# NON-RADIAL OSCILLATIONS OF COLOR SUPERCONDUCTING SELF-BOUND QUARK STARS.

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CSQCDIII Guarujá, Brazil
December 12 – 15, 2012

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## 1) INTRODUCTION

**Compact stars** (without rotation) can oscillate in different forms:

**Radial oscillations:** They are useful to study the stability of compact stars when perturbed.

**Non-radial oscillations:** They are sources of gravitational radiation and their observation is a very important tool to study the **internal composition of compact stars**.

## FAMILIES OF NON-RADIAL MODES

**f-modes:** frequency is proportional to the square root of the mean density of the star.

**p-modes**: pressure is the restoring force, frequencies are greater than those of the f-modes.

**g-modes:** buoyancy is the restoring force, frequencies are lower than those of the f-modes. They are present when there exist temperature gradients or density discontinuities.

**w-modes:** they are pure gravitational modes, fluid is not perturbed.

## How we calculate the f and p modes?

1) Choose an EoS (hadronic/quarks).

2) Construct an equilibrium model by solving the TOV equations

=> pressure, energy density profiles, etc: $p(r)$, $\rho(r)$,…

3) Solve the equations for the perturbed model (oscillation equations).

## 2) EQUATIONS OF STATE

# Hadronic matter

We use the relativistic field model to describe hadronic matter. We adopt the following Lagrangian [Glendenning & Moszkowski 1991]

$$
\begin{aligned}
\mathcal{L}_H = & \sum_B \bar{\psi}_B[\gamma_\mu(i\partial^\mu - g_{\omega B}\omega^\mu - \frac{1}{2}g_{\rho B}\vec{\tau}.\vec{\rho}^\mu) \\
& - (m_B - g_{\sigma B}\sigma)]\psi_B + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\
& - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{\rho}_{\mu\nu}.\vec{\rho}^{\mu\nu} \\
& + \frac{1}{2}m_\rho^2\vec{\rho}_\mu.\vec{\rho}^\mu - \frac{1}{3}bm_n(g_\sigma\sigma)^3 - \frac{1}{4}c(g_\sigma\sigma)^4 \\
& + \sum_L \bar{\psi}_L[i\gamma_\mu\partial^\mu - m_L]\psi_L,
\end{aligned}
$$

For matter composed by baryons, mesons, and leptons. For more details see [Lugones et al. 2010].

We use two different set of parameters shown in the following table [Glendenning & Moszkowski 1991, Lalazissis 1997]:

| Set | GM1 | NL3 |
|---|---|---|
| $m_\sigma$ (MeV) | 512 | 508.194 |
| $m_\omega$ (MeV) | 783 | 782.501 |
| $m_\rho$ (MeV) | 770 | 763 |
| $g_\sigma$ | 8.91 | 10.217 |
| $g_\omega$ | 10.61 | 12.868 |
| $g_\rho$ | 8.196 | 8.948 |
| $b$ | 0.002947 | 0.002055 |
| $c$ | -0.001070 | -0.002651 |
| $M_{max}$ | 2.32 | 2.73 |

**Strange Quark matter**

We use Witten hypothesis.

# Bag model and QCD corrections

We use the modified bag model [Alford et al. 2005, Weissenborn et al. 2011]

$$\Omega_{\rm QM} = \sum_{i=u,d,s,e} \Omega_i + \frac{3\mu^4}{4\pi^2}(1 - a_4) + B_{\rm eff}$$

Where $B_{eff}$ is the bag constant and the $a_4$ term is related to corrections from QCD.

$a_4 = 1$ corresponds to no QCD corrections.

Small values of $a_4 < 1$, corresponds to stronger corrections.

# Color Flavor Locked phase (Color superconductivity)

**There are Cooper pairs between quarks of different flavor and color.**

We use the bag model [G. Lugones & J. Horvath 2002 ]

$$\Omega_{\mathrm{CFL}} = \Omega_{\mathrm{free}} - \frac{3}{\pi^2}\Delta^2\mu^2 + B$$

$\Omega_{\mathrm{free}}$ includes the potential for free quarks: up, down, and strange.

Where **Δ** is the superconducting gap, and **B** is the bag constant.

**How we set the parameters ?**

1) We require non-strange quark matter to have binding energy per baryon higher than that of the most stable atomic nucleus $^{56}$Fe (Farhi & Jaffe 1984).

$$E = \left(\frac{\varepsilon}{n_B}\right)_{\mathrm{u,\,d}} \geq m_n = 939\mathrm{MeV}$$

2) We also implement the strange matter hypothesis (Bodmer 1971, Witten 1984):

$$E = \left(\frac{\varepsilon}{n_B}\right)_{\mathrm{SQM}} \leq m_n = 939\mathrm{MeV}$$

**We use set of parameters that allow masses greater than 2 solar masses in the light of recent observations. [Demorest et al. 2010]**

**By using the Lagrangian of hadronic matter and the potentials for quark matter we can obtain the equation of state: P=P(ρ)**

## 3) EQUILIBRIUM MODEL

We consider the following background metric:

$$ds^2 = -e^{\Phi(r)}dt^2 + e^{\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

and the stress-energy tensor of a perfect fluid:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

We obtain the TOV equations from Einstein's equations :

$$G_{\mu\nu} = T_{\mu\nu} \quad \xrightarrow{\; p = p(\rho) \;}$$

$$\frac{dp}{dr} = -\frac{\rho m}{r^2}\left(1 + \frac{p}{\rho}\right)\left(1 + \frac{4\pi p r^3}{m}\right)\left(1 - \frac{2m}{r}\right)^{-1}$$

$$\frac{d\Phi}{dr} = -\frac{2}{\rho}\frac{dp}{dr}\left(1 + \frac{p}{\rho}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

**From TOVs we can obtain m(r), p(r), and other necessary quantities to solve the perturbed model.**

## 4) PERTURBED MODEL

We consider the perturbed metric $g_{\mu\nu} = g^{(B)}_{\mu\nu} + h_{\mu\nu}$

$$h_{\mu\nu} = \begin{pmatrix} r^l \hat{H} e^{2\Phi} & i\omega r^{l+1}\hat{H}_1 & 0 & 0 \\ i\omega r^{l+1}\hat{H}_1 & r^l \hat{H} e^{2\Lambda} & 0 & 0 \\ 0 & 0 & r^{l+2}\hat{K} & 0 \\ 0 & 0 & 0 & r^{l+2}\hat{K}\sin^2\theta \end{pmatrix} Y^l_m \, e^{i\omega t}$$

And obtain the perturbed Einstein's tensor: $G_{\mu\nu} = \overset{(B)}{G}_{\mu\nu} + \delta G_{\mu\nu}$

We also consider perturbations in the fluid

$$\begin{aligned} \xi^r &= \frac{r^l}{r} e^{\Lambda} \hat{W} Y^l_m \, e^{i\omega t}, \\ \xi^\theta &= -\frac{r^l}{r^2} e^{\Lambda} \hat{V} \frac{\partial}{\partial\theta} Y^l_m \, e^{i\omega t}, \\ \xi^\phi &= -\frac{r^l}{r^2 \sin^2\theta} e^{\Lambda} \hat{V} \frac{\partial}{\partial\phi} Y^l_m \, e^{i\omega t} \end{aligned}$$

And obtain the perturbed stress-energy tensor: $T_{\mu\nu} = \overset{(B)}{T}_{\mu\nu} + \delta T_{\mu\nu}$

We put the perturbed quantities into the Einstein's equations and obtain the perturbed equations:

$$G_{\mu\nu} = \overset{(B)}{G}_{\mu\nu} + \delta G_{\mu\nu}$$

$$T_{\mu\nu} = \overset{(B)}{T}_{\mu\nu} + \delta T_{\mu\nu}$$

$$\Rightarrow \quad G_{\mu\nu} = T_{\mu\nu} \quad \Rightarrow \quad \delta G_{\mu\nu} = \delta T_{\mu\nu}$$

**We will consider the Cowling approximation: If the oscillations are present near the surface, the gravitational field is weakly perturbed, and we can set to zero the metric perturbations [MacDermott 1983].**

Then, we can obtain the oscillation equations [Sotani et al. 2011]

$$W' = \frac{d\rho}{dP}\left[\omega^2 r^2 e^{\Lambda - 2\Phi} V + \Phi' W\right] - \ell(\ell+1) e^{\Lambda} V$$

$$V' = 2\Phi' V - e^{\Lambda}\frac{W}{r^2}.$$

The coefficients are determined from the TOV equations (equilibrium model)

# Boundary conditions:

At the center we have the regularity conditions

$$V(r) = -Cr^{l} / l + O(r^{l+2})$$

$$W(r) = Cr^{l+1} + O(r^{l+3})$$

At the surface the lagrangian perturbation in the pressure is zero
Δp=0

$$\omega^2 r^2 e^{\Lambda - 2\Phi} V + \Phi' W = 0$$

## 5) NUMERICAL METHOD

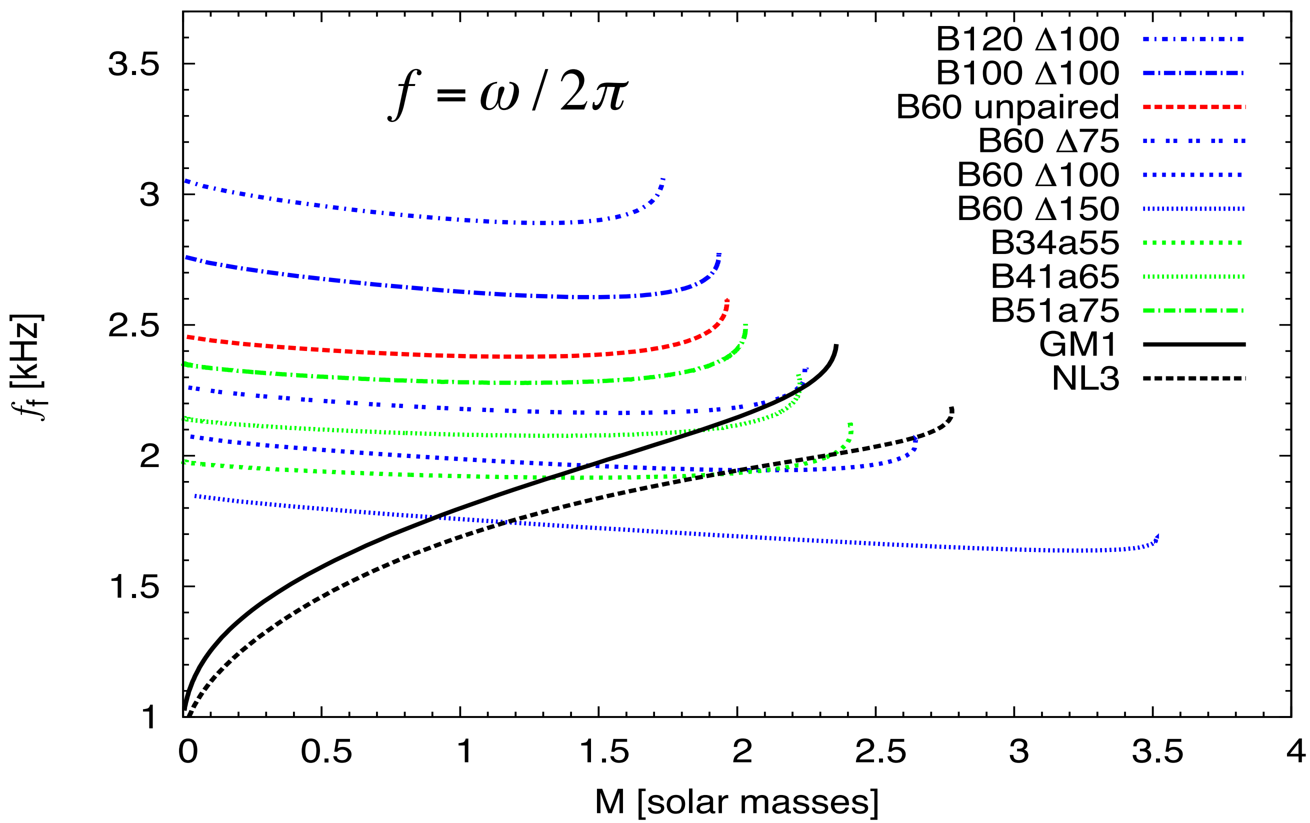
1) TOV equations are solved by a Runge Kutta method (integration until p is zero at the surface)
2) Oscillations equations are solved by a shooting method, which consists in:

a) Choose a trial frequency, use boundary conditions at r = 0.
b) Numerical integrate the Oscillations equations from the center to the surface.
c) Check if the boundary conditions at the surface are satisfied, if not, take again a new trial frequency. If yes we have calculated the frequency.

## 6) RESULTS

**f-mode (fundamental mode)**

**Blue lines for CFL stars, Green lines for Bag model with QCD Corrections, black lines for hadronic stars.**



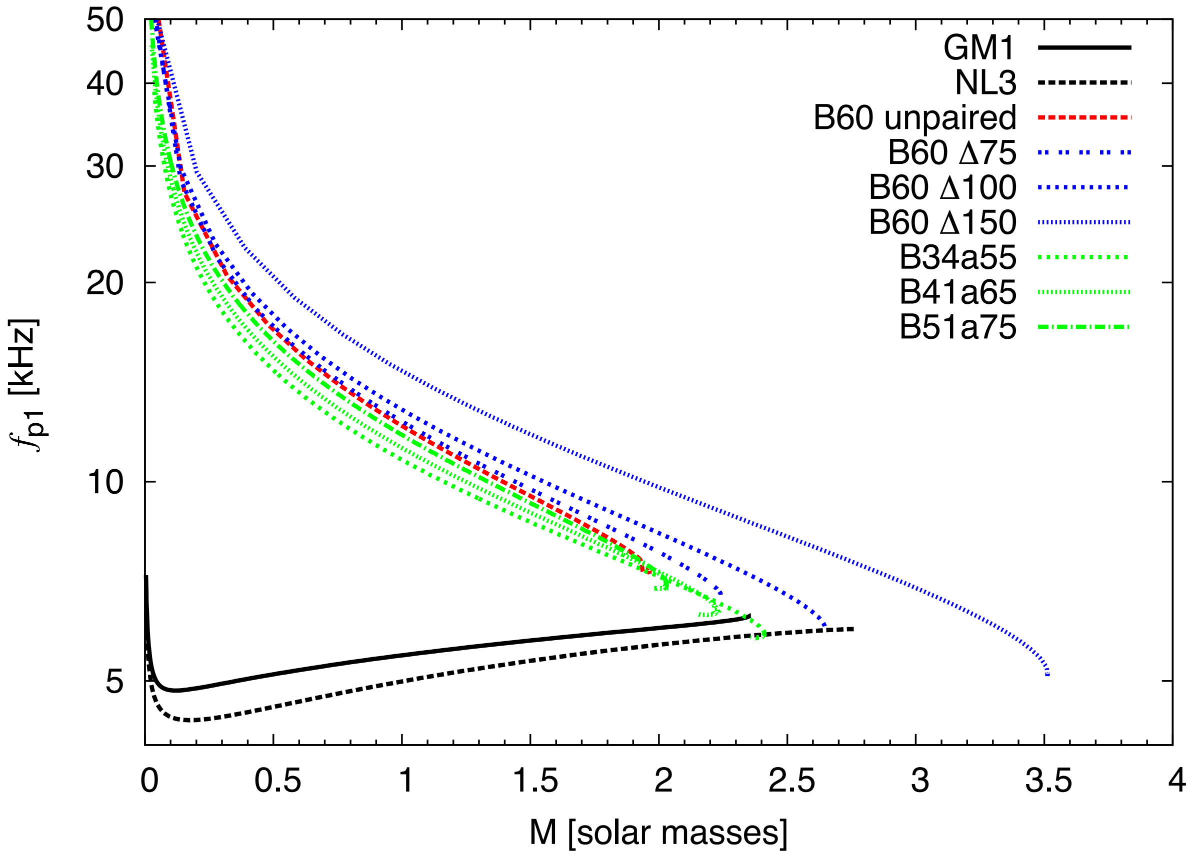

1) Large maximum masses for small B and large Δ.
2) For CFL stars $f_f$ decreases as Δ is increased.
3) CFL stars have $f_f \sim 2 – 3$ Khz.

2) Profiles for hadron stars and quark stars are qualitatively very different.

3) For hadron stars $f_f$ increases roughly linearly with the mass.

4) In contrast $f_f$ doesn't change considerably with the mass in the case of quark stars.

**p1-mode :**




1) Again there is a large difference between results for hadron and quark stars.

2) For a hadron stars the frequencies of the p1 modes are typically in the range 4 – 10 khz.

3) For quark stars frequencies  are in the same range for massive stars, but they increase significantly for low mass stars.

## 7) CONCLUSIONS

1) For quark stars the frequency of the fundamental mode has a small varitation with the mass.

2) For CFL stars $f_f$ decreases as $\Delta$ is increased.

3) We have found $f_f \sim 2 – 3$ Khz for parameters of the EoS that result stars with a maximum mass above 2 solar masses.

4) As in the case of $f_f$, the p1 modes also are very different to the corresponding modes of hadron stars.

5) Cowling approximations is a good tool to study qualitatively the effect of the EoS in the frequencies of the non-radial modes.

THANKS TO FAPESP AND UFABC FOR THE FINANCIAL SUPPORT.

# THANKS