Maximal electrically charged strange quark stars in nonlinear electrodynamics

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Abstract

We study the dynamical role played by an ultra strong nonlinear electric field on the hydrodynamical equilibrium (equation of state) of a Strange Quark Star (SQS) using the MIT Bag Model, in the context of the nonlinear electrodynamics (NLED) obtained as a reduction of the Abelian sector of low-energy (3+1) QCD theory by Pagels-Tomboulis , NPB 143, 485 (1978)

Ultrastrong electric field can appear on SQS surface in case it possesses a net electric charge. For ordinary strange matter: electric field ~ 10^{18} V/cm --- And up to 10^{19} V/cm if SQS forms a color superconductor

Astrophysical Motivation

- Just-born SQS may form during gravitational collapse of massive stars --- and NS phase transition
- Hypercritical magnetic field can permeate SQS
- It drives Vacuum Polarization !! (which should structurally distabilize the star) <----> Unstable Magnetar-like Objects Should Explode
- It may lead to SNe: Hypernovae, Quark-Novae, Short GRBs, Gravitational Wave Emission, etc.
- For such Superstrong B-Fields Maxwell Theory is not Reliable (nor physically appropriate)
- That is why NLED is needed !!

Some theories of NLED

- Born-Infeld (built on Special Relativity)
- Heisenberg-Euler (An infinite series in Maxwell Scalar $X = F_{ab} F^{ab}$)
- Novello-Bergliaffa-Salim (Limited Laurent-like series on X : positive , negative powers)
- Pagels-Tomboulis (A consequence of QCD (3+1) model)

Einstein field equations in NLED and SQS

--- **Pagels-Tomboulis Lagrangian:** $L(X) = -CX - \gamma |X|^{\delta}\,.$

--- **Action:**

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \frac{1}{4\pi} L(X) \right),$$

(1)

--- **Maxwell equations in presence of sources:**

$$\nabla_{\rho} F^{\rho\sigma} = 4\pi j^{\sigma} - \frac{\nabla_{\mu} L_X}{L_X} F^{\mu\sigma}\,,$$

(2)

$$\nabla_{\mu} F_{\nu\lambda} + \nabla_{\nu} F_{\lambda\mu} + \nabla_{\lambda} F_{\mu\nu} = 0\,.$$

Equation (2) can be recast in the form:

(3) $$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = 4\pi j^\nu - \frac{\gamma\delta(\delta-1)|X|^{\delta-2}|X|,\mu}{C+\gamma\delta|X|^{\delta-1}}F^{\mu\nu}.$$

NOTE: covariant derivative replaced by ordinary derivative because X is a scalar

We use spherically symmetric metric

(4) $$ds^2 = e^{\Phi(r)}c^2dt^2 - e^{\Lambda(r)}dr^2 - r^2 d\Omega,$$

With: $d\Omega = d\theta^2 + \sin^2\theta d\phi^2$

For this work in progress: Electric field non-null (Next phase of study B-Field non-zero)

- **This implies a nonvanishing electric field component:**

  F^{\mu \nu} = F^{01} = E_r(r) = E

  which renders:

  (5) $$X = F^2 = -2e^{\Lambda+\Phi} E^2 \,, \qquad Y = \frac{1}{4} F_{\mu\nu} {}^{*}\bar{F}^{\mu\nu}$$

- **And non-null current j^\mu component is: j^0 . Thus, Eq.(3) becomes:**

(6) $$\left(1 + \frac{2\gamma\delta(\delta-1)|X|^{\delta-2}|X|,_{\mu}}{C+\gamma\delta|X|^{\delta-1}}\right) E,_r + (\ln\sqrt{-g}),_r E = 4\pi j^0$$

Energy-Momentum Tensor

Two components:

- Strange matter term
- NLED contribution

(7) $$T_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(nl)} \, ,$$

(8) $$T_{\mu\nu}^{(m)} = (P + \rho c^2) u_\mu u_\nu + P g_{\mu\nu}$$

$u^\mu u_\mu = -1$ implies $u^0 = 1/\sqrt{g_{00}}$ and $u_0 = -\sqrt{g_{00}}$).

MIT Bag Model
Pressure vs. (Energy - B_{bag})

- Equation of State:

(9) $$P = \frac{1}{3}(\varepsilon - 4B)\,, \qquad B_{\rm bag} = -\frac{1}{2}\frac{\partial}{\partial r}(\Psi\bar{\Psi})$$

with $\varepsilon = \rho c^2$ and B=150 (MeV)^4

- In NLED

$$T^{(nl)}{}_{\nu}{}^{\mu} = \frac{1}{4\pi}\left[L_X F_{\nu\alpha}F^{\alpha\mu} + \frac{\delta^{\mu}_{\nu}}{4}L\right].$$

(10) $$T^{(nl)}{}_{\nu}{}^{\mu} = \frac{1}{4\pi}\left[C\left(F_{\nu\alpha}F^{\alpha\mu} + \frac{\delta^{\mu}_{\nu}}{4}X\right) + \gamma|X|^{\delta}\left(\delta\,|X|^{-1}F_{\nu\alpha}F^{\alpha\mu} + \frac{\delta^{\mu}_{\nu}}{4}\right)\right].$$

E-M Tensor Components

- Notice: **Its trace does not vanish** (due to conformal invariance)

(10)

$$T^{(nl)\,0}_{\;\;\;0} = T^{(nl)\,1}_{\;\;\;1} \;=\; = \frac{1}{4\pi}\left[\frac{C}{2}e^{\Lambda+\Phi}E^2 + \frac{\gamma}{2}\left(\frac{1}{2}-\delta\right)2^\delta e^{(\Lambda+\Phi)\delta}(E^2)^\delta\right]$$

$$T^{(nl)\,2}_{\;\;\;2} = T^{(nl)\,3}_{\;\;\;3} \;=\; \frac{1}{4\pi}\left[-\frac{C}{2}e^{\Lambda+\Phi}E^2 + \frac{\gamma}{4}2^\delta e^{(\Lambda+\Phi)\delta}(E^2)^\delta\right]$$

- **Eq.(3) admits th exact solution :**

(11)

$$E(r) = \frac{4\pi}{2\delta-1}e^{a(r)}\int_0^r j^0(r')e^{a(r')}dr' \;,$$

Where $a(r) \equiv \ln(\sqrt{-g})^{1/(2\delta-1)}$.

- By defining

$$\rho_{ch}^{(\delta)}(r) = j^0(r) \exp\left[\frac{\Phi(r)}{2}\frac{1}{2\delta-1}\right].$$

- Electric field assumes :

(12) $$E(r) = e^{-(\Lambda+\Phi)/2}\,\frac{Q_\delta(r)}{r^2}$$

where function Q reads:

(13) $$Q_\delta(r) = 4\pi G_\delta\left(\frac{r}{R}\right)\int_0^r \rho_{ch}^{(\delta)}(r')e^{\Lambda(r')/2}r'^2 F_\delta\left(\frac{r'}{R}\right)dr',$$

with $$G_\delta\left(\frac{r}{R}\right) = \frac{e^{-(\Lambda+\Phi)(\delta-1)/(2\delta-1)}}{2\delta-1}\left(\frac{R}{r}\right)^{4(\delta-1)/(2\delta-1)},$$

$$F_\delta\left(\frac{r}{R}\right) = e^{\Lambda(\delta-1)/(2\delta-1)}\left(\frac{r}{R}\right)^{4(\delta-1)/(2\delta-1)}.$$

F_\delta, G_\delta ---> 1 for \delta = 1 !!

Thus, Components E-M Tensor Read

- (13) $$T^{(nl)\,0}_{\ \ \ \ \ 0} = T^{(nl)\,1}_{\ \ \ \ \ 1} = \frac{\gamma}{8\pi}\left(\frac{1}{2}-\delta\right)|X|^{\delta} = \frac{Q^2}{8\pi r^4}\,\Gamma\,,$$

which allows to define total energy density!:

- (14) $$\frac{Q^2}{8\pi r^4}\Gamma = \frac{E^2_{\rm NLED}}{8\pi}$$

where

(15) $$T^{(nl)\,2}_{\ \ \ \ \ 2} = T^{(nl)\,3}_{\ \ \ \ \ 3} = \frac{\gamma}{4\pi}(1-\delta)|X|^{\delta} = \frac{Q^2}{8\pi r^4}\,\tilde{\Gamma}\,,$$

with

$$Q = Q_{\delta=1} = 4\pi\int_0^r \rho_{ch} r'^{\,2} e^{\Lambda/2}\,dr'$$

$$\rho_{ch} = \rho_{ch}^{(\delta=1)} = j^0 e^{\Phi/2}\,,$$

$$\Gamma = -(2\delta-1)\tilde{\Gamma}\,.$$

is similar as in Maxwell Theory !!

$$\tilde{\Gamma} \equiv \gamma 2^{\delta-1}\left(\frac{Q_\delta^{2\delta}}{Q^2}\right)\frac{1}{r^{4(\delta-1)}}$$

- With these definitions, Einstein field equations read

(16)
$$e^{-\Lambda}\left(\frac{1}{r^2}-\frac{1}{r}\frac{d\Lambda}{dr}\right)-\frac{1}{r^2}=\frac{\kappa}{c^4}\left(-\varepsilon+\frac{Q^2\Gamma}{8\pi r^4}\right),$$

(17)
$$e^{-\Lambda}\left(\frac{1}{r^2}+\frac{1}{r}\frac{d\Phi}{dr}\right)-\frac{1}{r^2}=\frac{\kappa}{c^4}\left(-P+\frac{Q^2\Gamma}{8\pi r^4}\right),$$

(18)
$$\frac{e^{-\Lambda}}{4}\left[\left(\frac{d\Phi}{dr}\right)^2-\frac{d\Phi}{dr}\frac{d\Lambda}{dr}+2\frac{d^2\Phi}{dr^2}+\frac{2}{r}\frac{d}{dr}(\Phi-\Lambda)\right]=\frac{\kappa}{c^4}\left(P+\frac{Q^2\tilde{\Gamma}}{8\pi r^4}\right).$$

- By defining:
$$e^{-\Lambda}=1-\frac{2Gm(r)}{c^2r}-\frac{G(Q^2\Gamma)}{c^4r^2}, \qquad m(r)=4\pi\int_0^r \rho(r')r'^{\,2}\,dr',$$

one gets:
$$\frac{dm}{dr}=\frac{4\pi r^2}{c^2}\varepsilon-\frac{1}{2c^2r}\frac{d(Q^2\Gamma)}{dr}.$$

Last term is mass-energy of Electric Field !

Deriving the hydrostatic equilibrium equation

- Solving for Eqs.(16) and (17) one gets

(v1)
$$r\Lambda' = 1 - e^{-\Lambda}\left[1 + \frac{\kappa}{c^4} r^2(-\varepsilon_{eff})\right] .$$

(v2)
$$r\Phi' = -1 + e^{\Lambda}\left[1 + \frac{\kappa}{c^4} r^2 P_{eff}\right] .$$

- Deriving Eq.(v2) with respect to *r* , and then multiplying by *r*, one gets

(v3)
$$r^2\frac{d^2\Phi}{dr^2} = 1 + \frac{\kappa}{c^4} e^{\Lambda}\left[2r^2 P_{eff} + r^3\frac{dP_{eff}}{dr}\right] - e^{2\Lambda}\left(1 - \frac{\kappa}{c^4} r^2\varepsilon_{eff}\right)\left(1 + \frac{\kappa}{c^4} r^2 P_{eff}\right) .$$

- After noticing that the square term in parenthesis in Eq.(18) can be written in the form:

(v4) $$\left(\frac{d\Phi}{dr}+\frac{2}{r}\right)\left(\frac{d\Phi}{dr}-\frac{d\Lambda}{dr}\right)+2\frac{d^2\Phi}{dr^2}$$

and that

(v5) $$\frac{d\Phi}{dr}+\frac{2}{r}=\frac{1}{r}\left[1+e^{\Lambda}\left(1+\frac{\kappa}{c^4}r^2P_{eff}\right)\right],$$

- then, one arrives to

(v6) $$\frac{d\Phi}{dr}-\frac{d\Lambda}{dr}=\frac{1}{r}\left[-2+e^{\Lambda}\left(2+\frac{\kappa}{c^4}r^2[-\varepsilon_{eff}+P_{eff}]\right)\right],$$

from which Eq.(19) follows immediately, with the form :

Finally, the hydrostatic equilibrium equation determines global structure of SQS

- (19)

$$\frac{dP}{dr} = -\frac{Gm(r)}{r^2}\left(\varepsilon_{eff} + P_{eff}\right)\frac{\left(1+\frac{P_{eff}}{\varepsilon_{eff}}\right)\left(1+\frac{4\pi r^3 P_{eff}}{m(r)c^2}\right)}{\left[1-\frac{2Gm(r)}{c^2 r}-\frac{G(Q^2\Gamma)}{c^4 r^2}\right]} + 4\frac{d}{dr}\left(\tilde{P}_{eff} - P_{eff}\right) - \frac{1}{8\pi}\frac{d}{dr}\left[\frac{(Q^2\Gamma)}{r^4}\right],$$

(20)

$$P_{eff} = P - \frac{(Q^2\Gamma)}{8\pi r^4}, \quad \text{with } \frac{E^2_{\rm NLED}}{8\pi} \equiv \frac{(Q^2\Gamma)}{8\pi r^4},$$

(21)

$$\tilde{P}_{eff} = P - \frac{(Q^2\tilde{\Gamma})}{8\pi r^4}, \quad \text{with } \frac{\tilde{E}^2_{\rm NLED}}{8\pi} \equiv \frac{(Q^2\tilde{\Gamma})}{8\pi r^4},$$

Where:

$$\varepsilon_{eff} = \varepsilon - \frac{(Q^2\Gamma)}{8\pi r^4}.$$

Electric charge distribution in SQS
[Negreiros et al. P.R.D 80, 083006 (2009)]
(It is still a matter of discussion in our set up)

- Gaussian centralized at SQS surface

  b: charge constant = gaussian width (2

  r_g: radial distance of centralized distribution

  sigma: normalization constant =~ q (charge)

  (related to total electric charge in Minkowski space

  but not for system where gravity is relevant! <-----> Q= Q(g_{\mu \nu})!)

$$\rho_{ch}(r) = k \exp\left[-\frac{(r - r_g)^2}{b^2}\right]$$

$$4\pi \int_{-\infty}^{+\infty} \rho_{ch}(r) r^2 dr = \sigma \,,$$

- **Normalization condition ----> (24)**

$$k = \frac{1}{8\pi} \frac{\sigma}{\frac{\sqrt{\pi} b^3}{4} + r_g b^2 + \frac{\sqrt{\pi} r_g^2 b}{2}} \,.$$

Fiducial SQS parameters [from Negreiros et al. P.R.D 80, 083006 (2009)]

- A wide parameter space is to be analyzed

TABLE I: Properties of electrically charged strange quark stars taken from Ref.[14].

σ	R(km)	M ($M_\odot$)	$Q(\times 10^{17}$C)	$E(\times 10^{19}$V/cm)
0	10.99	2.02	0	0
500	11.1	2.07	989	7.1
750	11.2	2.15	1486	10.5
10^3	11.4	2.25	1982	13.5

- $m(r = 0) = 0$, implying that the gravitational mass vanishes at the origin.
- $P(r = R) = 0$, which define the surface of the system at the boundary $r = R$, or simply, the radius of the matter distribution.

Boundary conditions for numerically solving for M vs. R and P vs. n_mass

- Epsilon(r = 0) = Epsilon_c , gives density at SQS center
- Q_\delta (r = 0) = 0, ----> Electric charge vanishes at the center !
- M(r = 0) = 0 , ----> Gravitational mass vanishes at the center
- P(r = R) = 0 , ----> surface of SQS <----> Defining radius of SQS !

Thanks all of you for your Attention

Other NLED Lagrangians

- And

$$L_\mu = -X - \frac{\mu^8}{X}, \qquad L_\mu = -X + \frac{(-\mu^8)}{\sqrt{X^2 + \beta^2}},$$

$$Y = \frac{1}{4} F_{\mu\nu} {}^*\bar{F}^{\mu\nu} \qquad {}^*F^{\mu\nu} = \frac{1}{2\sqrt{-g}} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

$$B_{\text{bag}} = -\frac{1}{2}\frac{\partial}{\partial r}(\Psi\bar{\Psi})$$