What does the ΛΛ interaction predict for ΛΛ hypernuclei?

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December 14th, 2012

In collaboration with J. Nieves and J. Amaro.
Based on Phys. Rev. Lett. 89 032501 (2002)

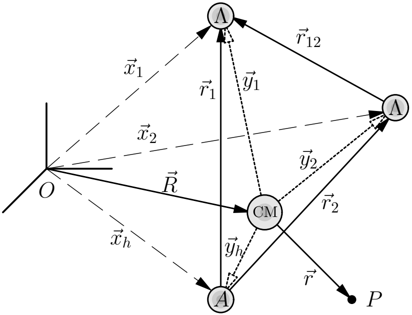
$\Lambda\Lambda$ Hypernuclei

Double lambda hypernuclei are systems of two Λ baryons bound to an atomic nuclei.

- There no exist scattering $\Lambda - \Lambda$ experiments
- Double lambda hypernuclei provide us with information on the interaction in the $S = -2$ sector

$$B_{\Lambda\Lambda} = -\left[M({}^{A+2}_{\Lambda\Lambda}Z) - M({}^{A}Z) - 2m_{\Lambda} \right]$$

Interaction model



$$H = h_{sp}(1) + h_{sp}(2) + V_{\Lambda\Lambda}(1,2) - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{M_A}$$

$$h_{sp}(i) = -\frac{\vec{\nabla}_i^2}{2\mu_A} + V_{\Lambda A}(|\vec{r}_i|)$$

Wave functions

$$\Phi(1,2) = \Psi(r_1, r_2, r_{12})\chi^{S=0}$$

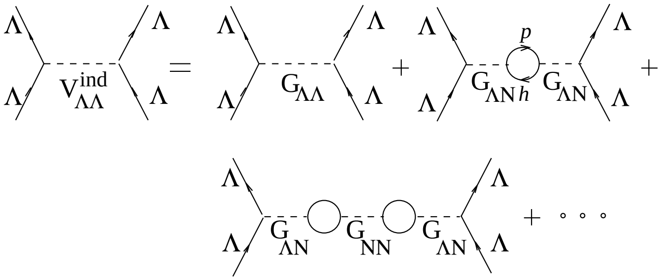
- We use the variational method: The fundamental state minimizes the expected value of the Hamiltonian

$$\delta < H >= \delta(\frac{< \Phi|H|\Phi >}{< \Phi|\Phi >}) = 0$$

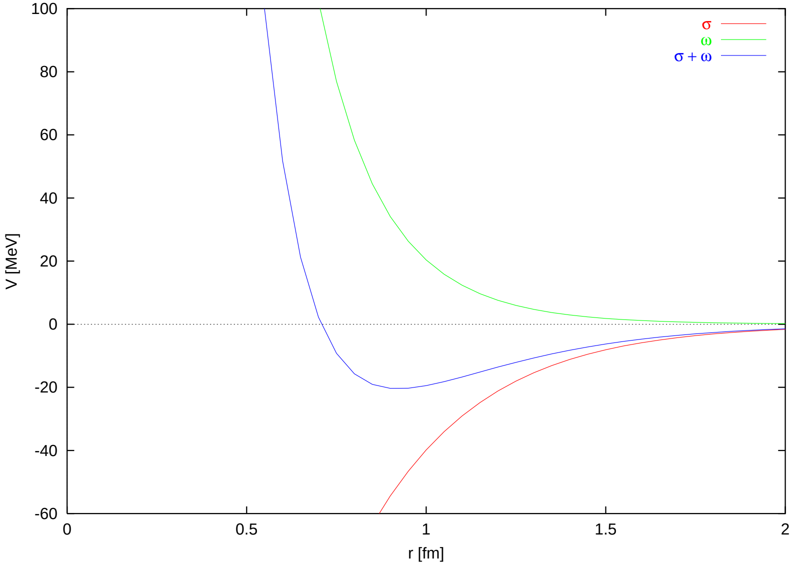
$$\Psi(r_1, r_2, r_{12}) = Nf(r_{12})\phi(r_1)\phi(r_2)$$

ΛΛ interaction

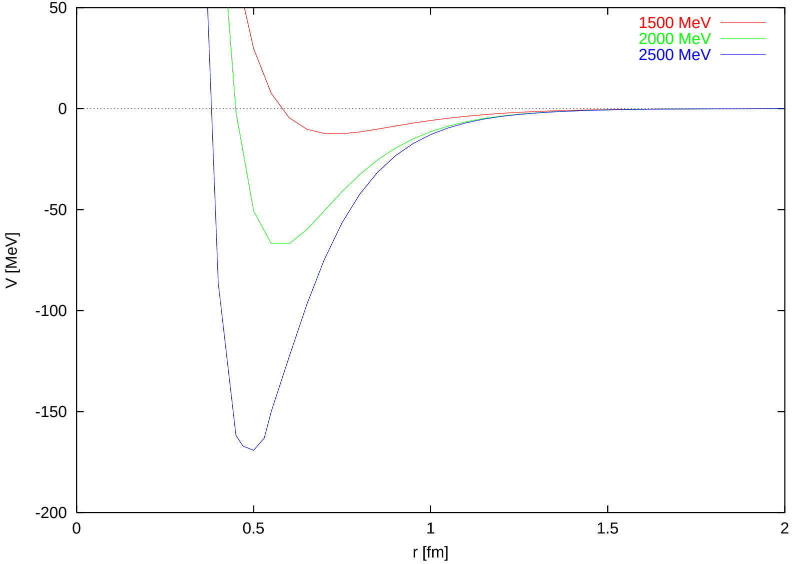
- $\Lambda\Lambda \to \Lambda\Lambda$
- $\Lambda\Lambda \to \Xi N \to \Lambda\Lambda$
- $\Lambda\Lambda \to \Sigma\Sigma \to \Lambda\Lambda$



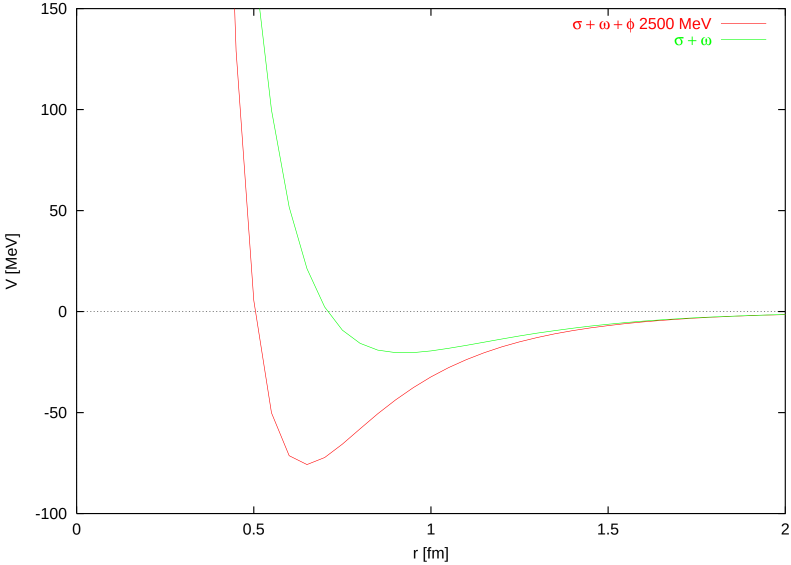
Vacuum ΛΛ interaction. σ and ω exchange.



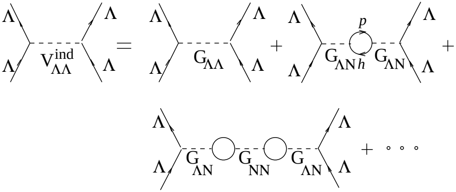
ϕ exchange.

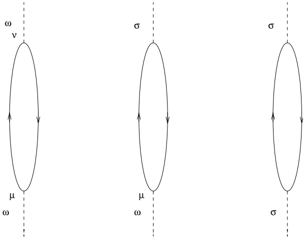


Comparison of σ, ω and ϕ exchange.



ΛΛ interaction in nuclear matter.





$$\mathcal{D}(Q) = \mathcal{D}^0(Q) + \mathcal{D}^0(Q)\Pi(Q)\mathcal{D}(Q)$$

$$\mathcal{D}^0(Q) = \left[\begin{array}{cc} D^{\omega}_{\mu\nu}(Q) & 0 \\ 0 & D^{\sigma}(Q) \end{array} \right]$$

$$\Pi = \left[\begin{array}{cc} \Pi(Q)_{\mu\nu} & \Pi(Q)_{\mu} \\ \Pi(Q)_{\nu} & \Pi(Q)_s \end{array} \right]$$

- We have approximated $G_{\Lambda N}$ y G_{NN} by the diagonal elements ΛN and NN interactions.
- We have considered $p-h$ excitations above Fermi level.
- We have worked with a non relativistic Fermi sea and we evaluate $p-h$ excitations in the static limit. The transferred four moment is $Q^{\mu}=(q^0=0,0,0,q)$

$$\Pi_{i,j} = U(0,q;\rho) C_i^N(q) C_j^N(q),\;\; i,j = 1,\ldots,5$$

$$C^B \equiv (g_{\omega BB}(q), 0, 0, 0, g_{\sigma BB}(q))$$

$$g_{\alpha BB}(q) = g_{\alpha BB}\frac{\Lambda^2_{\alpha BB} - m^2_\alpha}{\Lambda^2_{\alpha BB} + q^2},\;\; \alpha = \omega, \sigma,\;\; B = \Lambda, N$$

$$\Pi(q) = \begin{pmatrix} \Pi^{00}(|\vec{q}\,|) & 0 & 0 & 0 & \Pi^0(|\vec{q}\,|) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \Pi^0(|\vec{q}\,|) & 0 & 0 & 0 & \Pi_s(|\vec{q}\,|) \end{pmatrix} =$$

$$= U(q^0 = 0, |\vec{q}\,|) \begin{pmatrix} g^2_{\omega NN}(|\vec{q}\,|) & 0 & 0 & 0 & g_{\omega NN}(|\vec{q}\,|) g_{\sigma NN}(|\vec{q}\,|) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ g_{\omega NN}(|\vec{q}\,|) g_{\sigma NN}(|\vec{q}\,|) & 0 & 0 & 0 & g^2_{\sigma NN}(|\vec{q}\,|) \end{pmatrix}$$

$$\mathcal{D} = \left(1 - \mathcal{D}^0\Pi\right)^{-1}\mathcal{D}^0 =$$

$$= \frac{1}{\Delta}\begin{pmatrix} -\mathcal{D}^0_\omega\left(1 - \mathcal{D}^0_\sigma\Pi_s\right) & 0 & 0 & 0 & -\mathcal{D}^0_\sigma\mathcal{D}^0_\omega\Pi^0 \\ 0 & \Delta D^0_\omega & 0 & 0 & 0 \\ 0 & 0 & \Delta D^0_\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\mathcal{D}^0_\omega\mathcal{D}^0_\sigma\Pi^0 & 0 & 0 & 0 & \mathcal{D}^0_\sigma\left(1 + \mathcal{D}^0_\omega\Pi^{00}\right) \end{pmatrix}$$

$$\begin{aligned} \delta V^{RPA}_{\Lambda\Lambda}(q,\rho) &= \sum_{ij=1}^{5} C^\Lambda_i(q)\left[\mathcal{D}(Q) - \mathcal{D}^0(Q)\right]_{ij} C^\Lambda_j(q) \\ &= U(0,q;\rho)\frac{\left(W^\sigma_{\Lambda N} - W^\omega_{\Lambda N}\right)^2}{1 + U\left(W^\sigma_{NN} - W^\omega_{NN}\right)} \\ W^\alpha_{BB'} &= \frac{g_{\alpha BB}(q)g_{\alpha B'B'}(q)}{q^2 + m^2_\alpha} \end{aligned}$$

ΛΛ interaction in finite nuclei

- We asume straight line trajectories for the interactions carriers.
- We work in a local density approximation.
- We add a gap to the Lindhard function to take into account the minimun excitation energy of each nuclei.

$$\delta V_{\Lambda\Lambda}^{RPA}(1,2) = \int_0^1 d\lambda \delta V_{\Lambda\Lambda}^{RPA}(r_{12}, \rho(|\vec{r}_2 + \lambda \vec{r}_{12}|))$$

$$V_{\Lambda\Lambda}^{\mathrm{Ind}} = V_{\Lambda\Lambda}^{\mathrm{vacuum}} + \delta V_{\Lambda\Lambda}^{RPA}(1,2)$$

Λ-nucleus interaction

- Λ-nuclei interaction is fixed from single Λ hypernuclei.
- We have considered different potentials from literature.

Wave function

$$\Phi(1,2) = \Psi(r_1, r_2, r_{12})\chi^{S=0}$$
$$\Psi(r_1, r_2, r_{12}) = \Psi(r_2, r_1, r_{12})$$
$$\Psi(r_1, r_2, r_{12}) = Nf(r_{12})\phi(r_1)\phi(r_2)$$

- $f(r_{12}) \to constant$ si $r_{12} \to \infty$
- $f(r_{12}) \sim 0$ si $r_{12} \to 0$
- Must include a maximum in the proximities of the minimun of the potential.

$$f(r_{12}) = \left(1 + \frac{a_1}{1 + (\frac{r_{12}-R}{b_1})^2}\right) \prod_{i=2}^{3} \left(1 - a_i e^{-b_i^2 r_{12}^2}\right)$$

- If $V_{\Lambda\Lambda} \to 0$, then $\Psi(r_1, r_2, r_{12}) \sim \phi(r_1)\phi(r_2)$
  $\Rightarrow \phi(r)$ will be a solution for h_{sp}

$B_{\Lambda\Lambda}$ values from this model

		$B_{\Lambda\Lambda}^{\rm exp}$ no ϕ	$\Lambda_{\phi\Lambda\Lambda}$ [GeV] 1.5	2.0	2.5
$^{6}_{\Lambda\Lambda}$He	6.91 ± 0.13	6.34	6.41	6.82	7.33
$^{10}_{\Lambda\Lambda}$Be	11.9 ± 0.13	14.5	14.6	15.6	16.8
$^{13}_{\Lambda\Lambda}$B	23.30 ± 0.7	24.2	24.2	25.4	27.0
$^{42}_{\Lambda\Lambda}$Ca	–	38.3	38.2	39.1	40.1
$^{92}_{\Lambda\Lambda}$Zr	–	44.6	44.7	45.2	46.0
$^{210}_{\Lambda\Lambda}$Pb	–	53.4	53.4	53.7	54.1

Conclusions

- We have shown a method to calculate the binding energy of $\Lambda\Lambda$ hypernuclei.
- RPA series leads to an effective $\Lambda\Lambda$ potential. We have built it from free space Bonn-Jülich potentials.
- This method can be systematically improved.