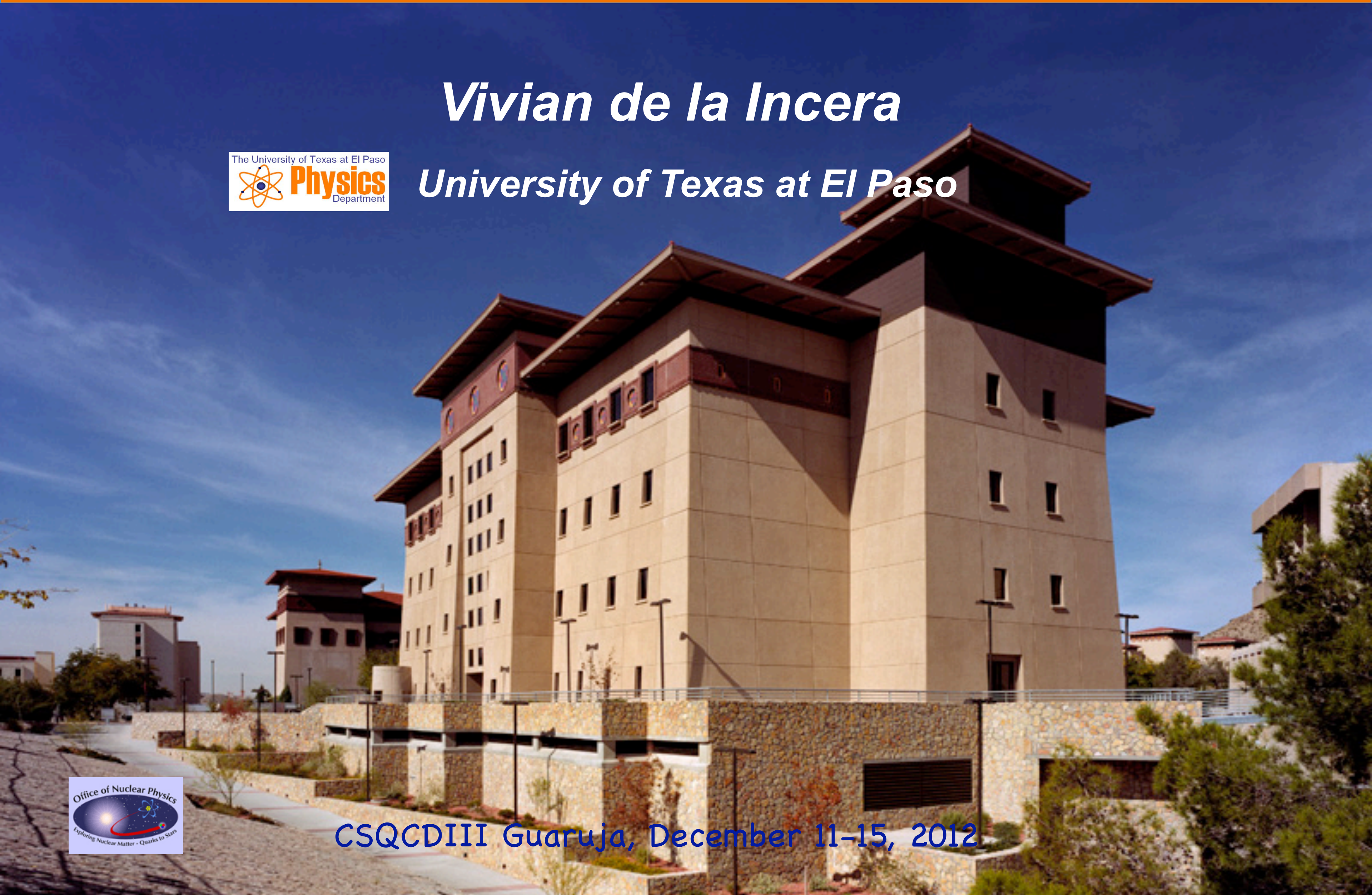


Office of Nuclear Physics — Exploring Nuclear Matter - Quarks to Stars

CSQCDIII Guaruja, December 11-15, 2012

OUTLINE

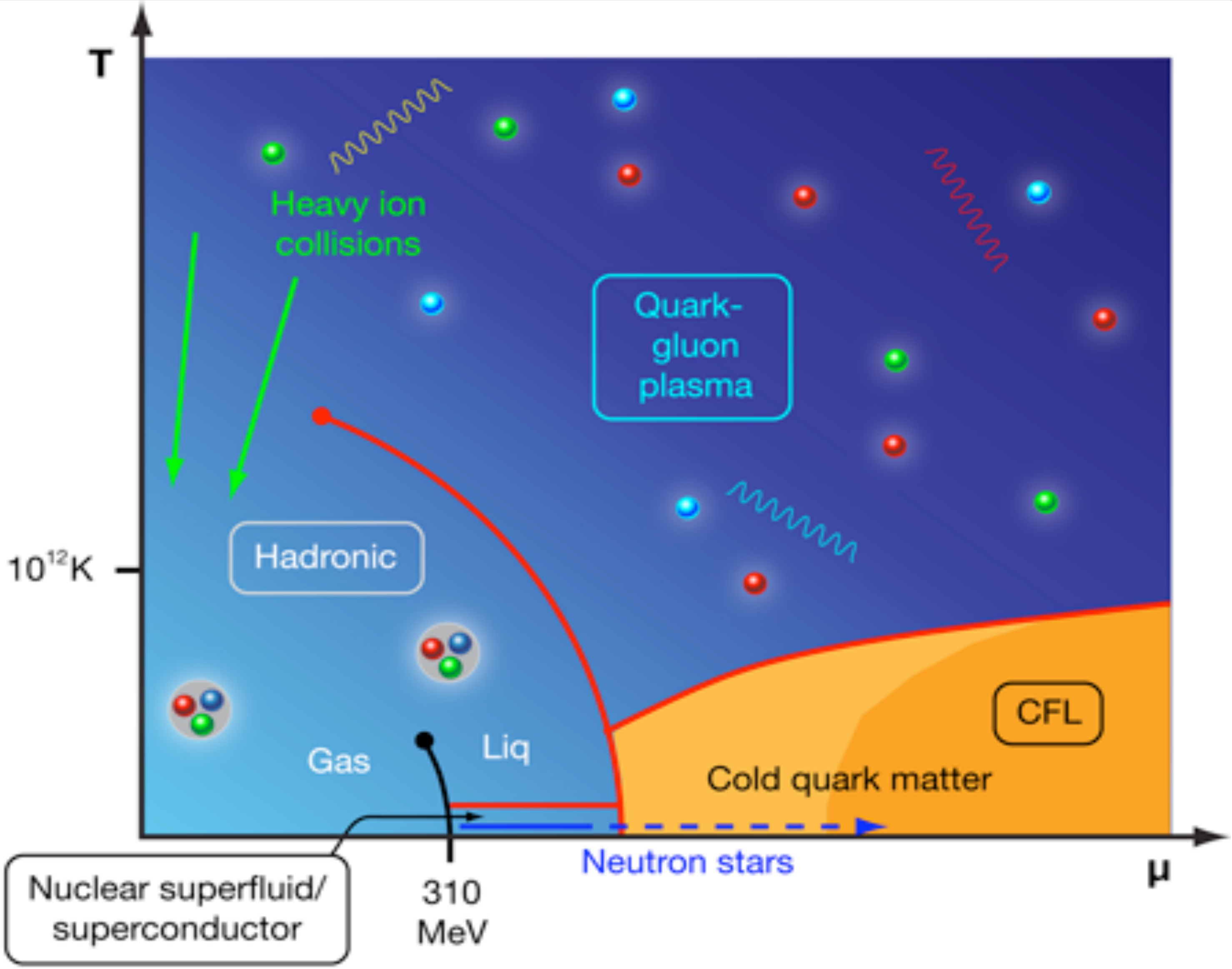
- ✓ ***Fermion Pairings, B, & QCD Map***
- ✓ ***Magnetoelectricity of the MCFL Phase***
- ✓ ***Quarkyonic Chiral Spirals in B:***

  Why B can "love" and "hate" pairing?
- ✓ ***Looking to the Future***

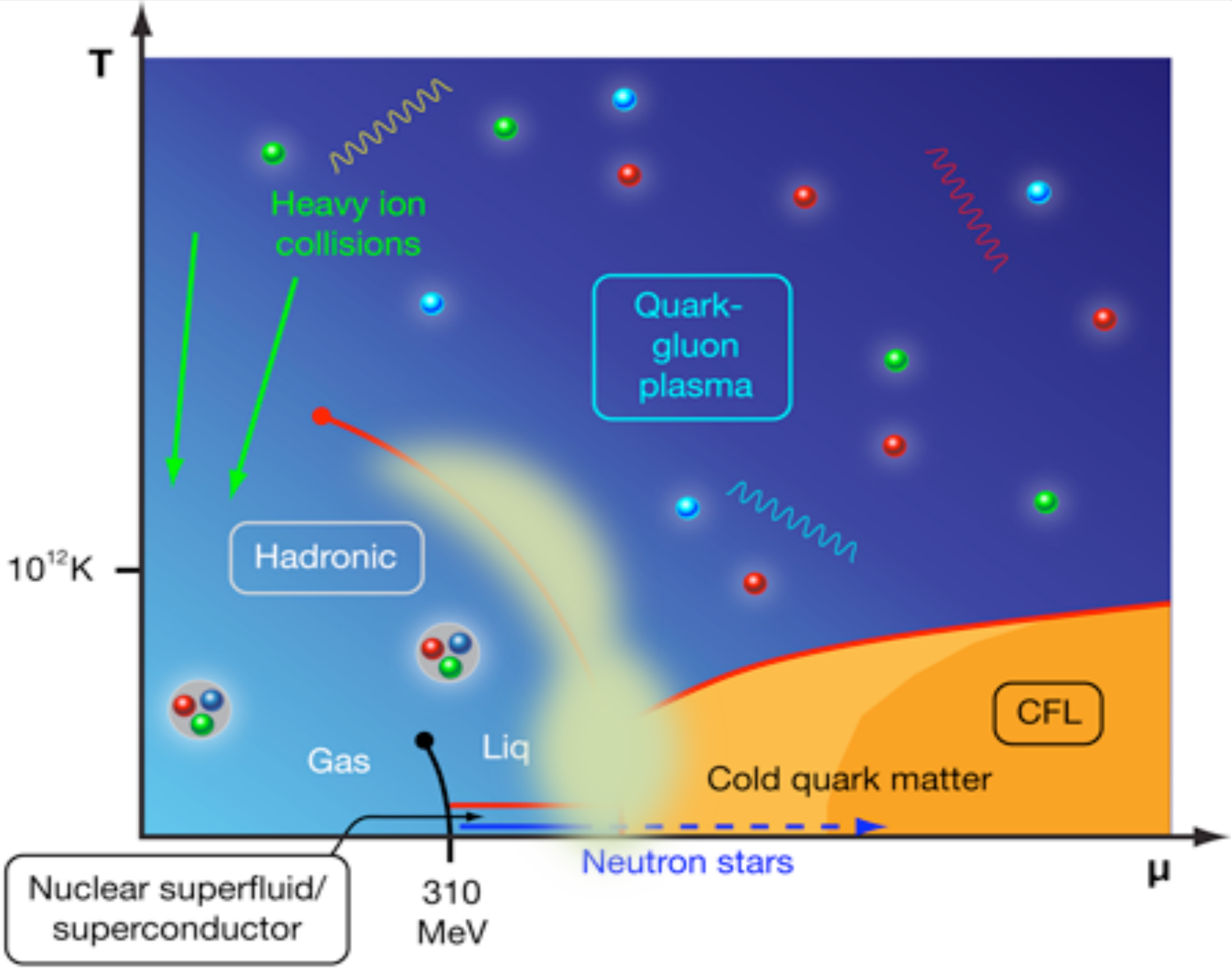
Collaborators:

Efrain J. Ferrer, Bo Feng, Angel Sanchez

CURRENT QCD PHASE MAP

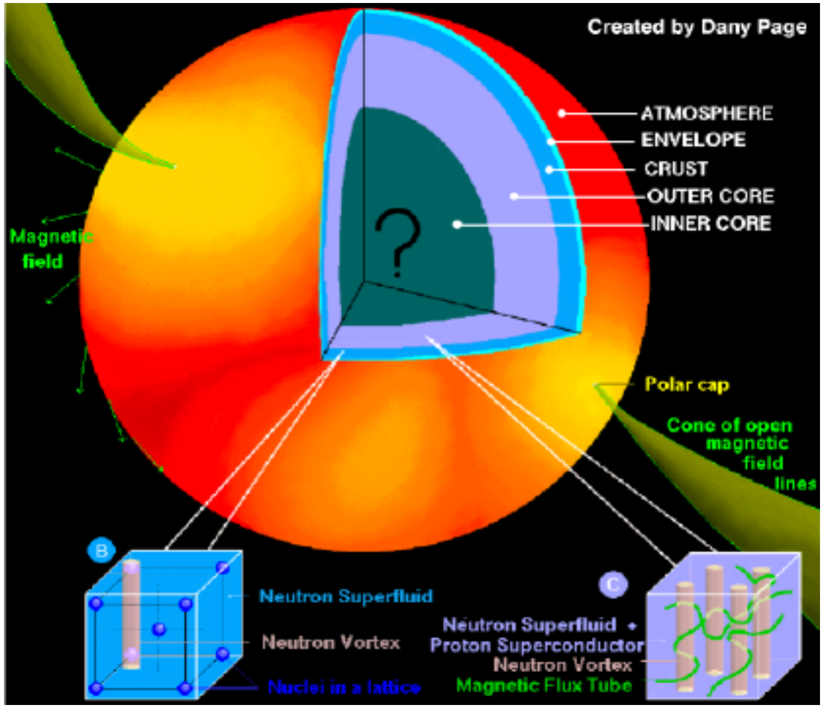


CURRENT QCD PHASE MAP



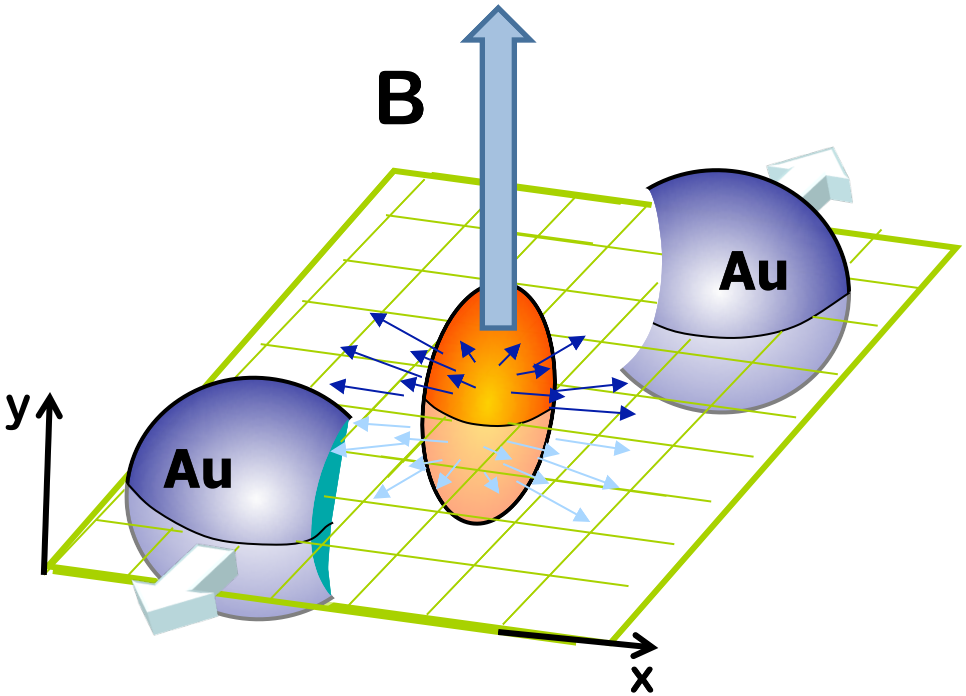
STRONGEST MAGNETIC FIELDS

NEUTRON STARS



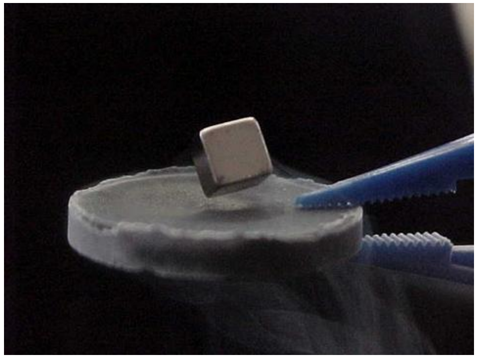
- Surface of magnetars 10^{14}-10^{15} G
- Core of NS, unknown, but could be ~ 10^{18}- 10^{19} G

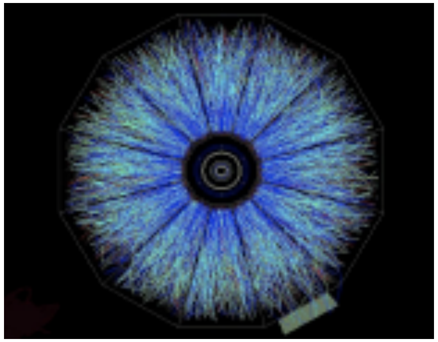
OFF-CENTRAL COLLISIONS AT RHIC



Relativistic collisions create magnetic fields ~ 10^{17}- 10^{18} G

FERMION PAIRINGS

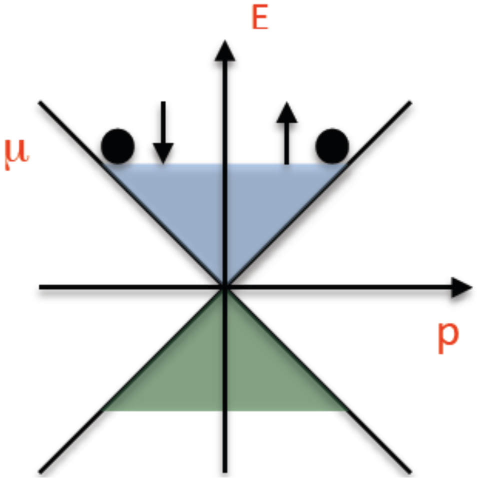


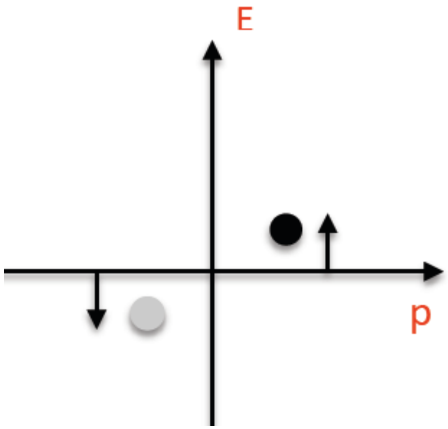


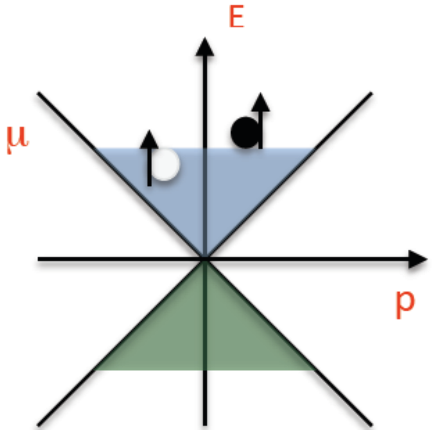
BCS Superconductivity

Mesons

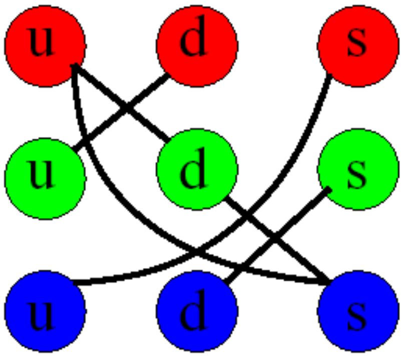
Quarkyonic Chiral Spirals







COLOR-FLAVOR LOCKED PHASE



Rapp, Schafer, Shuryak and Velkovsky, PRL'98
Alford, Rajagopal and Wilczek, PLB '98

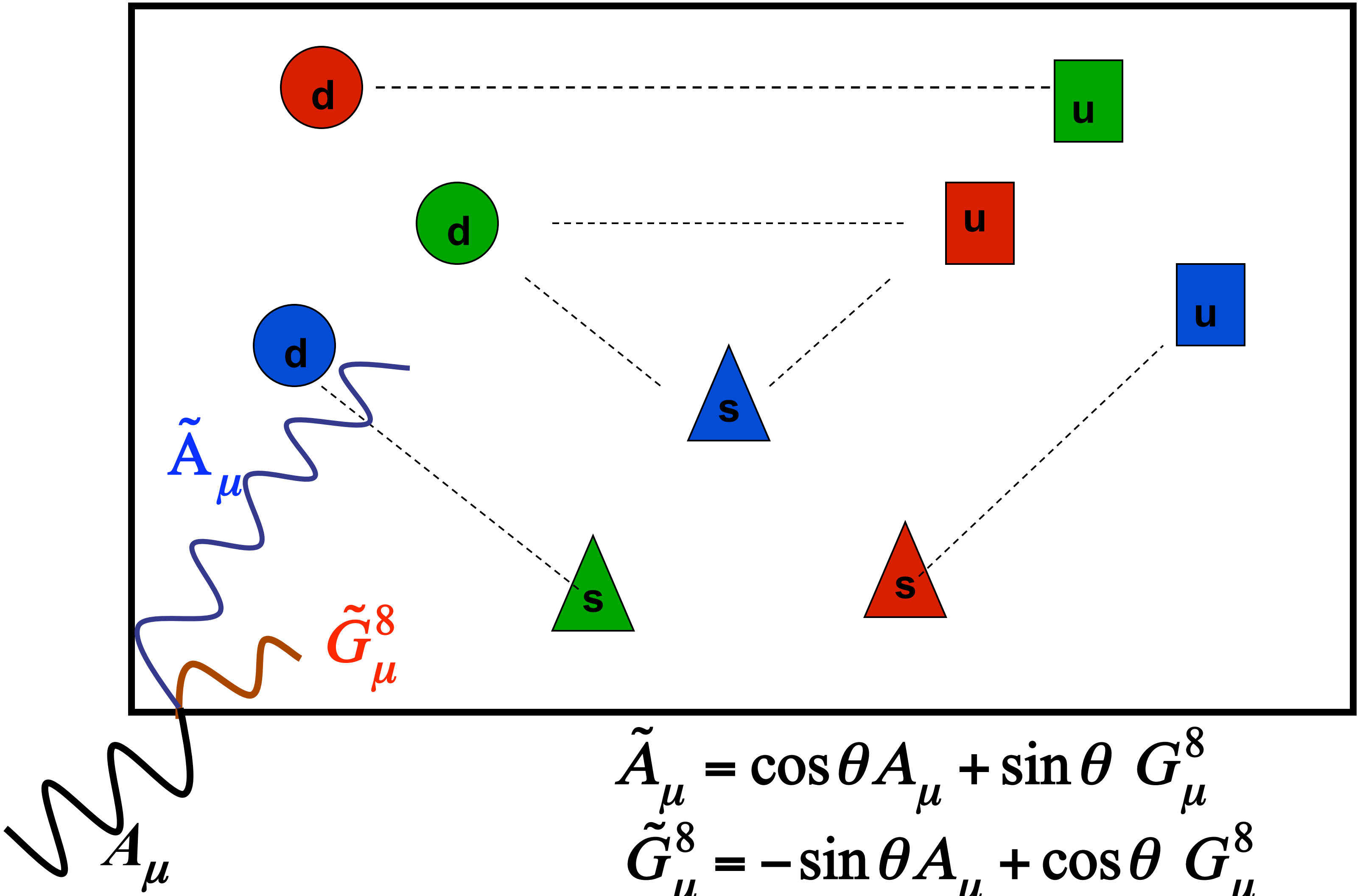
One Gap:

$$\langle \Psi_i^a C\gamma_5 \Psi_j^b \rangle = \Delta_{CFL} C\gamma_5 \epsilon^{abk} \epsilon^{ijk}$$

- ✓ ***All quark pair. No gapless fermions, no massless gluons.***
- ✓ ***Color superconductivity is more robust than conventional superconductivity (no need to resort to phonons). Hence is a high Tc superconductor.***
- ✓ ***Chiral symmetry is broken in an unconventional way: through the locking of flavor and color symmetries.***

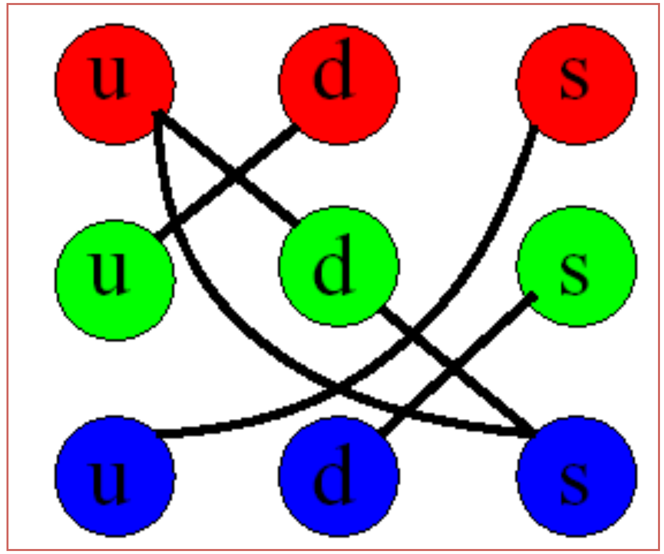
$$SU_C(3) \times SU_L(3) \times SU_R(3) \times U_B(1) \rightarrow SU(3)_{C+L+R}$$

ROTATED ELECTROMAGNETISM



$$\tilde{A}_\mu = \cos\theta A_\mu + \sin\theta \; G^8_\mu$$
$$\tilde{G}^8_\mu = -\sin\theta A_\mu + \cos\theta \; G^8_\mu$$

ROTATED CHARGES



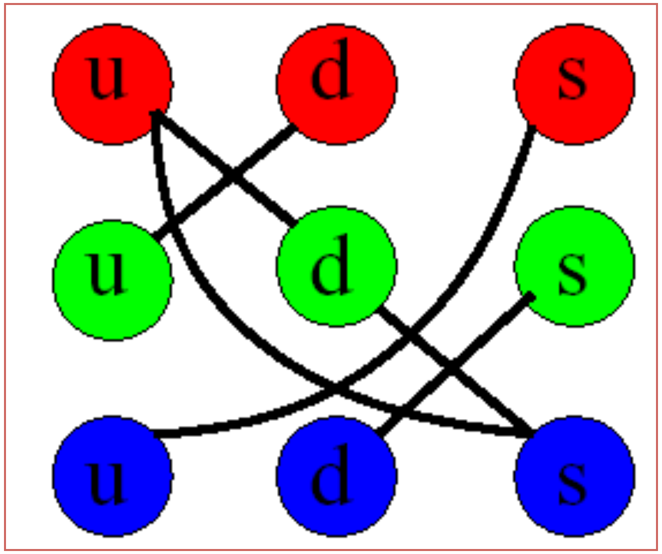
The pairs are all $\tilde{Q}$-neutral, but the quarks can be neutral or charged

$\tilde{Q}$ - CHARGES

s	s	s	d	d	d	u	u	u
0	0	-1	0	0	-1	1	1	0

All $\tilde{Q}$-charged quarks have integer charges

ROTATED CHARGES



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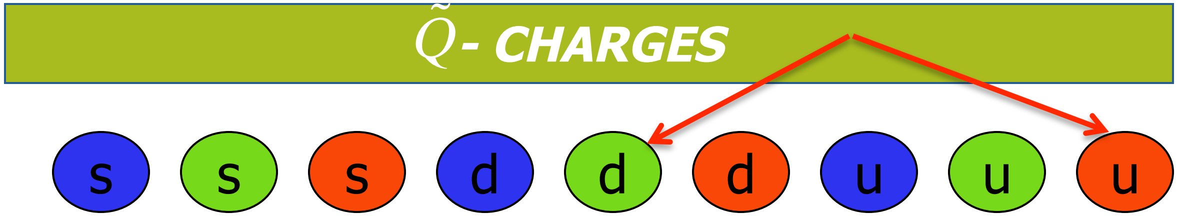

0	0	-1	0	0	-1	1	1	0

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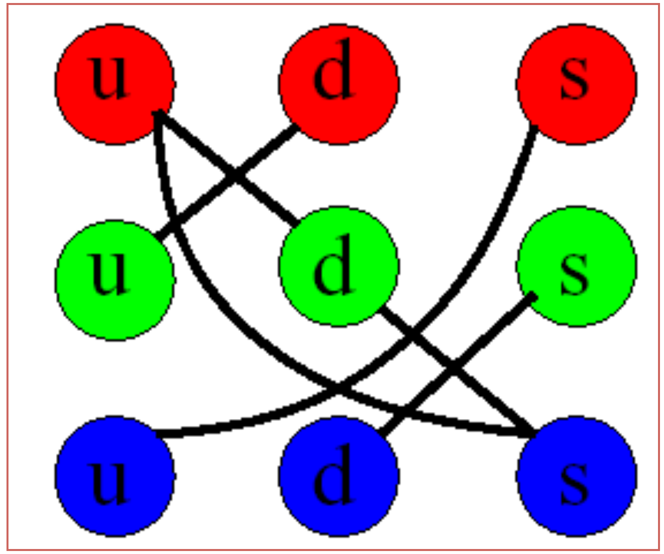

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$\tilde{Q}$- CHARGES

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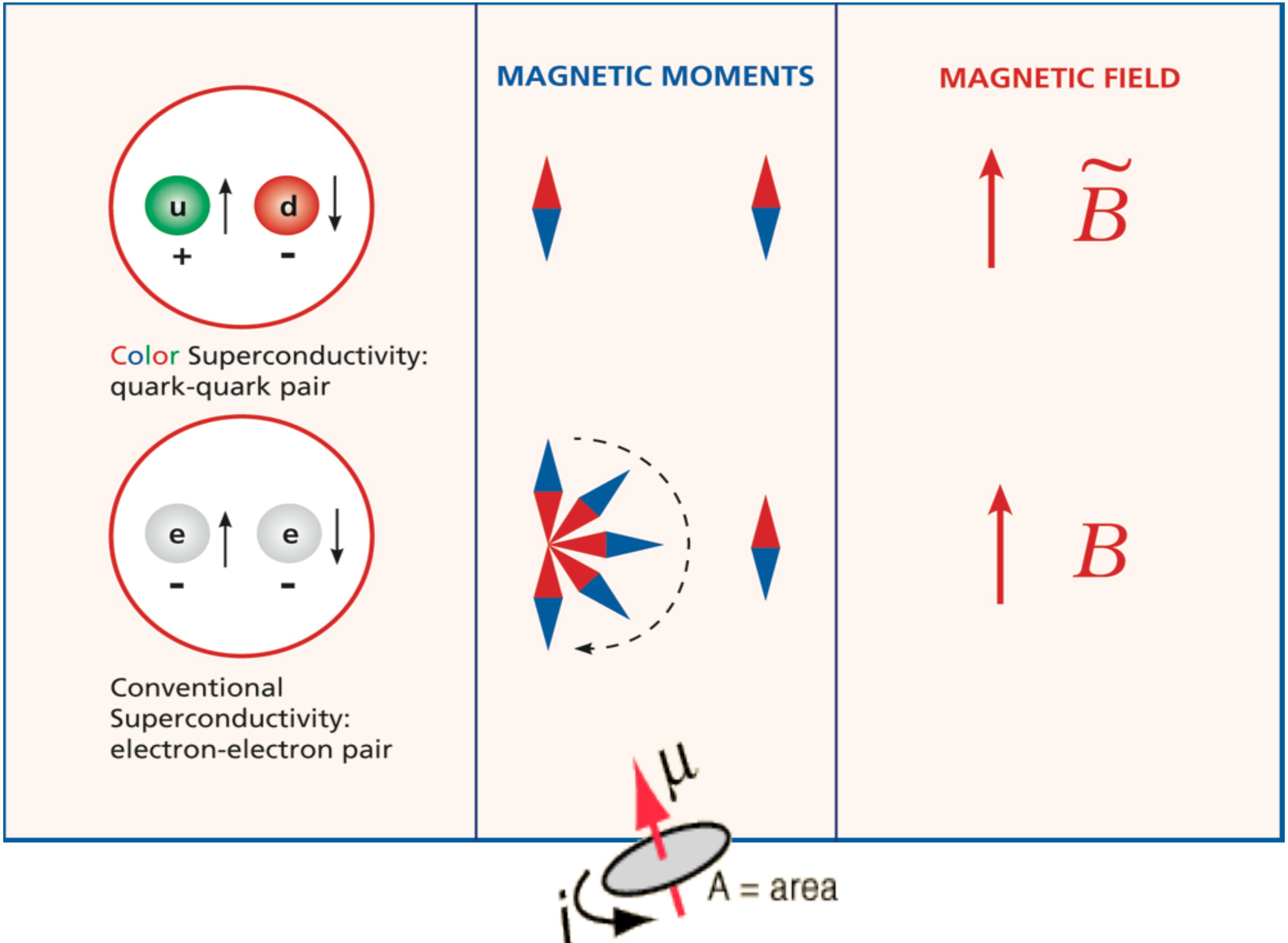
All $\tilde{Q}$-charged quarks have integer charges

Color Superconductivity & B



GAPS and GAP STRUCTURE IN MCFL

Instead of one gap like in CFL, the magnetic field leads to three gap parameters:

$\hat{\Delta}_H = \Delta_H C\gamma_5$ Gets contributions only of pairs of neutral quarks

$\hat{\Delta} = \Delta C\gamma_5$ Gets contributions of pairs of charged & neutral quarks

$\hat{\Delta}_M = i\Delta_M \gamma_5\gamma_1\gamma_2$ Magnetic moment condensate

$$\begin{pmatrix} 0 & \hat{\Delta} & \hat{\Delta}_B+\hat{\Delta}_M & 0 & 0 & 0 & 0 & 0 & 0 \\ \hat{\Delta} & 0 & \hat{\Delta}_B+\hat{\Delta}_M & 0 & 0 & 0 & 0 & 0 & 0 \\ \hat{\Delta}_B-\hat{\Delta}_M & \hat{\Delta}_B-\hat{\Delta}_M & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\hat{\Delta} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\hat{\Delta} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_B+\hat{\Delta}_M & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_B+\hat{\Delta}_M \\ 0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_B-\hat{\Delta}_M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_B-\hat{\Delta}_M & 0 & 0 \end{pmatrix}.$$

Ferrer, VI, and Manuel, PRL'05, NPB '06; Feng, Ferrer, & VI NPB'11

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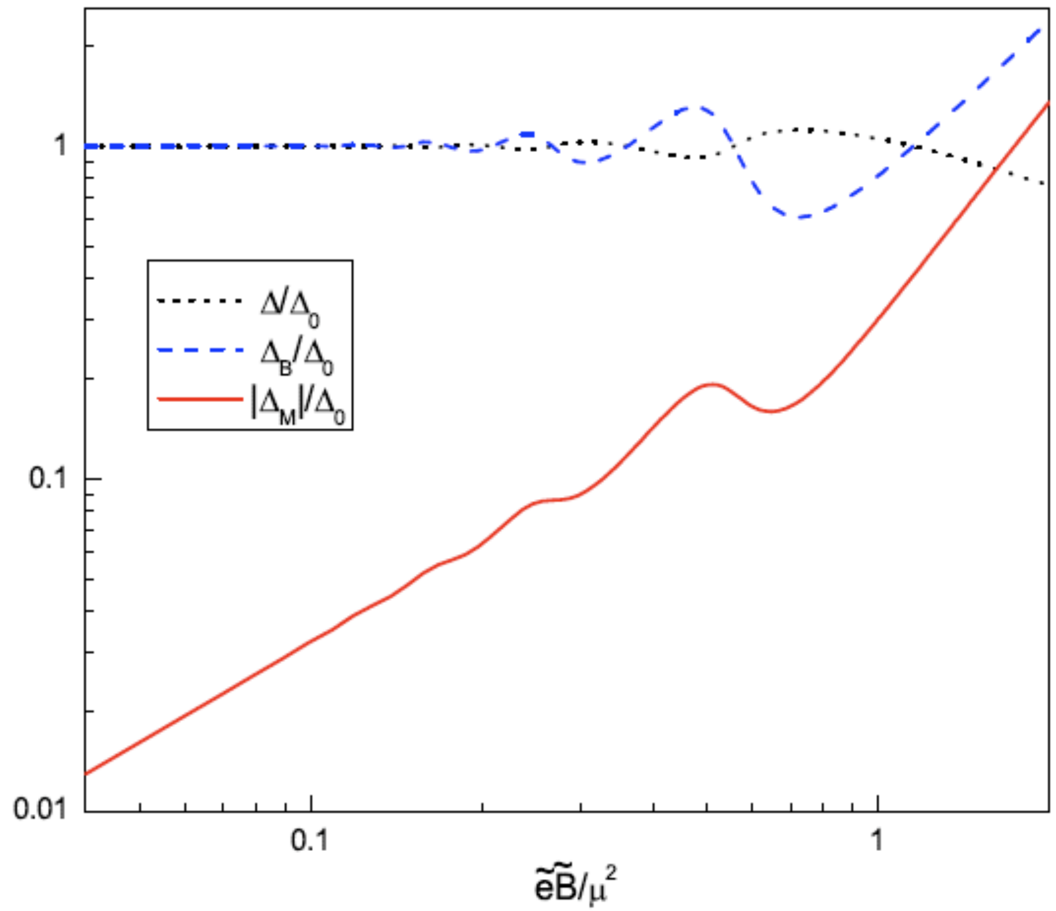
$$\begin{pmatrix}
0 & \hat{\Delta} & \hat{\Delta}_B+\hat{\Delta}_M & 0 & 0 & 0 & 0 & 0 & 0 \\
\hat{\Delta} & 0 & \hat{\Delta}_B+\hat{\Delta}_M & 0 & 0 & 0 & 0 & 0 & 0 \\
\hat{\Delta}_B-\hat{\Delta}_M & \hat{\Delta}_B-\hat{\Delta}_M & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\hat{\Delta} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\hat{\Delta} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_B+\hat{\Delta}_M & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_B+\hat{\Delta}_M \\
0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_B-\hat{\Delta}_M & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\hat{\Delta}_B-\hat{\Delta}_M & 0 & 0
\end{pmatrix}.$$

Ferrer, VI, and Manuel, PRL'05, NPB '06; Feng, Ferrer, & VI NPB'11

Gaps and Magnetic Moment vs B

Feng, Ferrer, & VI, NPB '11



- The field induces Hass van Alphen oscillations in the parameters
- LLL effect: The MM becomes comparable to the other gaps at large fields, when the LLL fills out

MAGNETOELECTRIC EFFECT

Magnetoelectricity: The electric polarization depends on an applied magnetic field or the magnetization depends on an applied electric field. Occurs in multiferroic materials.

At weak E and B fields, the free energy can be expanded in powers of the fields, so the polarization can be written as

$$P_i = -\frac{\partial F}{\partial \widetilde{E}_i} = \alpha_i + \gamma_{ij}\widetilde{B}_j + 2\eta_{ij}\widetilde{E}_j + 2\kappa_{ijk}\widetilde{E}_j\widetilde{B}_k + \lambda_{ijk}\widetilde{B}_j\widetilde{B}_k + 2\sigma_{ijkl}\widetilde{E}_j\widetilde{B}_k\widetilde{B}_l + \ldots$$

If tensor $\gamma \neq 0$, it is called the linear magnetoelectric effect.

At weak E but strong B, the free energy can be expanded in powers of E with coefficients that are functions of B,

$$F'(\widetilde{\mathbf{E}}, \widetilde{\mathbf{B}}) = F'_0(\widetilde{B}) - \alpha'_i \widetilde{E}_i - \eta'_{ij}\widetilde{E}_i\widetilde{E}_j - \ldots$$

If $\eta \neq 0$ the electric polarization will depend on the magnetic field, hence there is ME effect in the system. To find that coefficient we need to calculate the polarization operator

$$F'(\widetilde{\mathbf{E}}, \widetilde{\mathbf{B}}) - F'_0(\widetilde{\mathbf{B}}) \sim \frac{1}{V}\int \widetilde{A}_0(x_3)\Pi_{00}(x_3 - x'_3)\widetilde{A}_0(x'_3)dx_3dx'_3 = -\eta'\widetilde{E}^2,$$

Photon Polarization in MCFL

$$\Pi^{\mu\nu}(x,y) = \frac{\tilde{e}^2}{2} \sum_{\tilde{Q}=\pm} Tr\left[\Gamma^\mu \mathcal{S}_{(\tilde{Q})}(x,y)\Gamma^\nu \mathcal{S}_{(\tilde{Q})}(y,x)\right]$$

$$\tilde{S}^l_{(\tilde{Q})}(\bar{p}^{(\tilde{Q})}) = \begin{pmatrix} G^{+}_{(\tilde{Q})}{}^{l}(\bar{p}^{(\tilde{Q})}) & \Xi^{-}_{(\tilde{Q})}{}^{l}(\bar{p}^{(\tilde{Q})}) \\ \Xi^{+}_{(\tilde{Q})}{}^{l}(\bar{p}^{(\tilde{Q})}) & G^{-}_{(\tilde{Q})}{}^{l}(\bar{p}^{(\tilde{Q})}) \end{pmatrix} \qquad \Gamma^\mu = \begin{pmatrix} \tilde{Q}\gamma^\mu & 0 \\ 0 & \tilde{Q}\gamma^\mu \end{pmatrix}$$

In the strong field approximation only the LLL contributes

$$G^{\pm}_{(+)}{}^{l=0}(p^{\parallel}) = G^{\pm}_{(-)}{}^{l=0}(p^{\parallel}) = \sum_{e=\pm} \frac{p_0 \mp (\mu - e p_3)}{p_0^2 - [\epsilon^e_{p_3}]^2} \Lambda^e_{\mathbf{p}_3}\gamma_0$$

$$\Xi^{\pm}_{(+)}{}^{l=0}(p^{\parallel}) = \Xi^{\pm}_{(-)}{}^{l=0}(p^{\parallel}) = \pm \sum_{e=\pm} \frac{\Delta_0}{p_0^2 - [\epsilon^e_{p_3}]^2} \gamma_5 \Delta(+) \Lambda^{\mp e}_{\mathbf{p}_3}$$

MCFL's Magnetoelectricity

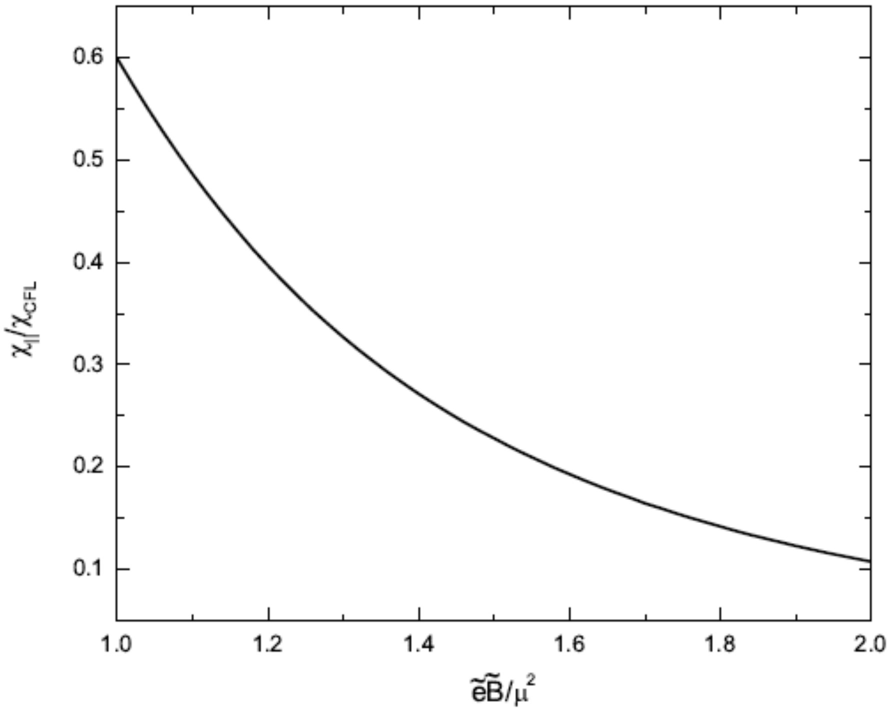
$$\Pi_{00}(p_0=0,\, p_3\to 0) \cong -\frac{\tilde{e}^2(\tilde{e}\tilde{B})p_3^2}{6\pi^2\Delta_0^2}$$

No Debye screening

$$\epsilon^{\perp}=1, \qquad \epsilon_{\|}=1+\chi_{\|}=1+\frac{2\tilde{\alpha}|\tilde{e}\tilde{B}|}{3\pi\Delta_0^2}$$

$$\Delta_0=\Delta_M-\Delta_B$$

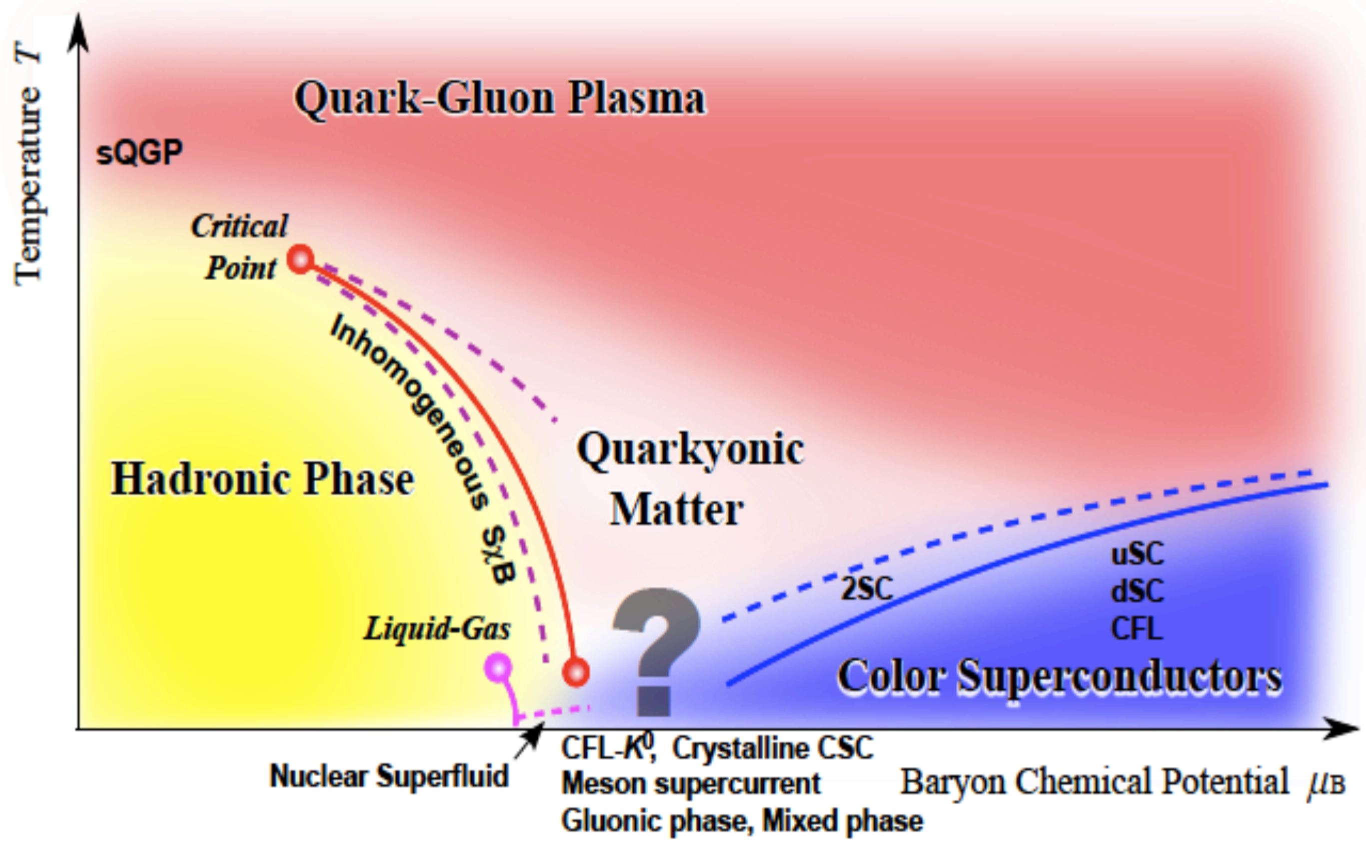
MCFL exhibits a Magnetoelectric Effect and a highly anisotropic electric polarization



The electric susceptibility decreases with the magnetic field.
The coherence length of the pairs is ~$1/\Delta_0$ and hence decreases with the field. Since the dipole length is of the order of the coherence length, it weakens with increasing magnetic field.

Feng, Ferrer, & VI, PLB '12

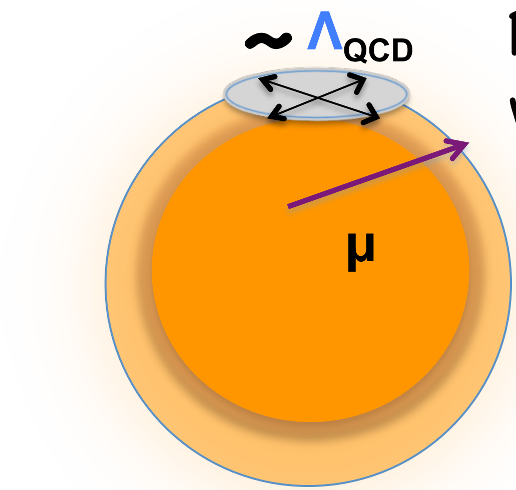
INTERMEDIATE PHASES



Quarkyonic Matter?

Deryaguin, Grigoriev, Rubakov, '92; Shuster and Son;'00; Kojo, Hidaka, McLerran, Pisarski, '09
Ferrer, VI, Sanchez, '12

QUARKYONIC MATTER



Relevant at high density & large N_c , where screening effects are negligible

$$m_D << \Lambda_{QCD} << \mu$$

Bulk Properties: perturbative

Excitations at the Fermi surface: confined

Large N_c : Gluon Propagator unaffected by quarks (same as in confined vacuum). Gribov-Zwanziger propagator:

$$D_{44}^{AB}(k) = -\frac{8\pi}{C_F} \times \frac{\sigma}{(\vec{k}^2)^2} \delta^{AB}$$

$$D_{ik}(\omega, \mathbf{q}) = \frac{\delta_{ik} - q_i q_k / q^2}{\omega^2 + \mathbf{q}^2}$$ (crossed out)

perturbative

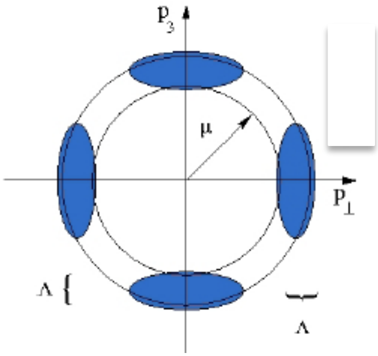
Valid in the Coulomb gauge and for $|p| \leq \Lambda_{QCD}$

McLerran & Pisarski, NPA'07; Kojo, et al NPA'10

DIMENSIONAL REDUCTION at B=0

$$\Sigma(p) = -\int \frac{d^4k}{(2\pi)^4} D^{AB}_{\mu\nu}(p-k)\,(\gamma_\mu t_A)\,S(k)\,(\gamma_\nu t_B)$$

$$\Sigma(p) = \int \frac{d^4k}{(2\pi)^4} \frac{8\pi\sigma}{((\vec{p}-\vec{k})^2)^2}\,\gamma_4\,S(k)\,\gamma_4$$



Can neglect $p_\perp$ in the quark propagator, because in the patch at the Fermi surface $|p_\perp| < \Lambda_{QCD} << \mu$ **and**

$$ip_4 \approx \delta p_z + \frac{\vec{p}_\perp^{\,2}}{2\mu} + \ldots$$ then,

4D QCD in Coulomb gauge reduces to 1+1 D QCD in axial gauge $A_z=0$

$$\Sigma(p_4, p_z, \vec{0}_\perp) \simeq \frac{N_c g^2_{2D}}{2} \int \frac{dk_4\,dk_z}{(2\pi)^2}\,\gamma_4\,S(k_4, k_z, \vec{0}_\perp)\,\gamma_4\,\frac{1}{(k_z-p_z)^2}$$

Deryagin, Grigoriev, & Rubakov '92; Shuster & Son, '99; Kojo et al, '10

EFFECTIVE 1+1 D THEORY AT B=0

Chiral rotation $\Phi = \exp\left(-i\,\mu\, z\,\Gamma^5\right)\Phi'$

$$\mathcal{L}_{\rm eff}^{\rm 2D} = \overline{\Phi}\left[i\,\Gamma^\mu\left(\partial_\mu + i g_{\rm 2D}\,A_\mu\right) + \mu\,\Gamma^0\right]\Phi = \overline{\Phi}'\left[i\,\Gamma^\mu\left(\partial_\mu + i g_{\rm 2D}\,A_\mu\right)\right]\Phi'$$

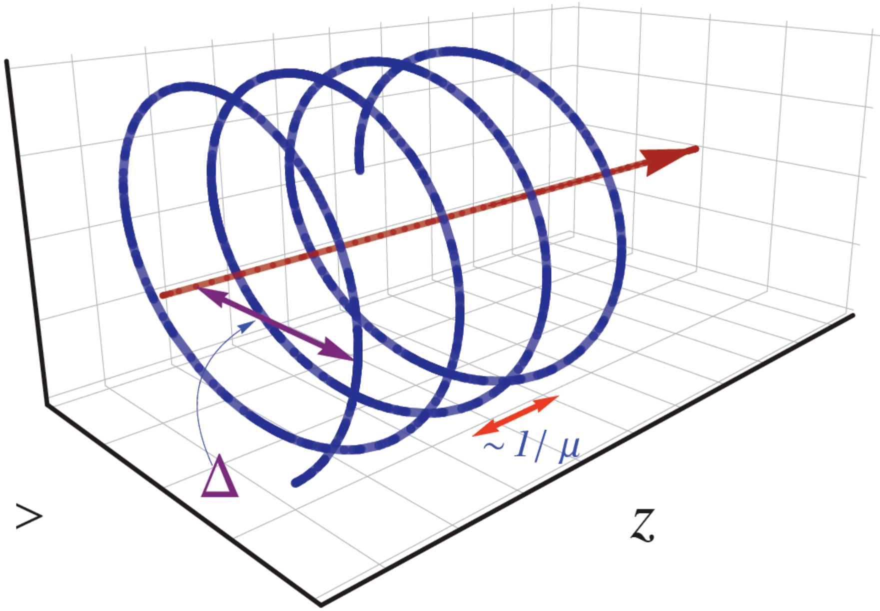
Chiral Spirals ← Chiral condensate $\langle\overline{\Phi}'\Phi'\rangle$

$\langle\overline{\Phi}\Phi\rangle$ $\langle\bar{\psi}\psi\rangle$

$= \langle\overline{\Phi}'\Phi'\rangle\cos(2\mu z)$ $<\overline{\Phi}\,\Phi>$

$\langle\overline{\Phi} i\Gamma^5\Phi\rangle$ $\langle\bar{\psi}\gamma^0\gamma^z\psi\rangle$

$= \langle\overline{\Phi}'\Phi'\rangle\sin(2\mu z)$ $<\overline{\Phi}\, i\Gamma^5\Phi>$



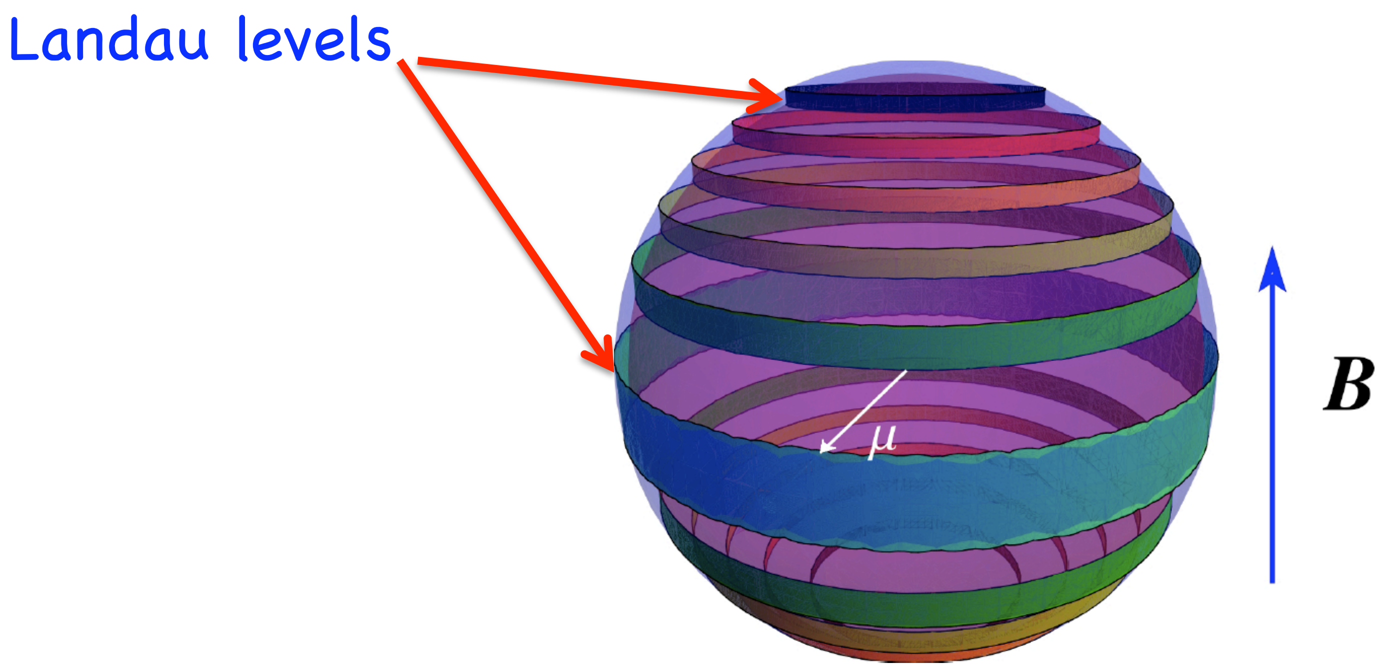
Kojo, et.al NPA'10

A NEW NEEDED INGREDIENT:B

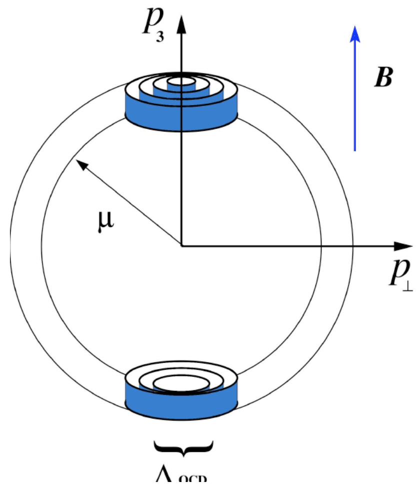
Possible realization of QCS:

Heavy ion collision experiments & neutron stars

Both environments typically have very strong magnetic fields, $B \leq \Lambda_{QCD}^2$



POLAR PATCHES



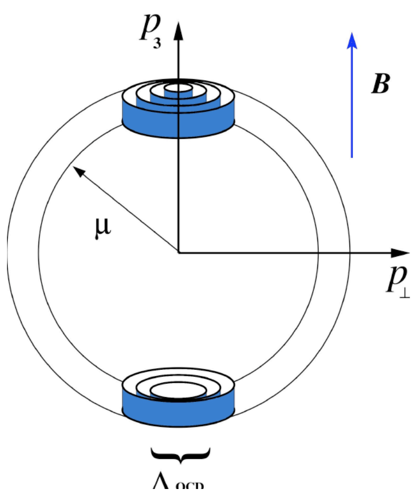
Assume $qB \leq (\Lambda_{QCD})^2$

One equation for each LL in the patch. The LLs become flavors l

$$
\begin{aligned}
\Sigma^l(p)\Pi(l) &= -i\tilde{g}^2\Pi(l)\int \frac{d^2q_{||}}{(2\pi)^2}\gamma_0\tilde{G}^l(\overline{p-q})\gamma_0\int\frac{d^2q_\perp}{(2\pi)^2}\frac{e^{-\hat{q}_\perp^2}}{(q_\perp^2+q_3^2)^2} \\
&= -i\tilde{g}^2\Pi(l)\int \frac{d^2q_{||}}{(2\pi)^2}\gamma_0\tilde{G}^l(\overline{p-q})\gamma_0\frac{1}{4\pi}\int_0^{\Lambda^2} dq_\perp^2\,\frac{e^{-\hat{q}_\perp^2}}{(q_\perp^2+q_3^2)^2} \\
&= -i\frac{\tilde{g}^2}{4\pi}\Pi(l)\int \frac{d^2q_{||}}{(2\pi)^2}\gamma_0\tilde{G}^l(\overline{p-q})\gamma_0\frac{1}{q_3^2}
\end{aligned}
$$

Only one spin contributes to the lowest LL

$$\Pi(l) = \Delta(+) + \Delta(-)(I - \delta_{l0})$$



Assume $qB \leq (\Lambda_{QCD})^2$

1+1-D theory with l_{max} flavors

$$L = \int d^2p_{||}\overline{\Phi}_0[\Gamma^\mu(p_\mu + g_{2D}A_\mu) + \mu\Gamma^0]\Phi_0 + \sum_{l=1}^{l_{max}} \int d^2p_{||}\overline{\Phi}_l[\Gamma^\mu(p_\mu + g_{2D}A_\mu) + \mu\Gamma^0]\Phi_l$$

$$l_{max} = [\Lambda^2 / 2eB]$$

zero Landau level

$$\Phi_0 = \begin{pmatrix} \varphi_{0\uparrow} \\ 0 \end{pmatrix}$$

higher Landau levels l>0

$$\Phi_l = \begin{pmatrix} \varphi_{l\uparrow} \\ \varphi_{l\downarrow} \end{pmatrix}$$

Quarkyonic Chiral Spirals at B≠0

Chiral rotation $\Phi_l = e^{(-i\mu z\Gamma_5)}\Phi'_l$

$$L = \overline{\Phi}_0[i\Gamma^\mu(\partial_\mu + ig_{2D}A_\mu) + \mu\Gamma^0]\Phi_0 + \sum_{l=1}^{l_{max}} \overline{\Phi}_l[i\Gamma^\mu(\partial_\mu + ig_{2D}A_\mu) + \mu\Gamma^0]\Phi_l$$

$$= \overline{\Phi}'_0[i\Gamma^\mu(\partial_\mu + ig_{2D}A_\mu)]\Phi'_0 + \sum_{l=1}^{l_{max}} \overline{\Phi'}_l[i\Gamma^\mu(\partial_\mu + ig_{2D}A_\mu)]\Phi'_l$$

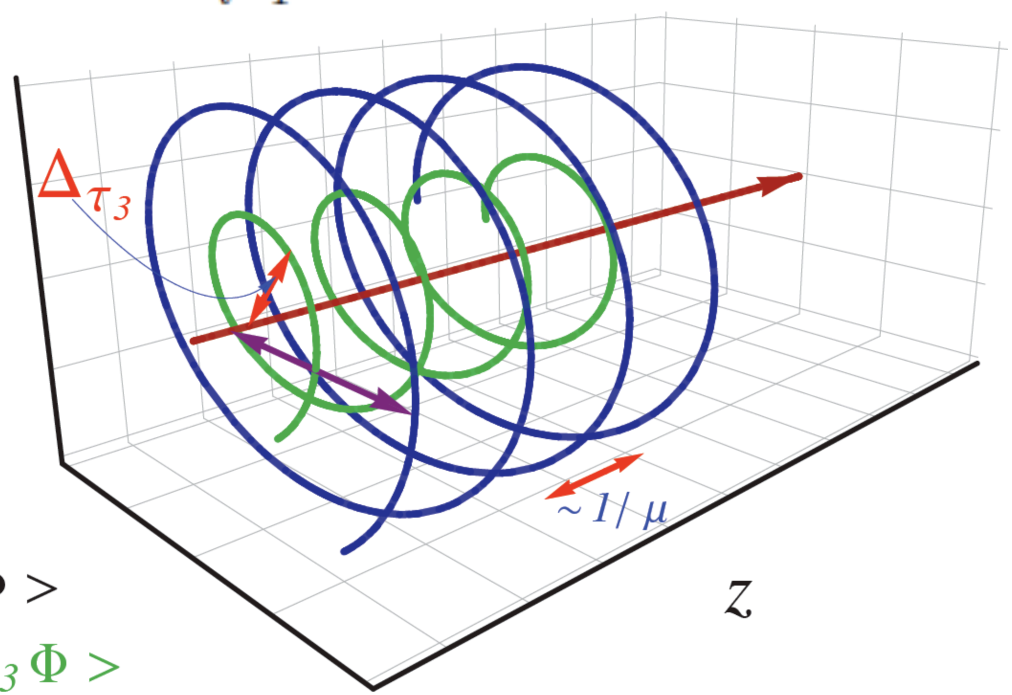
$\overline{\Phi}'\Phi'$ $\overline{\Phi}'\tau^3\Phi'$

$< \overline{\Phi}\ \Phi >$

$< \overline{\Phi}\ \tau_3 \Phi >$

$< \overline{\Phi}\ i\Gamma^5 \Phi >$

$< \overline{\Phi}\ i\Gamma^5 \tau_3 \Phi >$



Ferrer, V.I, and Sanchez, arXiv: 1205.4492

Chiral Spirals at B ≠ 0

$\langle\overline{\Phi}\Phi\rangle \longrightarrow \langle\bar{\psi}\psi\rangle$

$\langle\overline{\Phi}i\Gamma^5\Phi\rangle \longrightarrow \langle\bar{\psi}\gamma^0\gamma^z\psi\rangle$

Spiral is formed by chiral and electric dipole condensates

$\langle\overline{\Phi}\tau^3\Phi\rangle \longrightarrow \langle\overline{\psi}i\gamma^1\gamma^2\psi\rangle$

$\langle\overline{\Phi}\tau^3\Gamma^5\Phi\rangle \longrightarrow \langle\overline{\psi}\gamma^5\psi\rangle$

Spiral is formed by pion and magnetic moment condensates

Generation of Parallel & Inhomogeneous Electric and Magnetic fields

Ferrer, V.I, and Sanchez,
arXiv:1205.4492
Acta Phys. Pol. '12

Why the Double Spiral?

At B = 0

$$\overline{\Phi}'\Phi' = \overline{\varphi}'_{\uparrow}\varphi'_{\uparrow} + \overline{\varphi}'_{\downarrow}\varphi'_{\downarrow}$$

and

$$\overline{\Phi}'\tau^3\Phi' = \overline{\varphi}'_{\uparrow}\varphi'_{\uparrow} - \overline{\varphi}'_{\downarrow}\varphi'_{\downarrow} = 0$$

At B ≠ 0

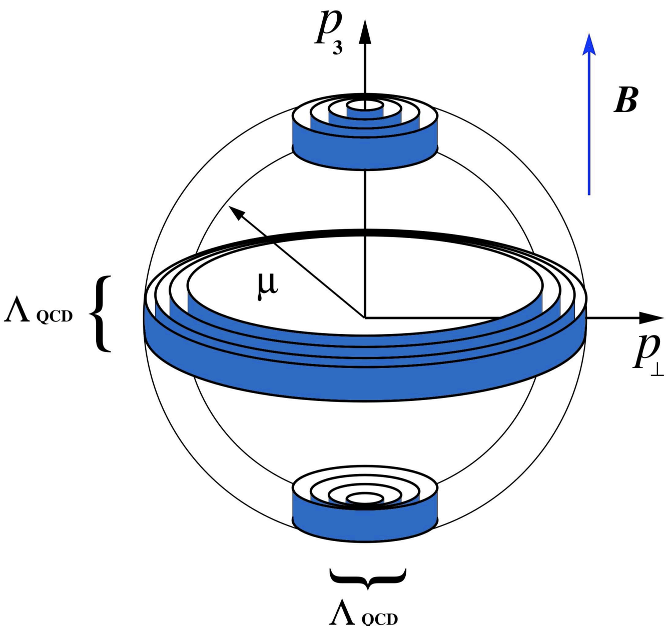
$$\overline{\Phi}'\Phi' = \overline{\varphi}'_{0\uparrow}\varphi'_{0\uparrow} + \sum_{l=1}^{l_{max}}[\overline{\varphi}'_{l\uparrow}\varphi'_{l\uparrow} + \overline{\varphi}'_{l\downarrow}\varphi'_{l\downarrow}]$$

and

$$\overline{\Phi}'\tau^3\Phi' = \overline{\varphi}'_{0\uparrow}\varphi'_{0\uparrow} + \sum_{l=1}^{l_{max}}[\overline{\varphi}'_{l\uparrow}\varphi'_{l\uparrow} - \overline{\varphi}'_{l\downarrow}\varphi'_{l\downarrow}]$$

The LLL makes it nonzero!!

EQUATORIAL PATCHES



$$\Sigma^l(\overline{p})\Pi_l =$$

$$= \frac{-ig^2 8\pi\tau}{2}\Pi_l \int \frac{d^2q_\parallel dq_\perp^2}{(2\pi)^3} e^{-\hat{q}_\perp^2} \frac{\gamma_0 G^l(\overline{p-q},\mu)\gamma_0}{(\vec{q}^2)^2}$$

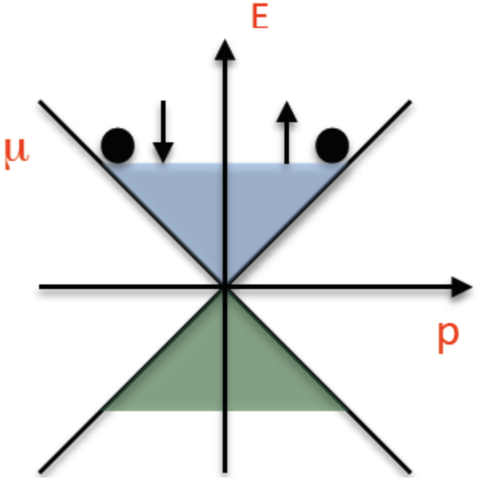
$$= -i\frac{\tilde{g}^2}{4\pi}\Pi(l) \int \frac{d^2q_{||}}{(2\pi)^2} \gamma_0 \tilde{G}^l(\overline{p-q})\gamma_0 \frac{1}{q_3^2}$$

$$\overline{p-q} \equiv (p_0 - q_0, 0, \mathrm{sgn}(e_f B)\sqrt{2|e_f B|l}, p_3 - q_3)$$

The term $\sqrt{2|e_f B|l}$ enters as an effective mass, then avoiding the pole instability. Hence, no spiral condensate can be formed in the equatorial patches.

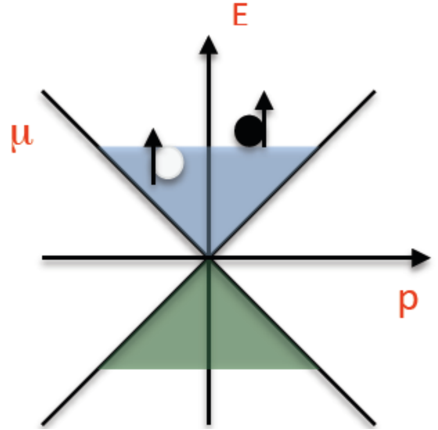
B EFFECTS IN FERMION PAIRINGS

Color Superconductivity



- ✓ Effects on the symmetry and de Hass van Alphen oscillations appear at mid Bs (scale $\sim \Delta^2_{CFL}$).
- ✓ Significant separation of the gaps requires large B (scale $\sim \mu^2$).
- ✓ MM condensate relevance determined by the LLL occupancy (significant only at large B (scale $\sim \mu^2$).

Quarkyonic Chiral Spirals

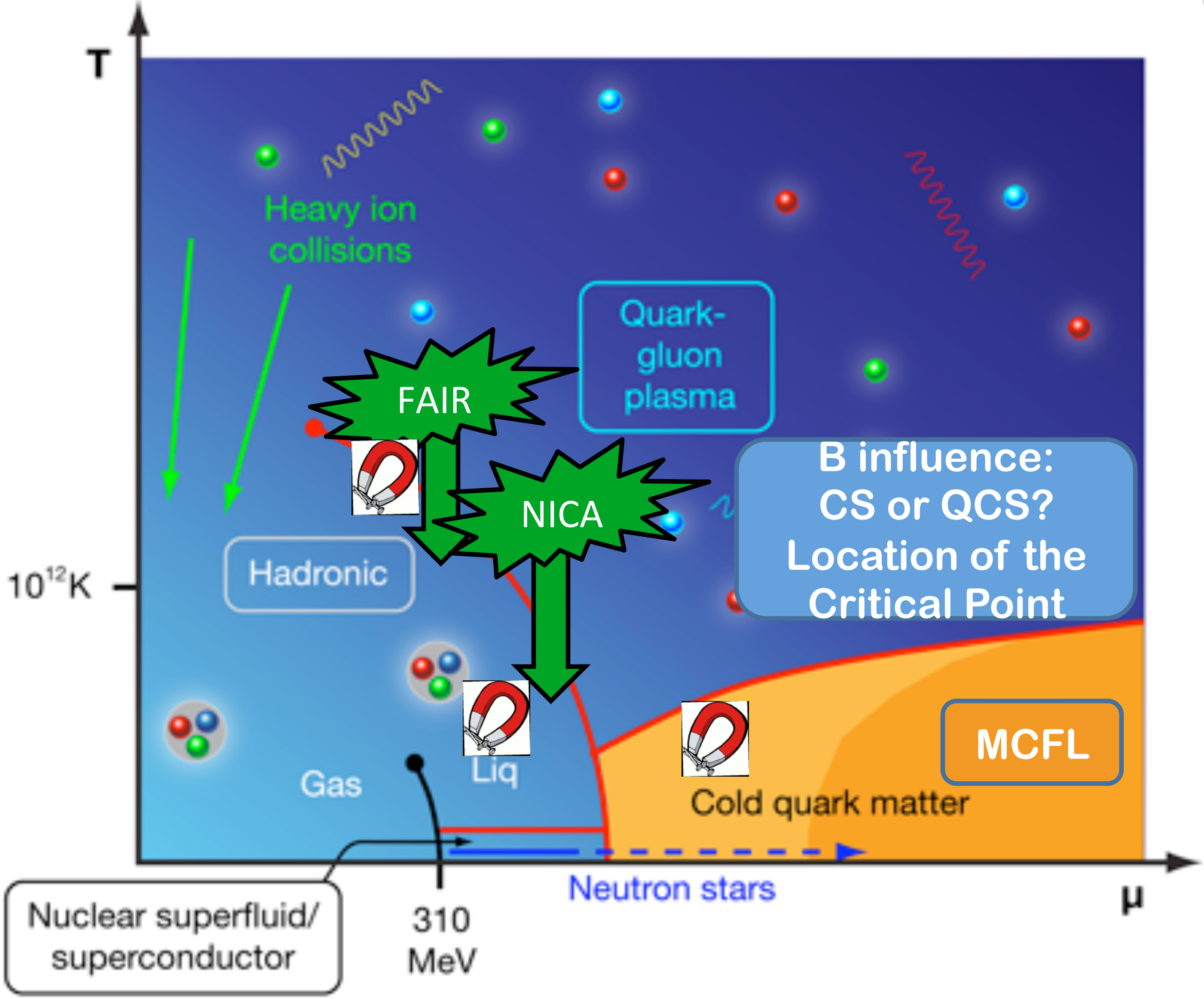


- ✓ Field effects important at mid fields: $B \sim \Lambda^2 << \mu^2$
- ✓ LLL essential to produce two QCS at the polar patches, but does not need to be $\sim \mu^2$
- ✓ QCS weakens at the lateral patches due to field-induced "effective mass" (2eBl).
- ✓ $2eBl \sim \mu^2 \Rightarrow$ no QCS at the equatorial patches.

CONCLUSIONS

- ✓ *Most B effects connected to LLL effects (even if B is not the largest scale)*
- ✓ *Higher LLs can weakens condensation because of 2eBl*
- ✓ *B ⟶ large anisotropic effects*
- ✓ *Color Superconductivity- Magnetoelectricity*
- ✓ *Chiral Spirals in B: Inhomogeneous electric and magnetic fields parallel to external B*
- ✓ *Outlook: Predicting signatures in stars and heavy-ion collisions*

LOOKING TO THE FUTURE



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