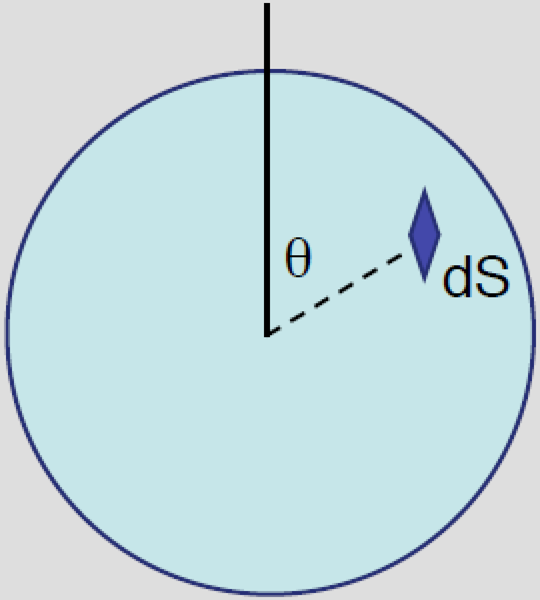
Cap4 - parte 2
RADIAÇÃO ELETROMAGNÉTICA E SUA INTERAÇÃO COM A MATÉRIA

The equation of radiative transfer

How does the intensity of radiation change in the presence of emission and / or absorption?

Definition of solid angle and steradian



Sphere radius r - area of a patch dS on the surface is:

$$dS = r d\theta \times r \sin\theta d\phi \equiv r^2 d\Omega$$

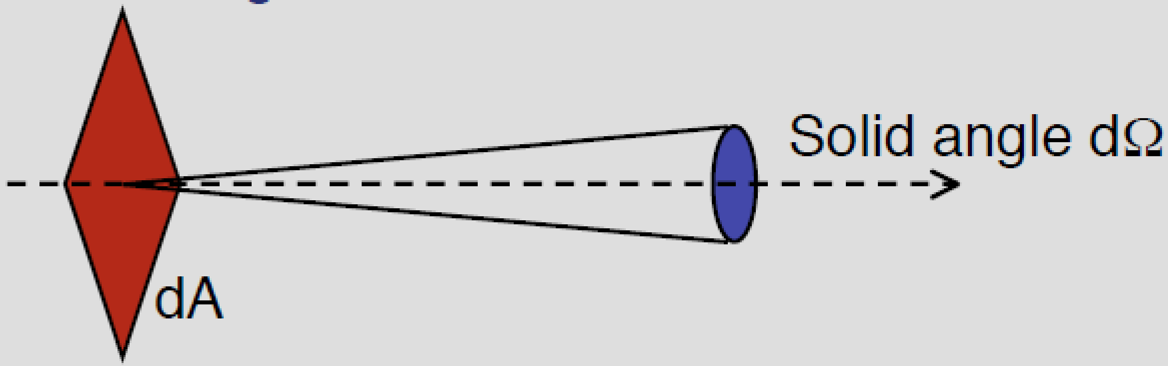
$d\Omega$ is the solid angle subtended by the area dS at the center of the sphere.

Unit of solid angle is the steradian.

4π steradians cover whole sphere.

Definition of the specific intensity

Construct an area dA normal to a light ray, and consider all the rays that pass through dA whose directions lie within a small solid angle dΩ.



The amount of energy passing through dA and into dΩ in time dt in frequency range dν is:

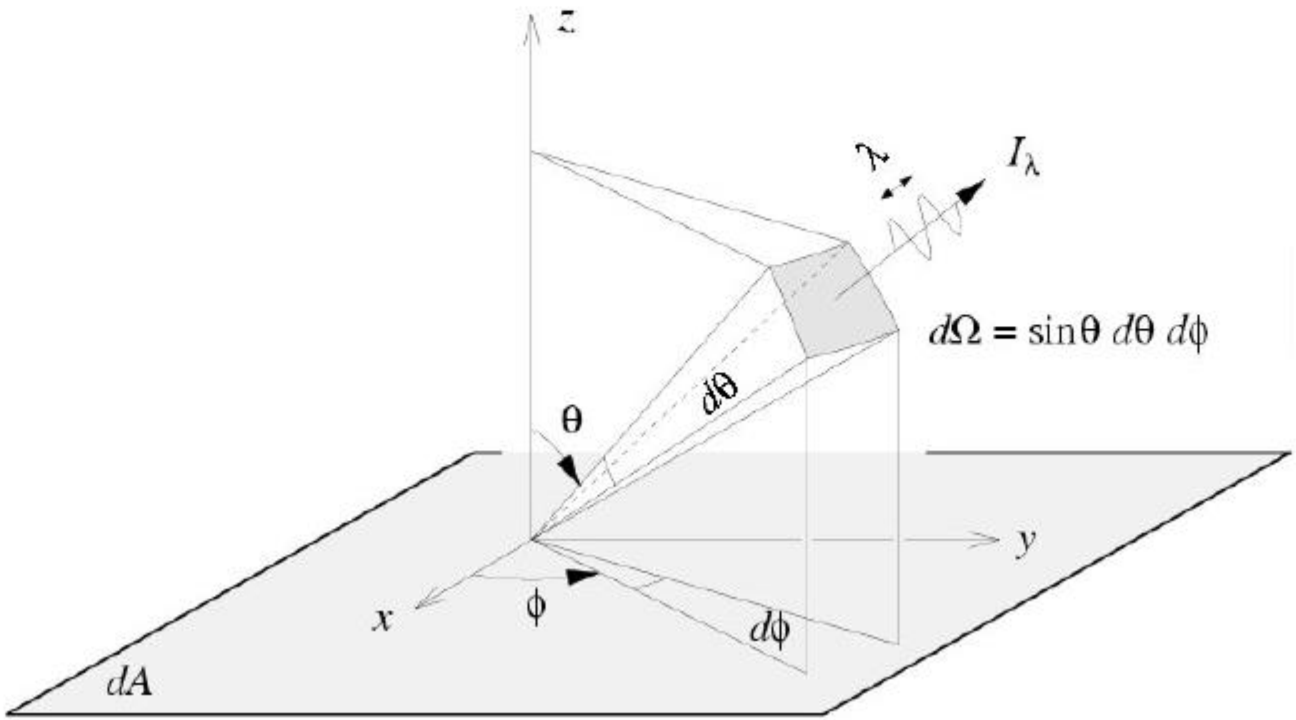
$$dE = I_\nu \, dA \, dt \, d\nu \, d\Omega$$

Specific intensity of the radiation.

Compare with definition of the flux: specific intensity is very similar except it depends upon direction and frequency as well as location.

Units of specific intensity are: $\underbrace{\text{erg s}^{-1}\ \text{cm}^{-2}\ \text{Hz}^{-1}}_{\text{Same as } F_\nu}\ \text{steradian}^{-1}$

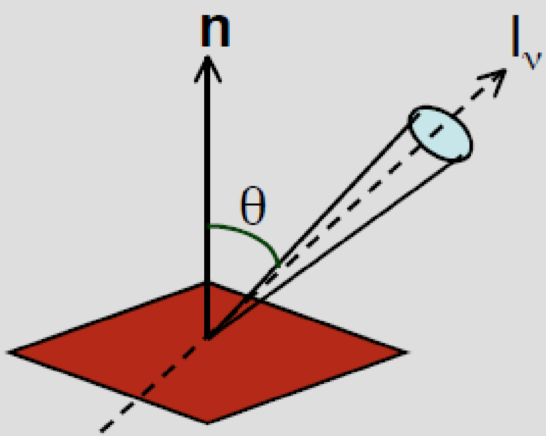
Another, more intuitive name for the specific intensity is **brightness**.



z
λ
I_λ
$d\Omega = \sin\theta \, d\theta \, d\phi$
$d\theta$
θ
y
x
ϕ
$d\phi$
dA

Simple relation between the flux and the specific intensity:

Consider a small area dA, with light rays passing through it at all angles to the normal to the surface **n**:



If $\theta = 90^o$, then light rays in that direction contribute **zero** net flux through area dA.

For rays at angle θ, foreshortening reduces the effective area by a factor of cos(θ).

Hence, net flux in the direction of **n** is given by integrating (the specific intensity x cos θ) over all solid angles:

$$F_\nu = \int I_\nu \cos\theta d\Omega$$

Note: to actually evaluate this need to express dΩ in terms of dθ and dϕ as before.

Real detectors are sensitive to a limited range of wavelengths. Need to consider how the incident radiation is distributed over frequency.

Total energy flux $$F = \int F_\nu(\nu)\, d\nu$$ Integral of F_ν over all frequencies

Units erg s^{-1} cm^{-2} Hz^{-1}

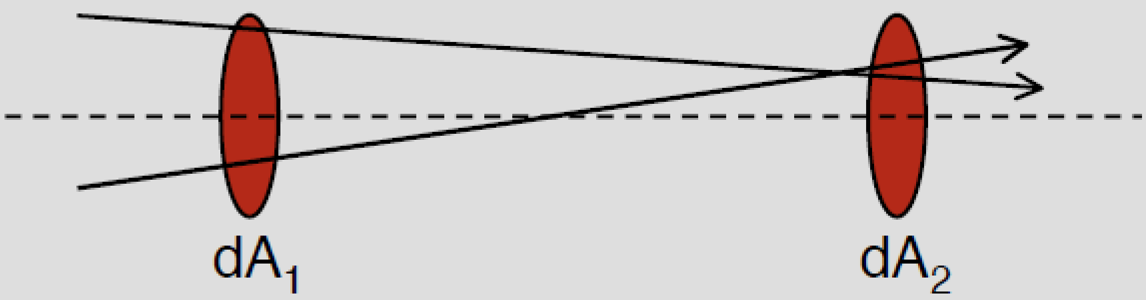
Radio astronomers use this (logical) way of measuring fluxes, though for convenience they define:

$$1 \text{ Jansky (Jy)} = 10^{-23} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$$

F_ν is often called the `flux density' - to get the power received one just multiplies by the area and by the bandwidth of the receiver (or integrates if F_ν varies significantly in that range).

How does specific intensity change along a ray

If there is no emission or absorption, specific intensity is just **constant** along the path of a light ray. Consider any two points along a ray, and construct areas dA_1 and dA_2 normal to the ray at those points. How much energy is carried by those rays that pass through **both** dA_1 and dA_2?



$$\left.\begin{aligned} dE_1 &= I_{\nu 1} dA_1 dt d\nu_1 d\Omega_1 \\ dE_2 &= I_{\nu 2} dA_2 dt d\nu_2 d\Omega_2 \end{aligned}\right\}$$

where $d\Omega_1$ is the solid angle subtended by dA_2 at dA_1 etc

The same photons pass through both dA_1 and dA_2, without change in their frequency. Conservation of energy gives:

$dE_1 = dE_2$ - equal energy

$d\nu_1 = d\nu_2$ - same frequency interval

Using definition of solid angle, if dA_1 is separated from dA_2 by distance r:

$$d\Omega_1 = \frac{dA_2}{r^2}, \quad d\Omega_2 = \frac{dA_1}{r^2}$$

Substitute:

$$I_{\nu 1} dA_1 dt d\nu_1 d\Omega_1 = I_{\nu 2} dA_2 dt d\nu_2 d\Omega_2 \qquad dE_1=dE_2$$

$$I_{\nu 1} dA_1 dt d\nu_1 \frac{dA_2}{r^2} = I_{\nu 2} dA_2 dt d\nu_2 \frac{dA_1}{r^2} \qquad d\nu_1=d\nu_2$$

$$I_{\nu 1} = I_{\nu 2}$$

Conclude: specific intensity remains the same as radiation propagates through free space.

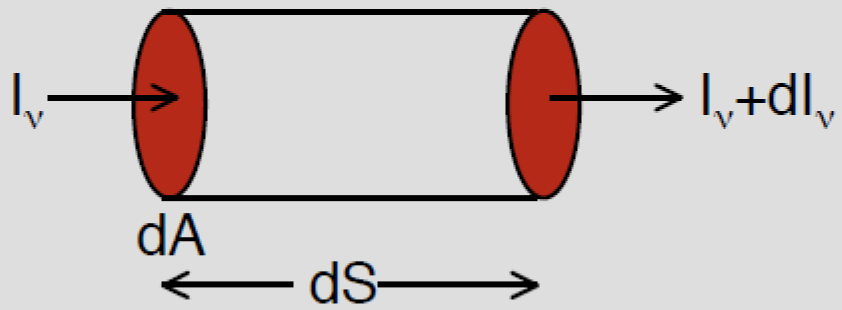
Justifies use of alternative term `brightness' - e.g. brightness of the disk of a star remains same no matter the distance - **flux** goes down but this is compensated by the light coming from a smaller area.

If we measure the distance along a ray by variable s, can express result equivalently in differential form:

$$\frac{dI_\nu}{ds} = 0$$

Emission

If the radiation travels through a medium which is itself emitting radiation, that will add to the energy:



Spontaneous **emission coefficient** is the amount of energy emitted per unit time, per unit solid angle, per unit frequency interval, and per unit volume:

$$dE = j_\nu dV d\Omega dt d\nu$$

In going a distance ds, beam of cross-section dA travels through a volume dV = dA x ds.

Change (increase) in specific intensity is therefore:

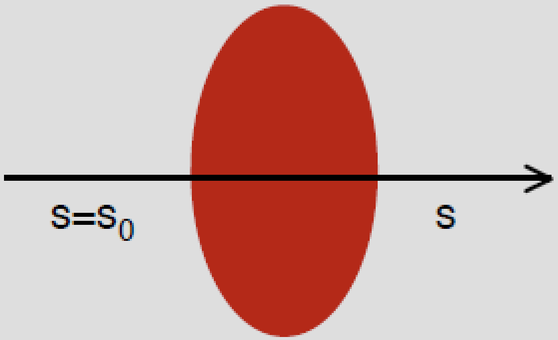
$$dI_\nu = j_\nu ds$$

Equation of radiative transfer for pure emission becomes:

$$\frac{dI_\nu}{ds} = j_\nu$$

If we know what j_ν is, can integrate this equation to find the change in specific intensity as radiation propagates through the gas:

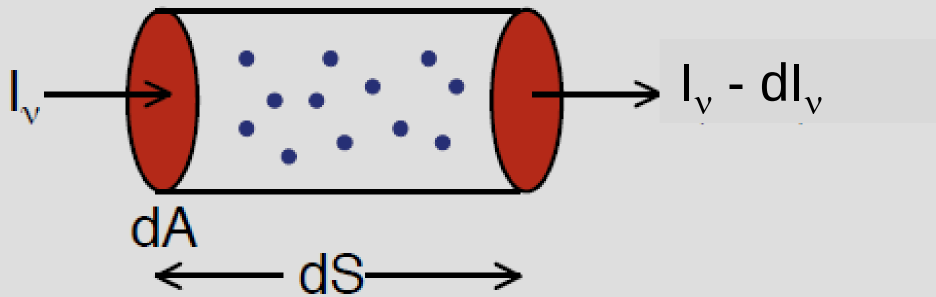
$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$$



i.e. add up the contributions to the emission all along the path.

Absorption

If the radiation travels through a medium which absorbs (or scatters) radiation, the energy in the beam will be reduced:



Number density of absorbers (particles per unit volume) = n
Each absorber has cross-sectional area = σ_ν (units cm^2)

If beam travels through ds, total area of absorbers is:

$$\text{number of absorbers} \times \text{cross-section} = n dA ds \times \sigma_\nu$$

Fraction of radiation absorbed = fraction of area blocked:

$$\frac{dI_\nu}{I_\nu} = -\frac{ndAds\sigma_\nu}{dA} = -n\sigma_\nu ds$$

$$dI_\nu = -n\sigma_\nu I_\nu ds \equiv -\alpha_\nu I_\nu ds$$

absorption coefficient (units cm^{-1})

Can also write this in terms of mass:

$$\alpha_\nu \equiv \rho\kappa_\nu$$

κ_ν is called the mass absorption coefficient or the **opacity**.

Opacity has units of $cm^2\ g^{-1}$ (i.e. the cross section of a gram of gas).

Example: Thomson scattering

A free electron has a cross section to radiation given by the Thomson value:

$$\sigma_\nu^T = 6.7 \times 10^{-25} \text{ cm}^2$$

…independent of frequency. The opacity is therefore:

$$\kappa_\nu = \frac{n}{\rho}\sigma_\nu = \underbrace{N_A \sigma_\nu = 0.4 \text{ cm}^2}_{\text{If the gas is pure hydrogen (protons and electrons only)}} \text{ g}^{-1}$$

If the gas is pure hydrogen
(protons and electrons only)

(note: really should distinguish between absorption and scattering, but don't need to worry about that here…)

Equation of radiative transfer for pure absorption. Rearrange previous equation:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

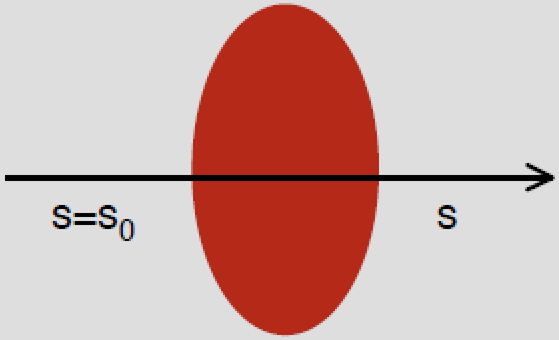
Different from emission because depends on how much radiation we already have.

Integrate to find how radiation changes along path:

$$\int_{s_0}^{s} \frac{dI_\nu}{I_\nu} = -\int_{s_0}^{s} \alpha_\nu(s')ds'$$

$$\left[\ln I_\nu\right]_{s_0}^{s} = -\int_{s_0}^{s} \alpha_\nu(s')ds'$$

$$I_\nu(s) = I_\nu(s_0)e^{-\int_{s_0}^{s} \alpha_\nu(s')ds'}$$



e.g. if the absorption coefficient is a constant (example, a uniform density gas of ionized hydrogen):

$$I_\nu(\Delta s) = I_0 e^{-\alpha_\nu \Delta s}$$

Specific intensity after distance Δs

Initial intensity

Radiation exponentially absorbed with distance

Radiative transfer equation with both absorption and emission:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

absorption

emission

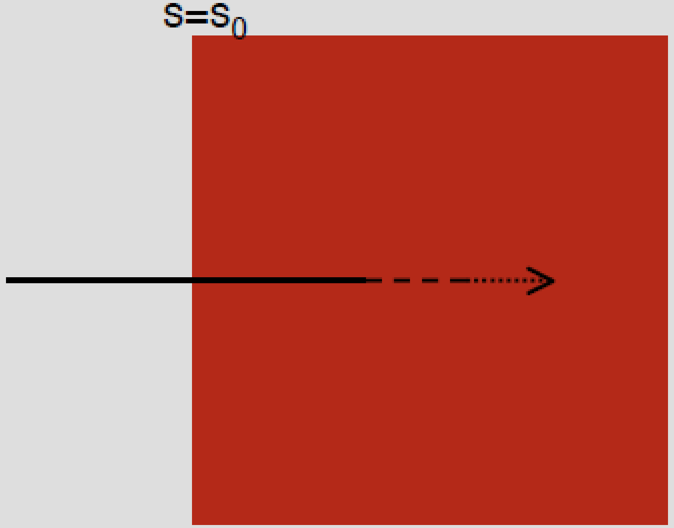
Optical depth

Look again at general solution for pure absorption:

$$I_\nu(s) = I_\nu(s_0) e^{-\int_{s_0}^{s} \alpha_\nu(s')ds'}$$

Imagine radiation traveling into a cloud of absorbing gas, exponential defines a scale over which radiation is attenuated.

When: $\int_{s_0}^{s} \alpha_\nu(s')ds' = 1$



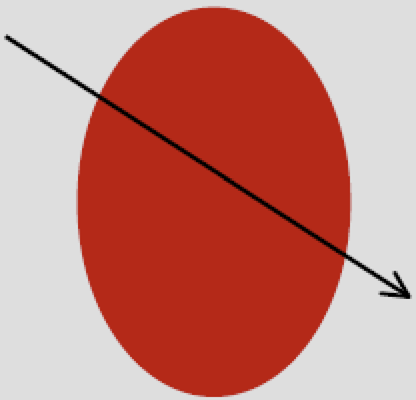
…intensity will be reduced to 1/e of its original value.

Define **optical depth** τ as:

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')ds'$$

or equivalently $d\tau_\nu = \alpha_\nu ds$

A medium is **optically thick** at a frequency ν if the optical depth for a typical path through the medium satisfies:



$$\tau_\nu \geq 1$$

Medium is said to be **optically thin** if instead:

$$\tau_\nu < 1$$

Interpretation: an optically thin medium is one which a typical photon of frequency ν can pass through without being absorbed.

Can rewrite the radiative transfer equation using the optical depth as a measure of `distance' rather than s:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{\alpha_\nu ds} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

divide by the absorption coefficient

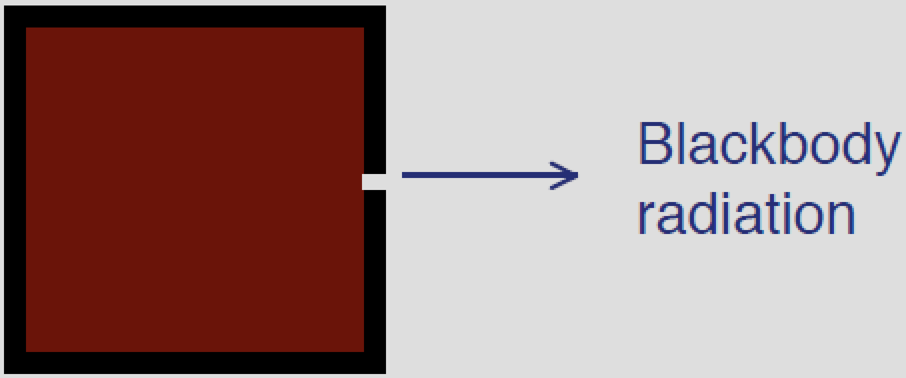
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

…where $S_\nu = j_\nu / \alpha_\nu$ is the **source function**. An alternative and sometimes more convenient way to write the equation.

Sources of radiation

Most important type of radiation is **blackbody radiation**. This is radiation that is in thermal equilibrium with matter at some temperature T.

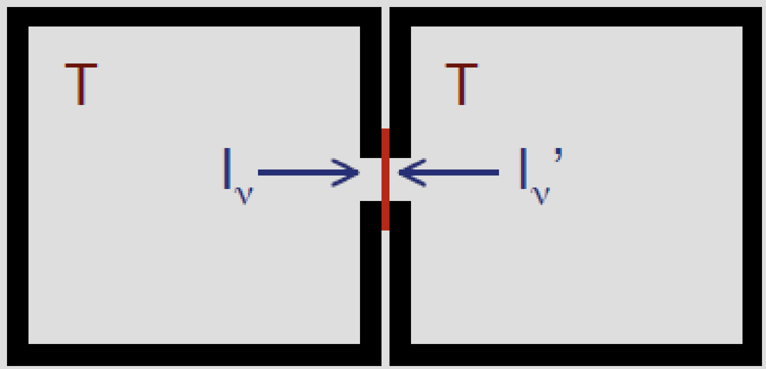
Lab source of blackbody radiation: hot oven with a small hole which does not disturb thermal equilibrium inside:



Important because:

- Interiors of stars (for example) are like this
- Emission from many objects is roughly of this form.

A general principle in physics is that **energy cannot flow spontaneously from a cold body to a hot one** (a statement of the second law of thermodynamics). Implies that the intensity of blackbody radiation can only depend on T:



Imagine two blackbody cavities at the same temperature T, separated by a filter that transmits only radiation with frequency close to ν. If:

$$I_\nu \quad I'_\nu$$

…there would be a net energy flow, in violation of the 2nd law. Hence I_ν can only depend on T.

Spectrum of blackbody radiation

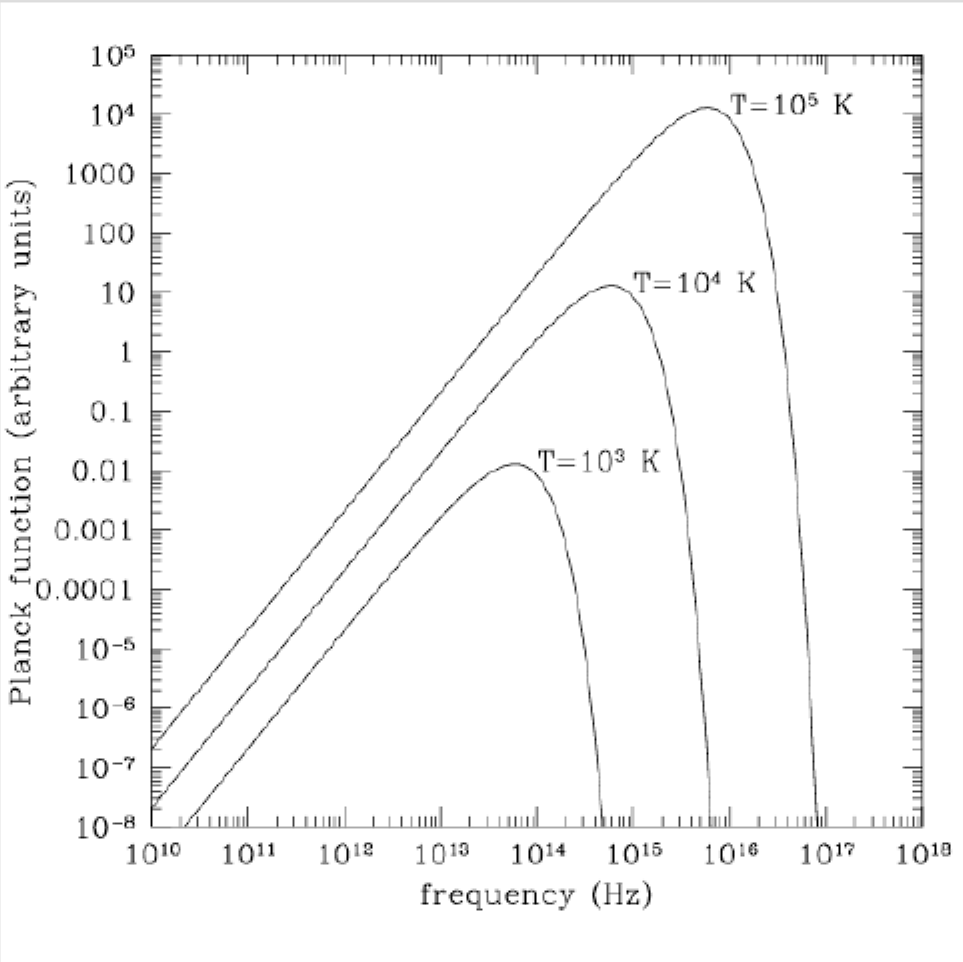
The frequency dependence of blackbody radiation is given by the **Planck function**:

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

- $h = 6.63 \times 10^{-27}$ erg s is Planck's constant
- $k = 1.38 \times 10^{-16}$ erg K^{-1} is Boltzmann's constant

Same units as specific intensity: erg s^{-1} cm^{-2} $steradian^{-1}$ Hz^{-1}

Properties of blackbody radiation



Plot $B_\nu(T)$:

- Continuous spectrum
- Increasing T increases B_ν at **all** frequencies
- Higher temperature shifts the peak to higher frequency / shorter wavelength.

Thermal radiation:

$$S_\nu = B_\nu$$

Blackbody radiation:

$$I_\nu = B_\nu$$

- **Lei de Planck** : todos os objetos opacos aquecidos emitem um espectro característico de radiação eletromagnética, e esse espectro está concentrado em comprimentos de onda maiores para os objetos mais frios.

- A tabela abaixo resume as temperaturas de corpo negro necessárias para termos picos de radiação emitida nas diferentes regiões do espectro.

Region	Wavelength (centimeters)	Energy (eV)	Blackbody Temp (K) T
Radio	> 10	$< 10^{-5}$	< 0.03
Microwave	10 - 0.01	10^{-5} - 0.01	0.03 - 30
Infrared	0.01 - 7×10^{-5}	0.01 - 2	30 - 4100
Visible	7×10^{-5} - 4×10^{-5}	2 - 3	4100 - 7300
Ultraviolet	4×10^{-5} - 10^{-7}	3 - 10^{3}	7300 - 3×10^{6}
X-Rays	10^{-7} - 10^{-9}	10^{3} - 10^{5}	3×10^{6} - 3×10^{8}
Gamma Rays	$< 10^{-9}$	$> 10^{5}$	$> 3 \times 10^{8}$

ALTA OPACIDADE E A APARIÇÃO DAS ESTRELAS:

Radiação térmica e emissão de Corpo Negro (CN)

- As regras da radiação térmica funcionam somente se o objeto é OPACO.
  - T > 0 $\rightarrow$ emissão de fotons
  - Radiação térmica é criada através da interação do foton com a matéria
  - Se os fotons não encontram resistência à propagação (ou seja, o meio é transparente), não há emissão térmica.

- Como as estrelas são **opacas** em essencialmente todos os comprimentos de onda $\Rightarrow$ elas emitem como corpos negros ideais.

- Radiação (Intensidade) de corpo negro vinda de um perfeito emissor/absorvedor é descrita pela **função de Planck:**

$$\boxed{B_\lambda(T) = \frac{2\,h\,c^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}}$$

Potência / unid. Área, largura de banda e ângulo sólido.

h = constante de Planck

k = constante de Boltzmann

$[B_\lambda] = \text{erg}\,\text{cm}^{-2}\ \text{cm}^{-1}\ \text{str}^{-1}$

- Lembre-se que : $B_\nu d\nu = -B_\lambda d\lambda, \quad d\nu = -\frac{c}{\lambda^2}d\lambda$

$B_\lambda(\lambda,\ T)\ \mathrm{d}\lambda = -B_\nu(\nu(\lambda),\ T)\ \mathrm{d}\nu$, which leads to $B_\lambda(\lambda,\ T) = -\frac{\mathrm{d}\nu}{\mathrm{d}\lambda}B_\nu(\nu(\lambda),\ T).$

Lei de Planck em função de λ e ν

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_\mathrm{B}T}} - 1}$$

or

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_\mathrm{B}T}} - 1}$$

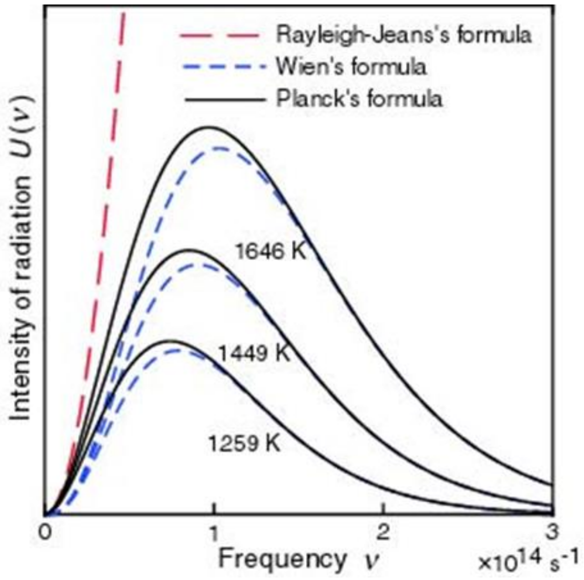
Comportamento Assimptótico

- No vermelho: domínio de Rayleigh-Jeans

$$\frac{h\nu}{kT} << 1 \qquad \exp\left(\frac{h\nu}{kT}\right) \approx 1 + \frac{h\nu}{kT}$$

$$B_\nu(\nu,T) = \frac{2k\nu^2 T}{c^2}$$

$$\boxed{B_\nu(\lambda,T) = \frac{2ckT}{\lambda^4}}$$



$$\frac{h\nu}{kT} >> 1 \qquad \exp\left(\frac{h\nu}{kT}\right) - 1 \approx \exp\left(\frac{h\nu}{kT}\right)$$

- No azul: domínio de Wien

$$B_\nu(\nu,T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{kT}\right)$$

$$\boxed{B_\nu(\lambda,T) = \frac{2hc^2}{\lambda^5}\exp\left(-\frac{hc}{\lambda kT}\right)}$$

Integração sobre frequências

$$B(T) = \int_0^\infty B_\nu(T)d\nu = \int_0^\infty \frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]^{-1} d\nu$$

$$= \frac{2k^4}{c^2h^3}T^4 \underbrace{\int_0^\infty \frac{x^3}{e^x-1}dx}_{=\pi^4/15} = \frac{2}{15}\frac{\pi^4k^4}{c^2h^3}T^4$$

$$= \frac{\sigma}{\pi}T^4 \quad \text{with} \;\; \sigma = \frac{2}{15}\frac{\pi^5k^4}{c^2h^3} = 5.669\cdot10^{-5}\,\text{erg cm}^{-2}\ \text{s}^{-1}\ \text{deg}^{-4}$$

- Stefan-Boltzmann law

- Energy density of blackbody radiation:

$$\boxed{u = \frac{4\pi}{c}\int_0^\infty J_\nu(\nu)d\nu = \frac{4\pi}{c}B(T) = \frac{4\sigma}{c}T^4}$$

- O fluxo total emitido em todas as direções em todos os comprimentos de onda por um CN é:

$$F = \int\int d\lambda\, d\omega \cos\theta\, B_\lambda(T) = 2\pi \int_0^\infty d\lambda \int_0^\pi d\theta \cos\theta \sin\theta\, B_\lambda(T) = \pi B$$

Mas,
$$B(T) = \int_0^\infty B_\lambda d\lambda = \int_0^\infty B_\nu d\nu = \frac{A}{\pi} \times T^4, \;\; A = \frac{2k^4}{c^2 h^3} \frac{\pi^5}{15}$$

Como
$$B(T) = \frac{\sigma}{\pi} T^4 \implies$$

$$\boxed{F = \sigma T^4}$$

Lei de Wien

$$\frac{d}{d\nu}B_\nu(\nu,T) = \frac{d}{d\nu}\left[\frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]^{-1}\right]$$

$x := h\nu / kT$

$$= B_\nu\left[\frac{3}{\nu}+\frac{-1}{\mathrm{e}^{\mathrm{x}}-1}\frac{x}{\nu}\mathrm{e}^{\mathrm{x}}\right]$$

$$\frac{d}{d\nu}B_\nu = 0 \rightarrow 3 - x_{\max}\,\mathrm{e}^{\mathrm{x}_{\max}}/\left(\mathrm{e}^{\mathrm{x}_{\max}}-1\right) = 0$$

$$\rightarrow x_{\max} - 3\left(1-\mathrm{e}^{-\mathrm{x}_{\max}}\right) = 0$$

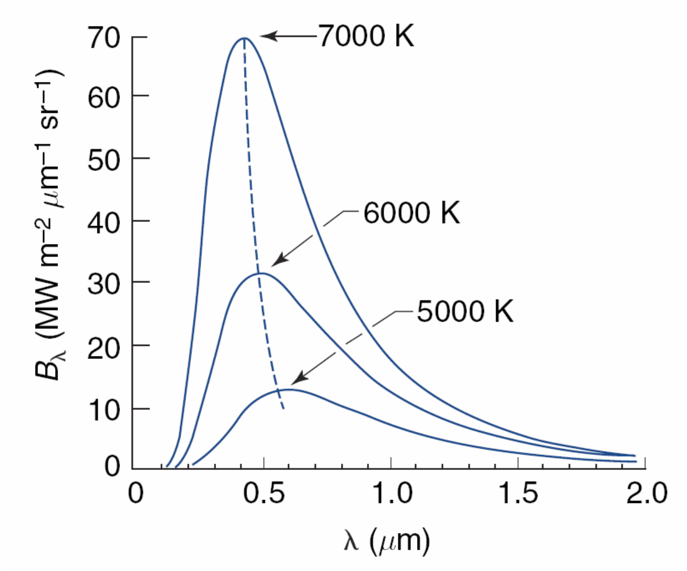
Máximo de B_ν

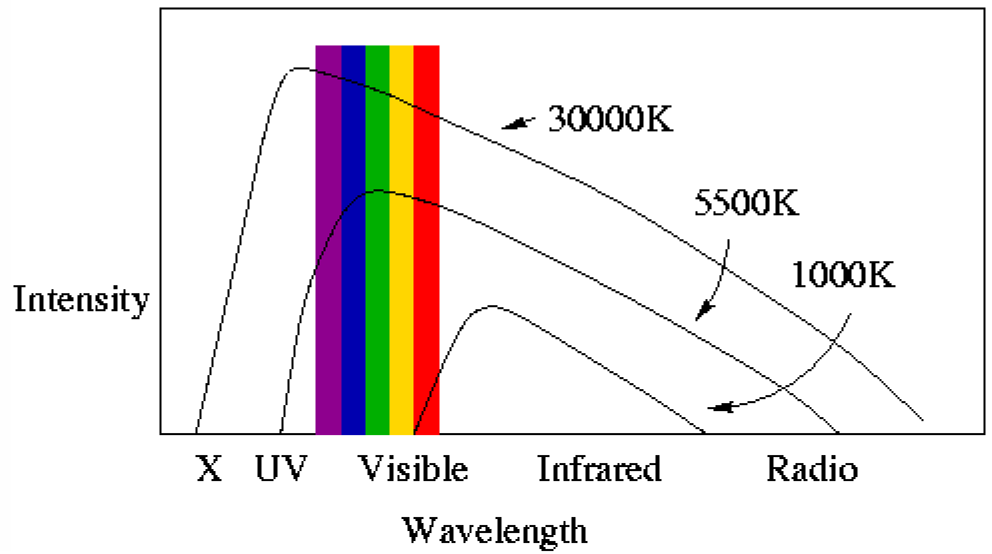
numerical solution: $\boxed{x_{\max} = 2.821 = \frac{h\nu_{\max}}{kT}}$ $\lambda_{\max}T = 0.5100\,\text{cm deg}$

$$\frac{d}{d\lambda}B_\lambda = 0 \rightarrow x_{\max} - 5\left(1-\mathrm{e}^{-\mathrm{x}_{\max}}\right) = 0$$

Máximo de B_λ

numerical solution: $\boxed{x_{\max} = 4.965 = \frac{hc}{\lambda_{\max}kT}}$ $\lambda_{\max}T = 0.2897\,\text{cm deg}$



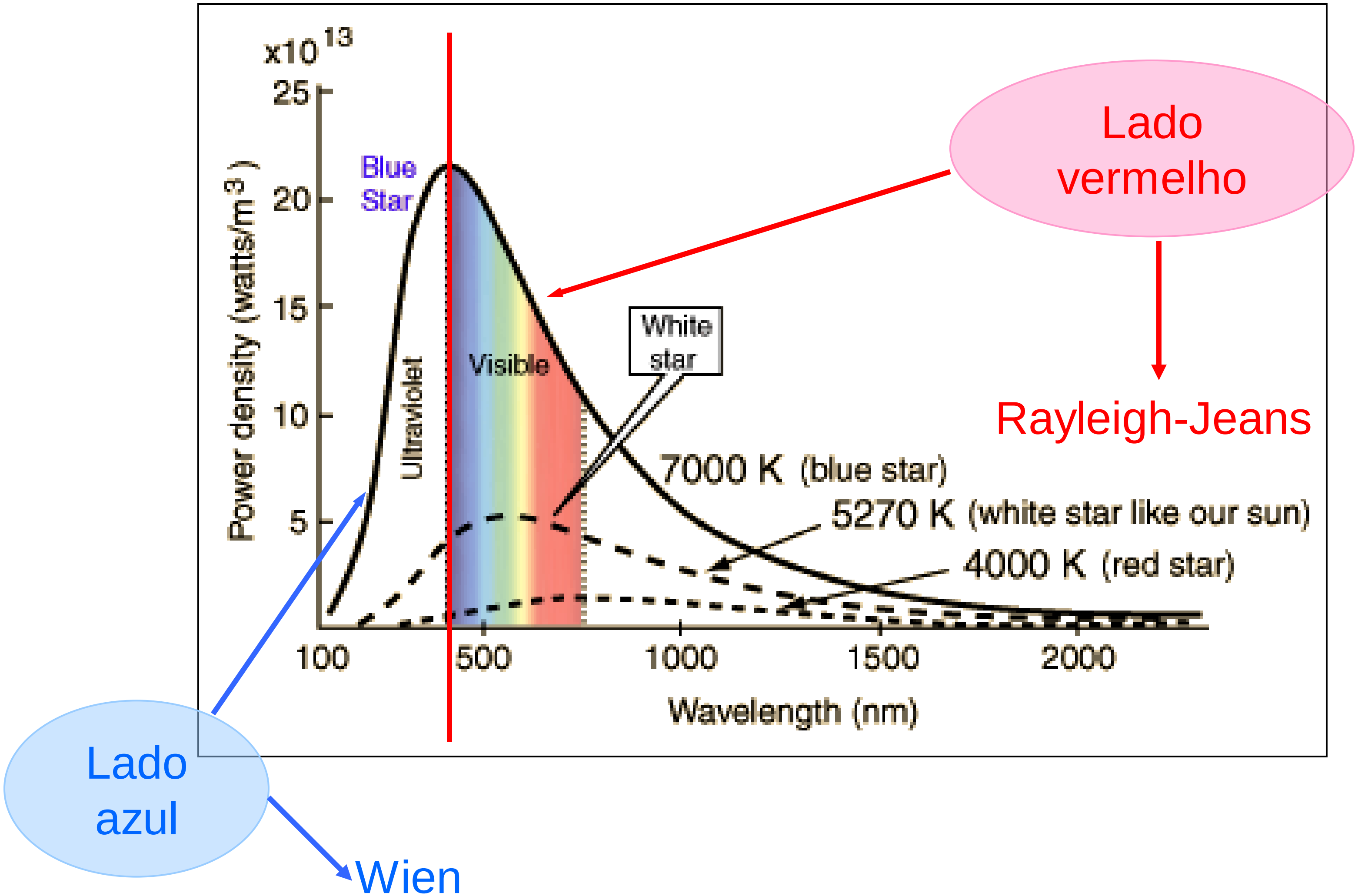


- **Lei de Wien** $\rightarrow$ B_λ (T) é máximo em:

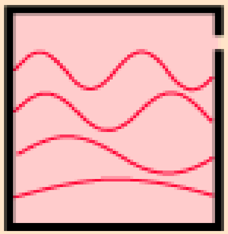
$$\boxed{\lambda_{max} = \frac{0.29}{T}}$$

[λ_{max}] = cm, [T] = K

Obs.: O máximo de uma função pode ser encontrado calculando-se a derivada primeira da função e igualando-a a zero

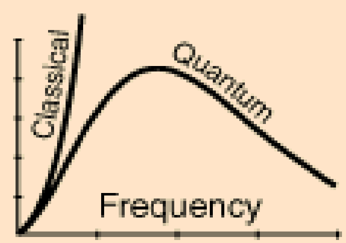

x10^13
25
20
15
10
5
Power density (watts/m³)
Blue Star
Ultraviolet
Visible
White star
7000 K (blue star)
5270 K (white star like our sun)
4000 K (red star)
100
500
1000
1500
2000
Wavelength (nm)
Lado vermelho
Rayleigh-Jeans
Lado azul
Wien

Radiation modes in a hot cavity provide a test of quantum theory

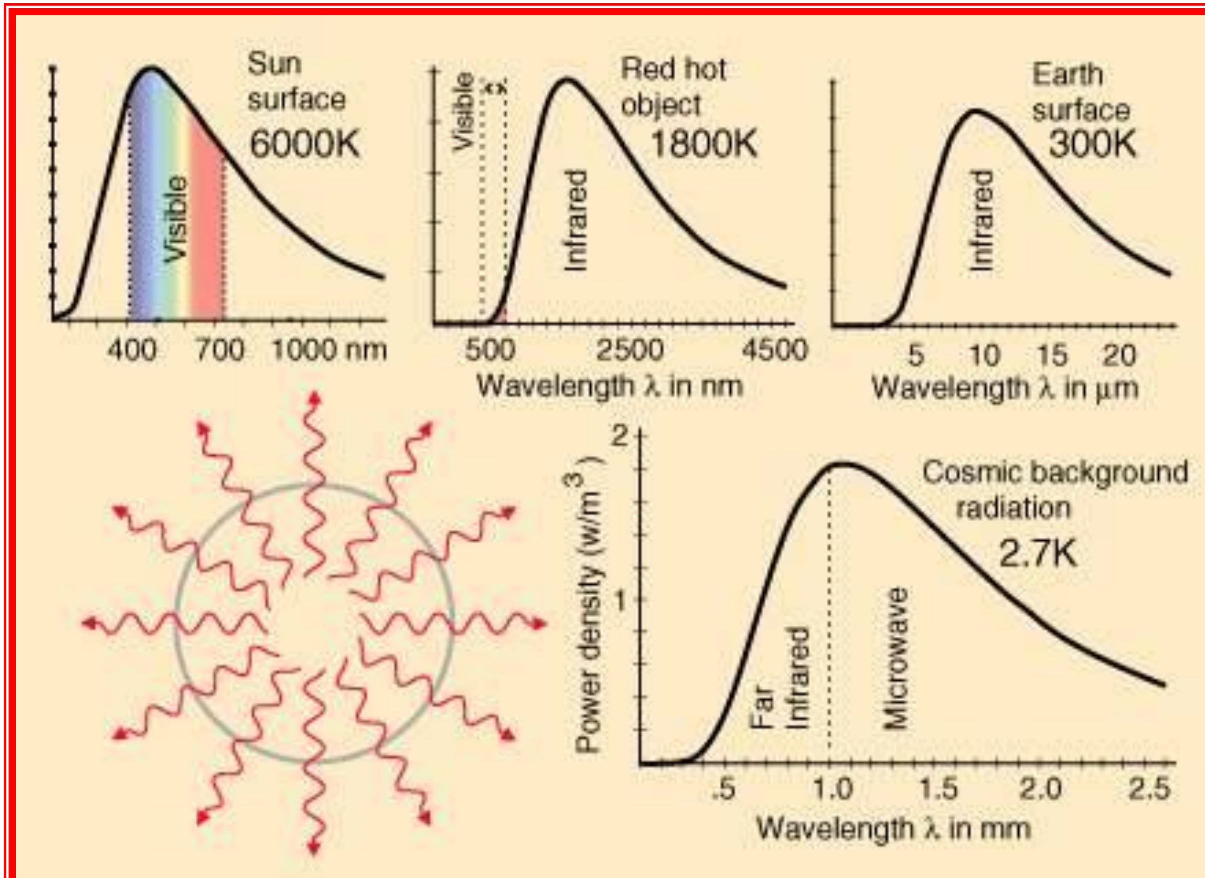


	#Modes per unit frequency per unit volume	Probability of occupying modes	Average energy per mode
CLASSICAL	$\dfrac{8\pi\nu^2}{c^3}$	Equal for all modes	kT
QUANTUM	$\dfrac{8\pi\nu^2}{c^3}$	Quantized modes: require $h\nu$ energy to excite upper modes, less probable	$\dfrac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$

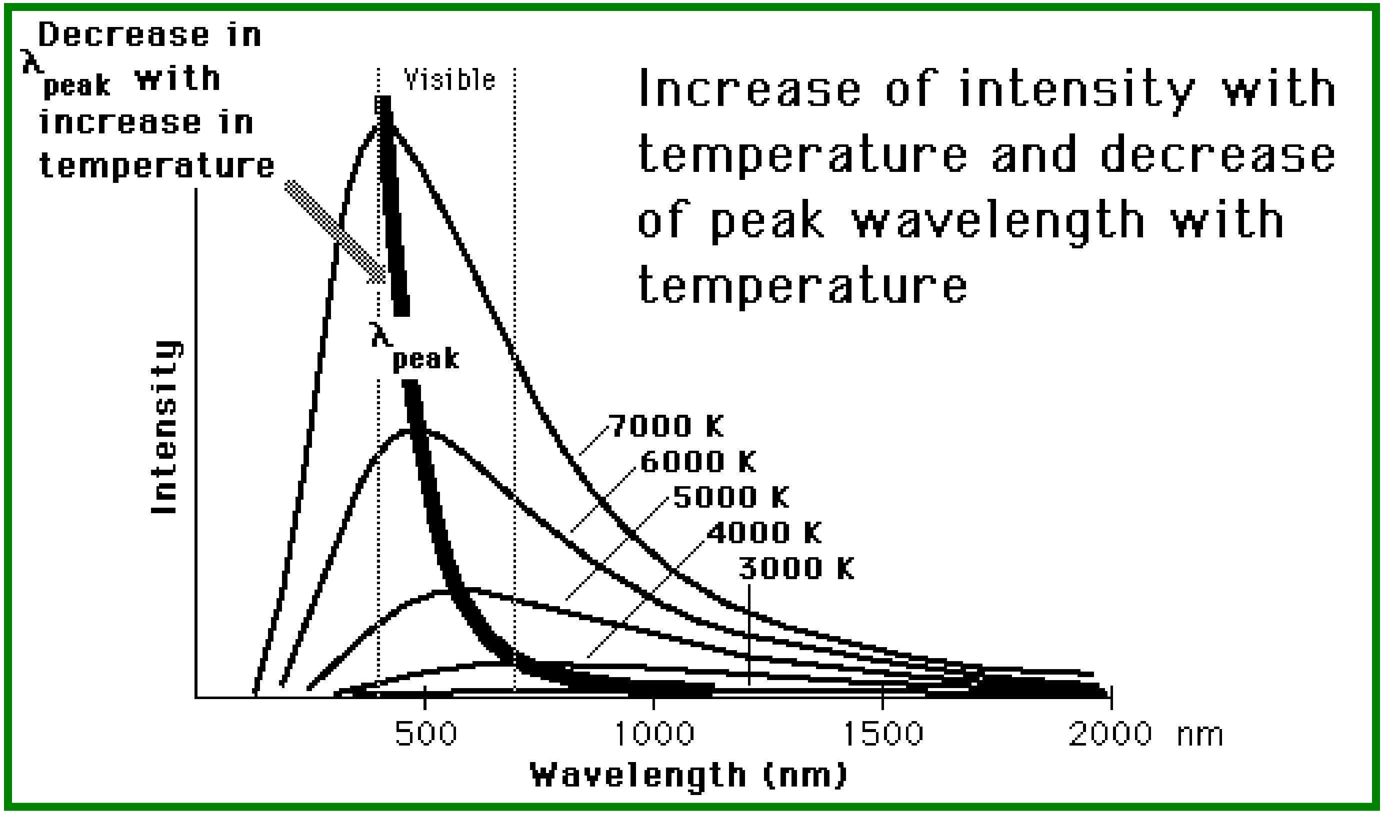
The amount of radiation emitted in a given frequency range should be proportional to the number of modes in that range. The best of classical physics suggested that all modes had an equal chance of being produced, and that the number of modes went up proportional to the square of the frequency.



But the predicted continual increase in radiated energy with frequency (dubbed the "ultraviolet catastrophe") did not happen. Nature knew better.



Sun
surface
6000K
Visible
400
700
1000 nm
Red hot
object
1800K
Visible
Infrared
500
2500
4500
Wavelength λ in nm
Earth
surface
300K
Infrared
5
10
15
20
Wavelength λ in µm
Cosmic background
radiation
2.7K
Power density (w/m³)
2
1
Far
Infrared
Microwave
.5
1.0
1.5
2.0
2.5
Wavelength λ in mm



• O λ do pico da curva de radiaçãode um CN decresce linearmente com o aumento da temperatura (lei de Wien). Essa variação linear não é evidente a partir do gráfico mostrado acima, já que a intensidade aumenta com T^4 (lei de Stefan-Boltzmann).

• A natureza da mudança de comprimento de onda do pico pode ser evidenciada graficando-se a 4ª potência da intensidade.

EXEMPLO 1

Suponha que duas estrelas vermelhas fazem parte de um sistema . A estrela A é 9 vezes mais brilhante que a estrela B. O que podemos dizer sobre seus tamanhos relativos e temperaturas?

SOLUÇÃO: Como ambas são vermelhas (mesma cor), os espectros apresentam picos no mesmo comprimento de onda. Pela lei de Wien:

$$\lambda_{peak} = \frac{2900}{T}$$

λ_{peak} in μm

T in K

e, então, elas têm a mesma temperatura.

Pela lei que rege a luminosidade, raio e temperatura de um objeto:

$$L = 4\pi R^2 \sigma T^4$$

Watts

A estrela A deve ser maior em tamanho (porque ela tem a mesma temperatura, mas é 9 vezes mais luminosa). Quão maior?

$$\frac{L_A}{L_B} = \frac{4\pi R_A^2 \sigma T_A^4}{4\pi R_B^2 \sigma T_B^4} \Rightarrow 9 = \frac{R_A^2}{R_B^2}$$

Assim, a estrela A é 3 vezes maior que a B.

$$\Rightarrow R_A = 3R_B$$

EXEMPLO 2

Suponha duas estrelas C e D que formam um par binário.

- Estrela C tem um pico de emissão em 3500 A (0.35 mm)
- Estrela D tem um pico de emissão em 7000 A (0.70 mm)

Qual a temperatura das estrelas?

SOLUÇÃO: Pela lei de Wien: $\lambda_{peak} = \frac{2900}{T}$ λ_{peak} in μm, T in K

Então,

para a estrela C, $T_C = \frac{2900}{\lambda_{peak}} = \frac{2900}{0.35} = 8300K$

e, para a estrela D, $T_D = \frac{2900}{\lambda_{peak}} = \frac{2900}{0.70} = 4150K$

Se ambas as estrelas são iguais em brilho (neste caso, elas têm mesma luminosidade porque são parte de um par que está a mesma distância, quais são os tamanhos de C e D?

$$\frac{L_C}{L_D} = \frac{4\pi R_C^2 \sigma T_C^4}{4\pi R_D^2 \sigma T_D^4} \quad \Rightarrow 1 = \frac{R_C^2}{R_D^2} \frac{8300^4}{4150^4}$$

$$\Rightarrow R_D^2 = 16 R_C^2 \quad \Rightarrow R_D = 4R_C$$

Assim, C é 4 vezes menor que D.

Resumindo:

Differentiate $B_\nu(T)$ with respect to frequency and set resulting expression to zero to find where the Planck function peaks.

$$h\nu_{\max} = 2.82kT$$

$$\nu_{\max} = 5.88 \times 10^{10} T \text{ Hz K}^{-1}$$

Wien displacement law - peak shifts linearly with increasing temperature to higher frequency.

Rayleigh-Jeans law: for low frequencies $h\nu \ll kT$:

$$B_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

Often valid in the radio part of the spectrum, at frequencies far below the peak of the Planck function.

The energy density of blackbody radiation:

$$u(T) = aT^4$$

…where a = 7.56×10^{-15} erg cm^{-3} K^{-4} is the radiation constant.

The emergent flux from a surface emitting blackbody radiation is:

$$F = \sigma T^4$$

…where $\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1} is the Stefan-Boltzmann constant.

e.g. star, radius R, temperature T, emitting as a blackbody, has a luminosity:

$$L = 4\pi R^2 \sigma T^4$$

Emission from most astronomical sources is only roughly described by the Planck function (if at all).

Source has emergent flux F (integrated over all frequencies), define the **effective temperature** T_e via:

$$F \equiv \sigma T_e^4$$

Effective temperature is the temperature of a blackbody that emits same flux. e.g. for the Sun:

$$L_{sun} = 4\pi R_{sun}^2 \sigma T_e^4$$

…find T_e = 5770 K.

Note: effective temperature is well-defined even if the spectrum is nothing like a blackbody.

Which objects have blackbody spectra?

Radiation will be blackbody radiation wherever we have matter in thermal equilibrium with radiation - i.e. at large optical depth. To show this, consider transfer equation with $S_\nu = B_\nu(T)$, and assume T is constant:

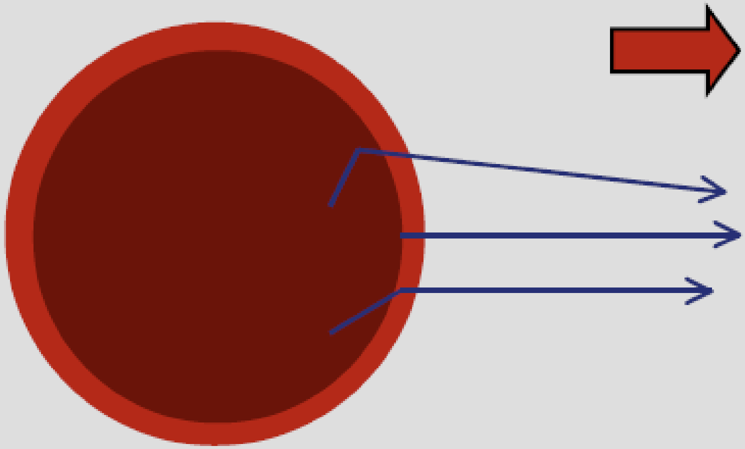
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(T)$$

Can integrate this equation, with solution:

$$I_\nu(\tau) = B_\nu + \underbrace{e^{-\tau}\left[I_0 - B_\nu\right]}_{\text{-> 0 as } \tau \text{ becomes large}}$$

where I_0 is the value of I_ν at $\tau = 0$. Conclude $I_\nu = B_\nu$ at high optical depth, e.g. in the center of a star.

Recall interpretation of optical depth: at $\tau = 1$ there is (very roughly speaking) a 50% chance that a photon headed toward us will suffer an absorption or scattering along the way.

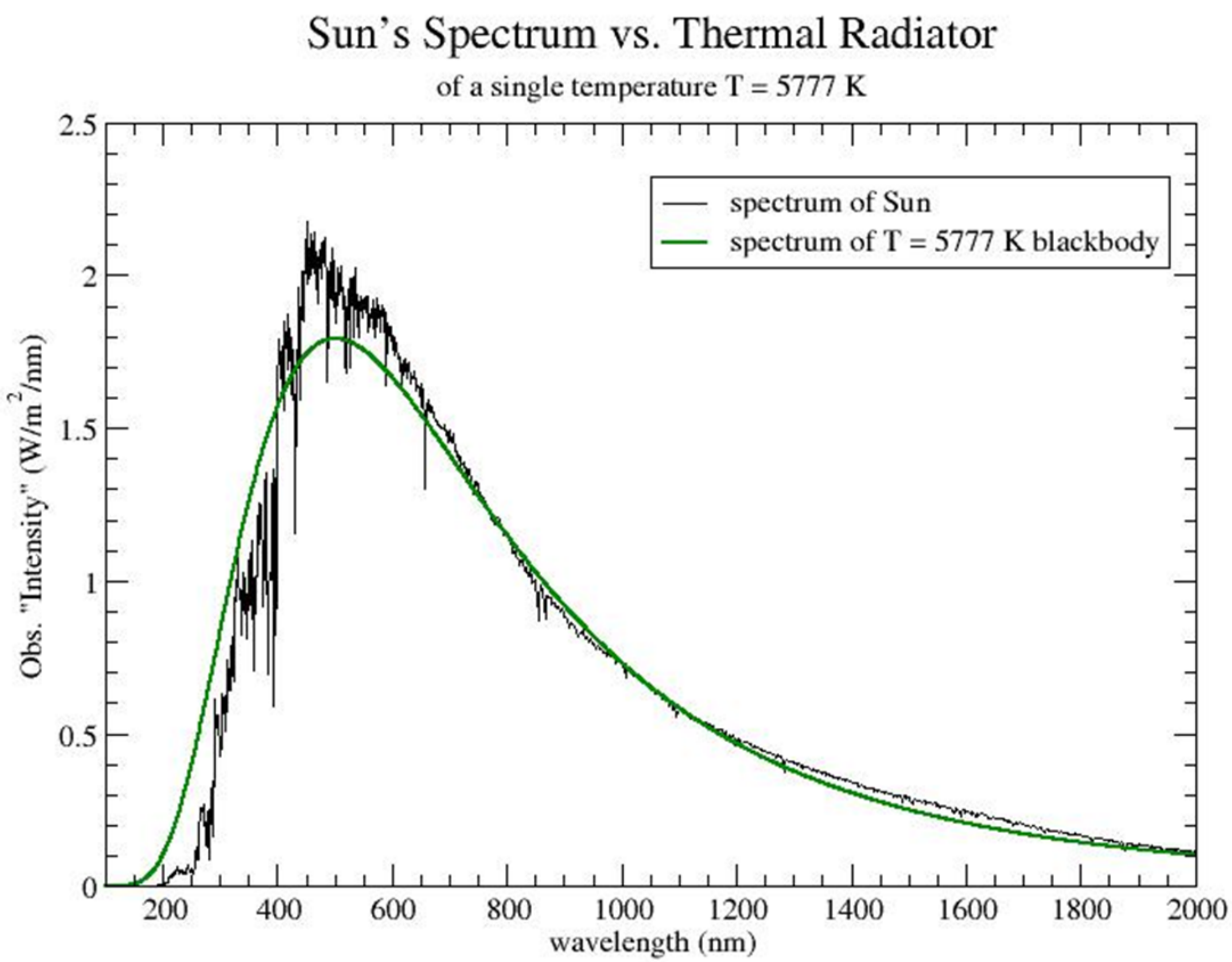


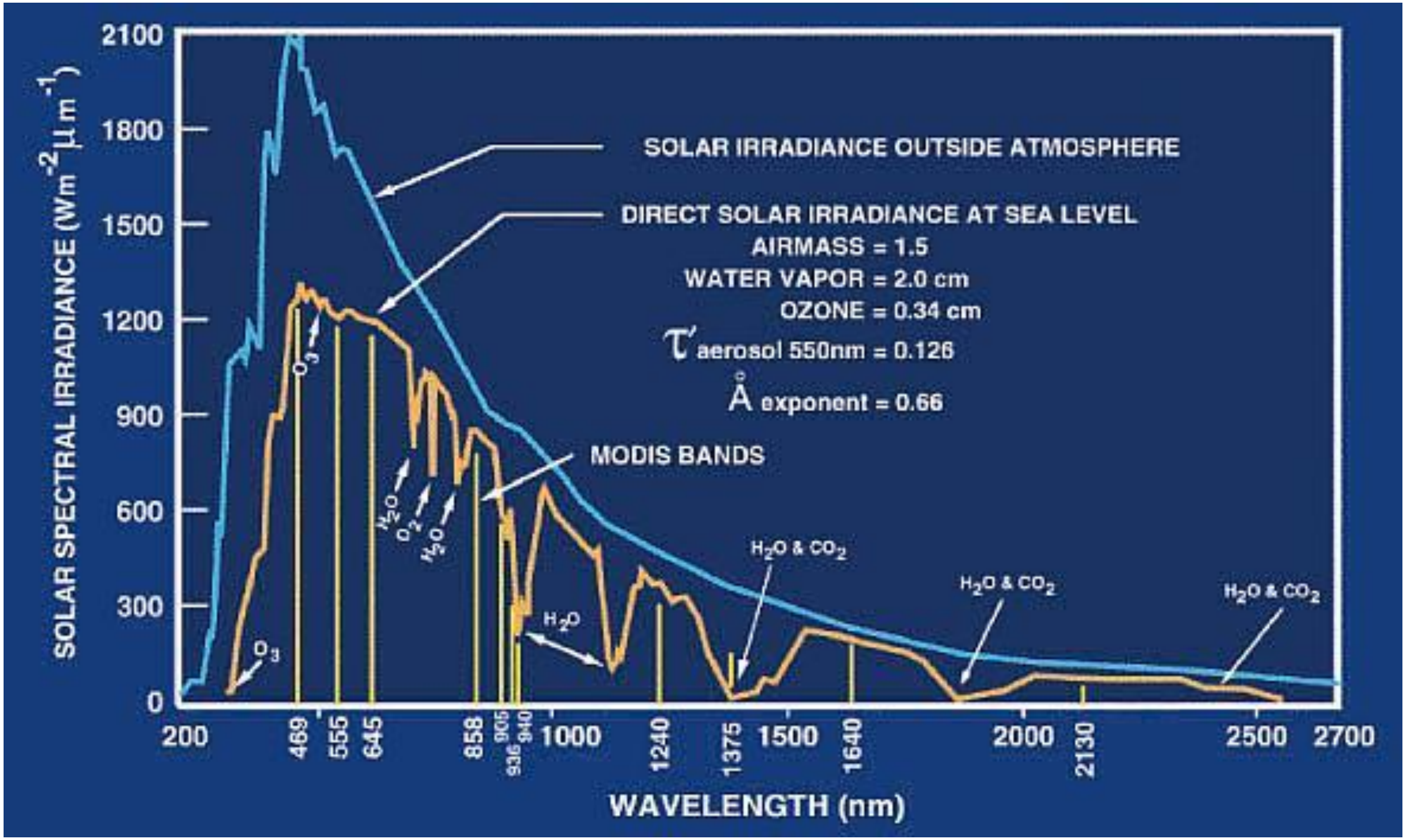
By definition, photons we can observe from an optically thick source come from near the $\tau = 1$ surface.

Since $\tau = 1$ is *not* `high optical depth', observed radiation from an optically thick source is not necessarily blackbody spectrum.

Deviations will depend upon the frequency dependence of the opacity around the $\tau = 1$ surface (the photosphere).

Solar spectrum - approximately of blackbody form. Very cool stars show larger departures from thermal spectra.




SOLAR IRRADIANCE OUTSIDE ATMOSPHERE
DIRECT SOLAR IRRADIANCE AT SEA LEVEL
AIRMASS = 1.5
WATER VAPOR = 2.0 cm
OZONE = 0.34 cm
τ′aerosol 550nm = 0.126
Å exponent = 0.66
MODIS BANDS
O_3
H_2O
O_2
H_2O & CO_2
SOLAR SPECTRAL IRRADIANCE ($Wm^{-2}\mu m^{-1}$)
WAVELENGTH (nm)
2100
1800
1500
1200
900
600
300
0
200
469
555
645
858
905
936
940
1000
1240
1375
1500
1640
2000
2130
2500
2700

Big bang model - Universe was hot, dense, and in thermal equilibrium between matter and radiation in the past.

Relic radiation from this era is the **cosmic microwave background** radiation. Best known blackbody:

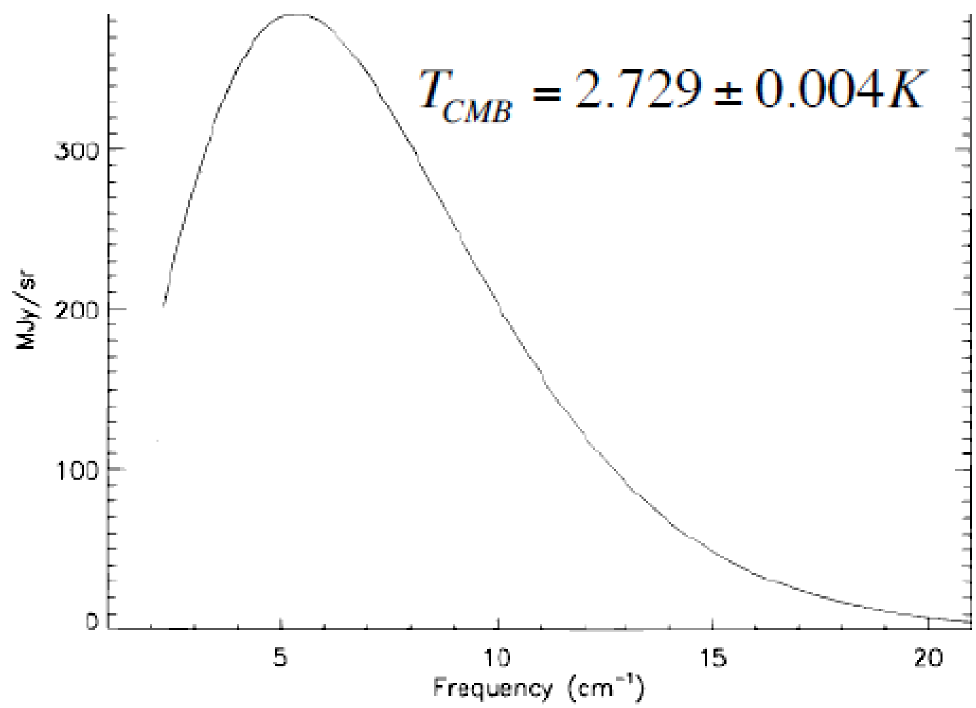


FIG. 4.—Uniform spectrum and fit to Planck blackbody (T). Uncertainties are a small fraction of the line thickness.

No known distortions of the CMB from a perfect blackbody!

Thermal Radiation Reprocessing: e.g., Planets

Two sources of radiation:
- Directly reflected Sun light
- Absorbed Solar radiation, reradiated as a blackbody

e.g. Jupiter: $L_{sun} = 3.86 \times 10^{33}$ erg s^{-1}

$a_J = 7.8 \times 10^{13}$ cm Jupiter orbital radius

$\mathrm{R_J} = 7.1 \times 10^{9}$ cm Jupiter radius

Solar radiation incident on the planet is:

$$L_J = \frac{\pi R_J^2}{4\pi a_J^2} \times L_{sun} \approx 2 \times 10^{-9} L_{sun}$$

Suppose planet directly reflects 10% - in the optical Jupiter is ~10^{10} times fainter than the Sun as seen from far away.

Absorb and reradiate as a blackbody:

$$L_J = 4\pi R_J^2 \times \sigma T_J^4$$

If all Sunlight is absorbed, estimate T = 120 K. Use:

$$h\nu_{\mathrm{max}} = 2.82kT$$

Find $\nu_{max} = 7 \times 10^{12}$ Hz, i.e., $\lambda \sim 40\ \mu m$
Best wavelengths to look for planets in direct emission are in the mid-IR, as the star's own spectrum drops

Another curiosity: dilluted starlight has an energy density much lower than that corresponding to its temperature - i.e., has a lower entropy. Important for the photo-bio-chemistry?

Processos Radiativos

Processos físicos de absorção e emissão da radiação eletromagnética (EMR)

– Leis de Kirchoff
– Processos (quânticos) discretos
– Processos contínuos (no contínuo)

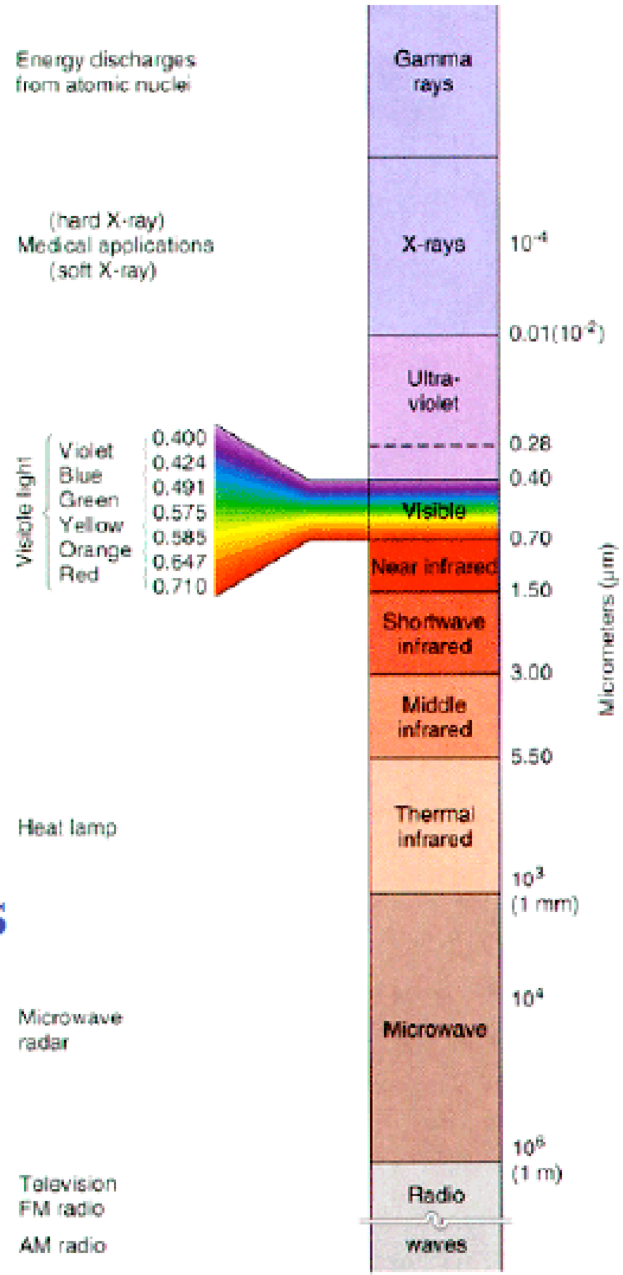
Primary Astrophysical Processes

When charged particles change direction (i.e., they are accelerated), they emit radiation

- Discrete (quantum transitions)
- Continuum
  - Thermal (i.e., blackbody)
  - Nonthermal: Synchrotron
    Free-free
    Cherenkov

Which one(s) will dominate,
depends on the physical conditions of the gas/plasma.
Thus, EMR is a *physical diagnostic*.

Different Physical Processes Dominate at Different Wavelengths



source: Christopherson (2000) Geosystems

Nuclear energy levels

Inner shells of heavier elements

Atomic energy levels (outer shells)

Molecular transitions

Hyperfine transitions

Plasma in typical magnetic fields

Kirchoff's Laws

1. **CONTINUOUS SPECTRUM**: Any hot opaque body (e.g., hot gas/plasma) produces a continuous spectrum or complete rainbow
2. **EMISSION SPECTRUM**: A hot transparent gas will produce an emission line spectrum
3. **ABSORPTION SPECTRUM**: A (relatively) cool transparent gas in front of a source of a continuous spectrum will produce an absorption line spectrum

Modern atomic/quantum physics provides a ready explanation for these empirical rules

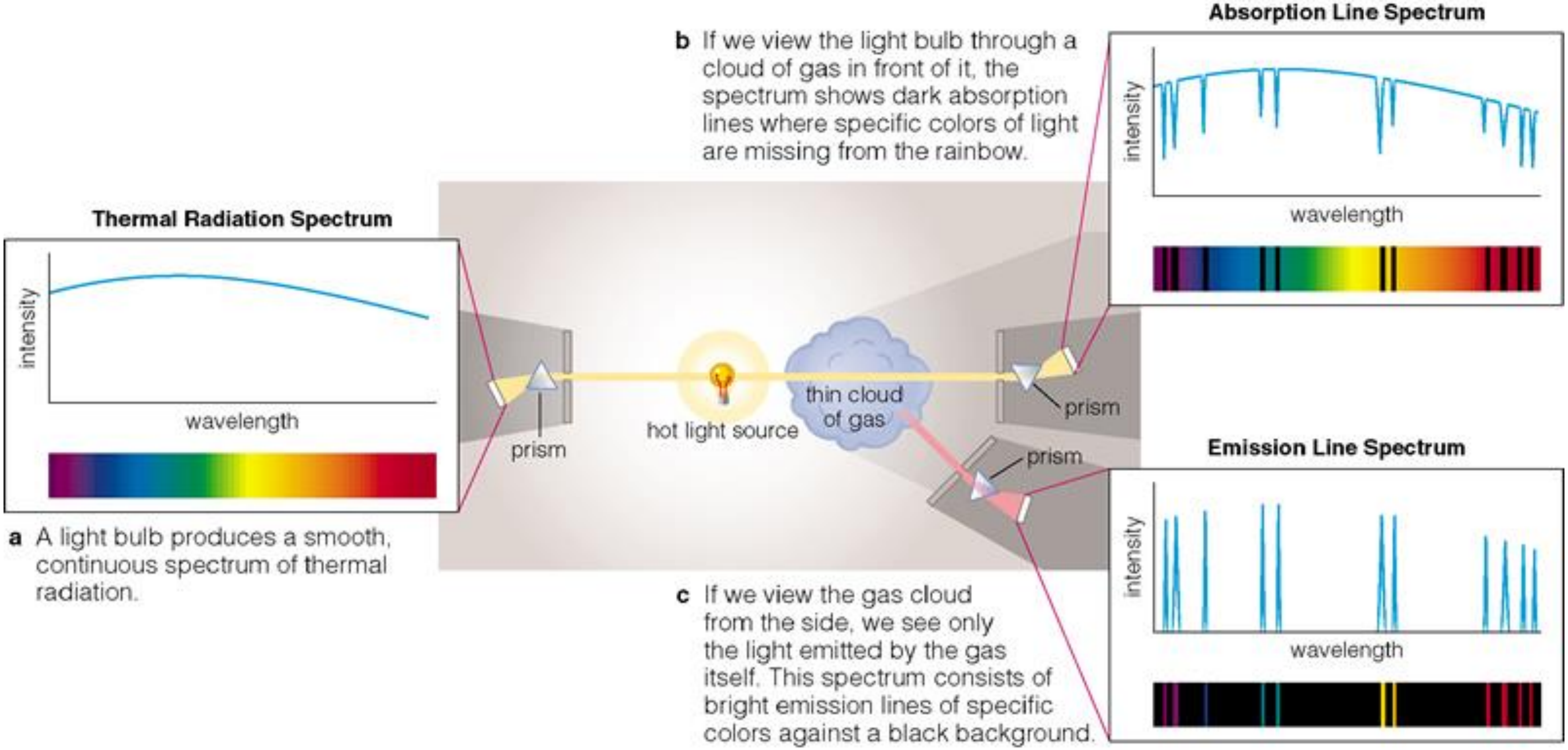
Espectros Astronômicos

Há três tipos básicos de espectros:

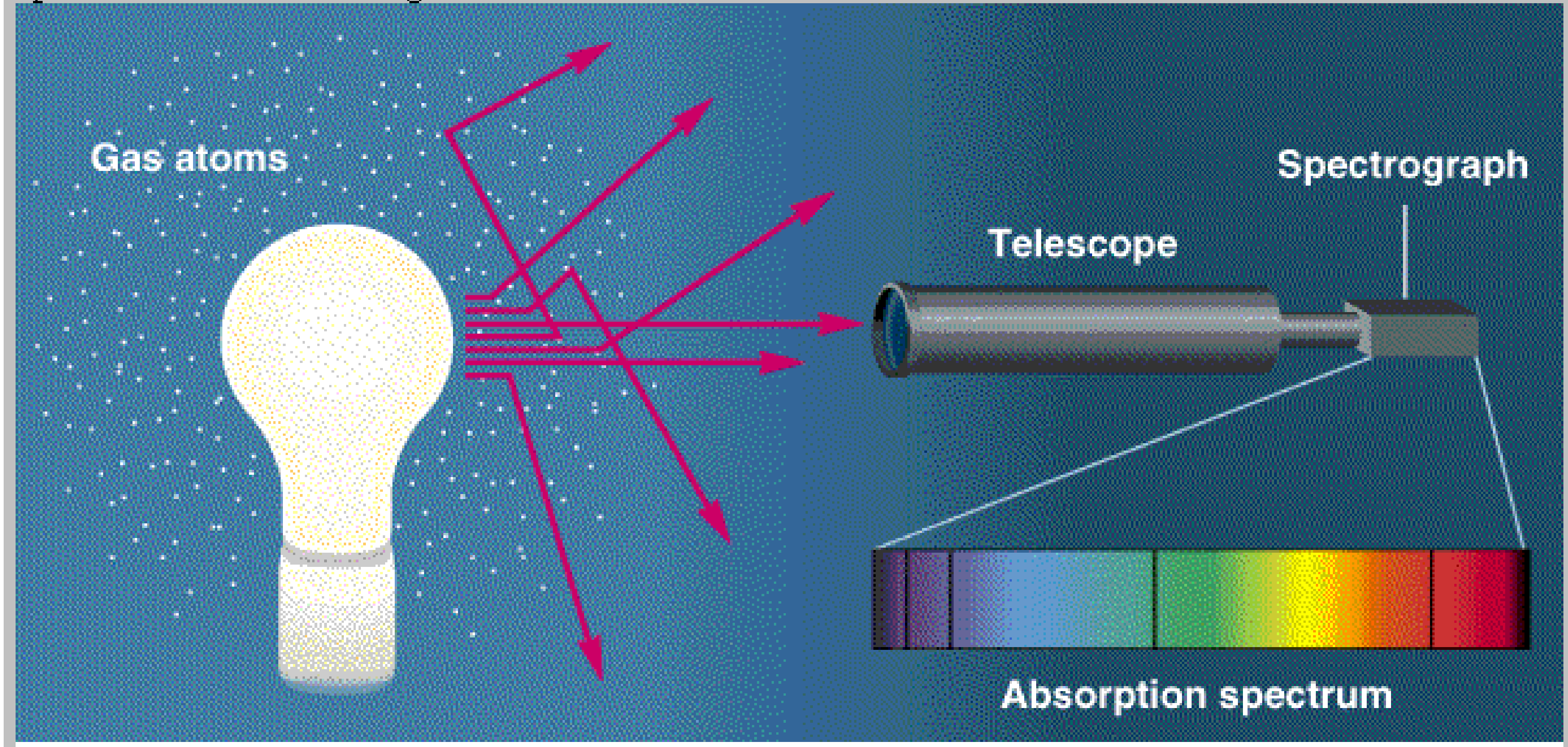
a. Espectro de radiação térmica: *Todos objetos com temperatura finita emitem radiação térmica.*

b. Espectro de Linha de Absorção:

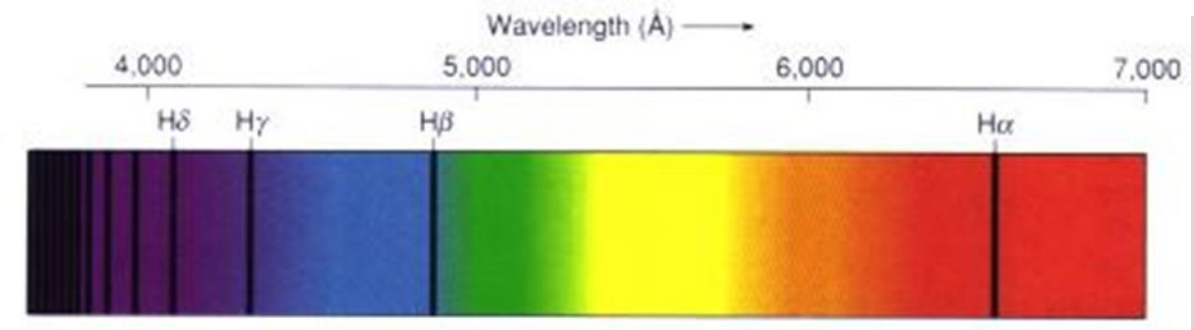
c. Espectro de Linha de Emissão:



Generally the absorption spectrum looks as follows. Photons of only the proper wavelengths can be absorbed by the gas atoms and then are re–emitted in random directions. Most of these re–emitted photons will not reach the telescope and the spectrum will be missing some "colors".



Leis de Kircchoff em ação:



Dark hydrogen absorption lines appear against a continuous visual spectrum, the light in the spectrum absorbed by intervening hydrogen atoms. From "Astronomy! A Brief Edition," J. B. Kaler, Addison-Wesley, 1997

- Espectros de laboratório → identificação de linhas em laboratório
- Análise de espectros → abundâncias químicas + condições físicas (temperatura, pressão, gravidade, fluxo ionizante, campos magnéticos)

\+ Velocidades

Atomic Processes

Radiation can be emitted or absorbed when electrons make transitions between different states:

Bound-bound: electron moves between two bound states (orbitals) in an atom or ion. Photon is emitted or absorbed.

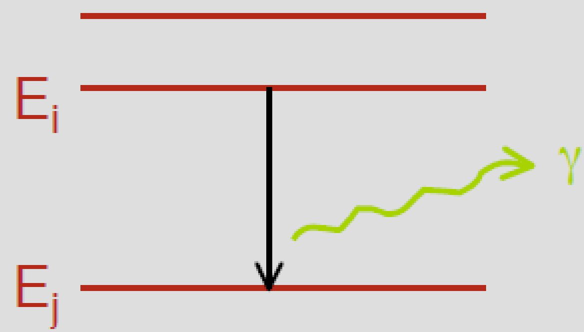
Bound-free:

- Bound -> unbound: **ionization**
- Unbound -> bound: **recombination**

Free-free: free electron gains energy by absorbing a photon as it passes near an ion, or loses energy by emitting a photon. Also called **bremsstrahlung**.

Bound-bound transitions

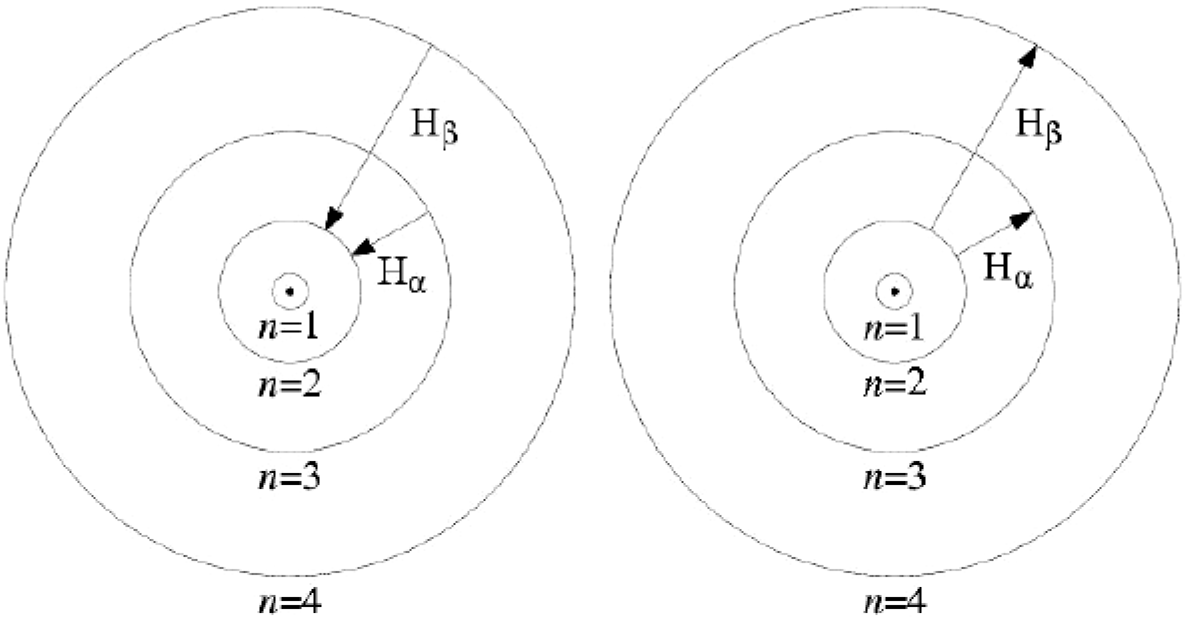
Transitions between two atomic energy levels:



Energy of the emitted / absorbed photon is the difference between the energies of the two levels:

$$h\nu = \left| E_i - E_j \right|$$

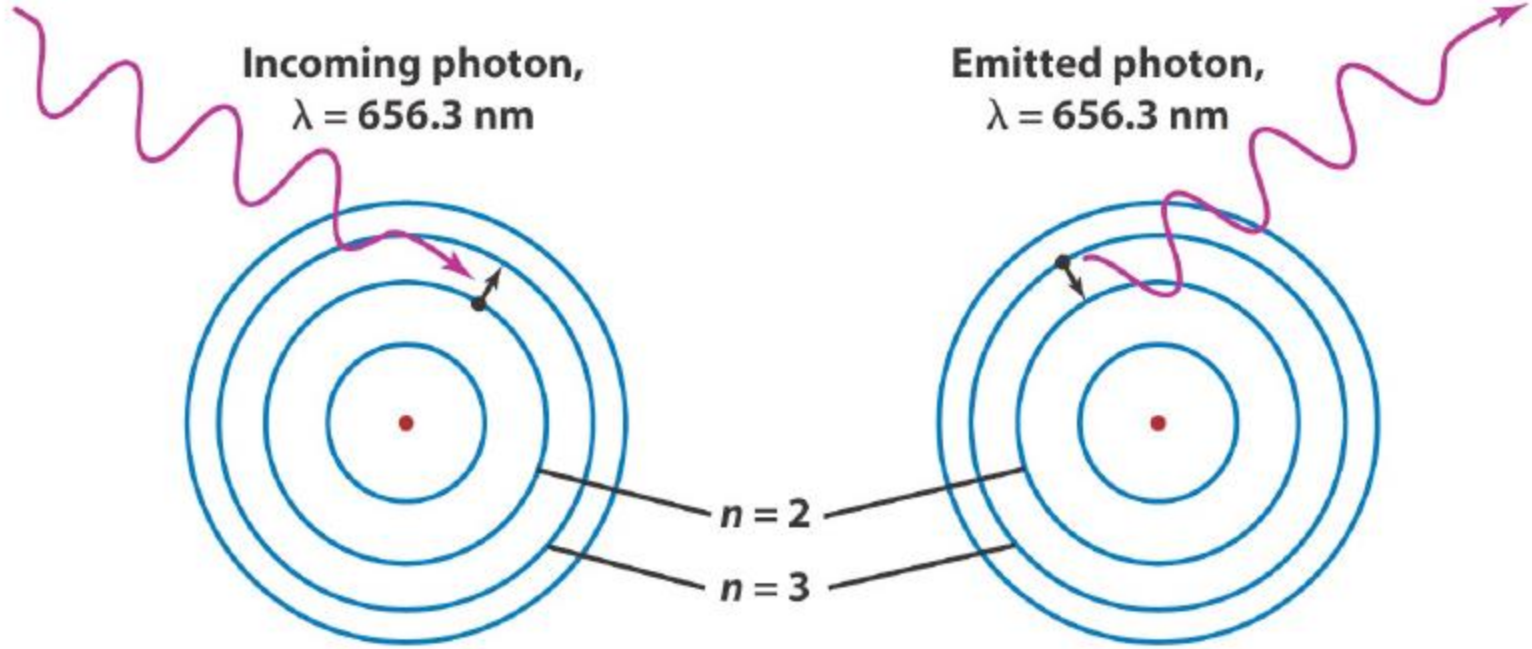
Energy Transitions: The Bohr Atom



Atoms transition from lower to higher energy levels (**excitation / de-excitation**) in discrete quantum jumps. The energy exchange can be **radiative** (involving a photon) or **collisional** (2 atoms)

Example of an Atomic Energy Transition: Hydrogen Atom

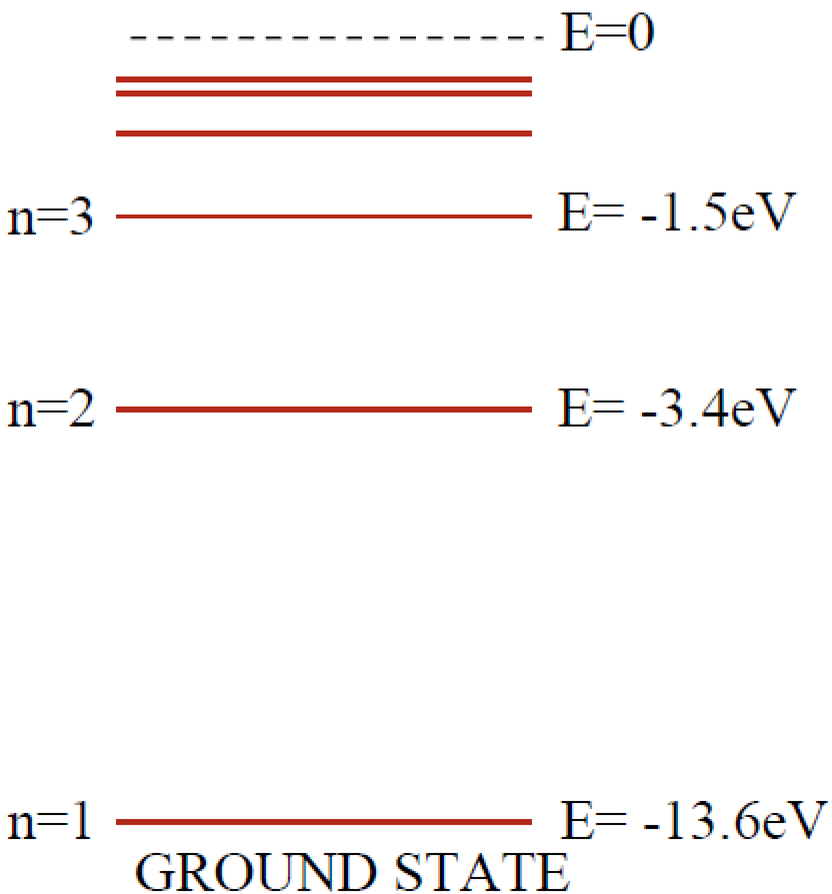
Photon energy: $h\nu = \left|E_i - E_j\right|$



(a) Atom absorbs a 656.3-nm photon; absorbed energy causes electron to jump from the $n = 2$ orbit up to the $n = 3$ orbit

(b) Electron falls from the $n = 3$ orbit to the $n = 2$ orbit; energy lost by atom goes into emitting a 656.3-nm photon

Hydrogen Energy Levels



Energy levels are labeled by n - the *principal Quantum number.*

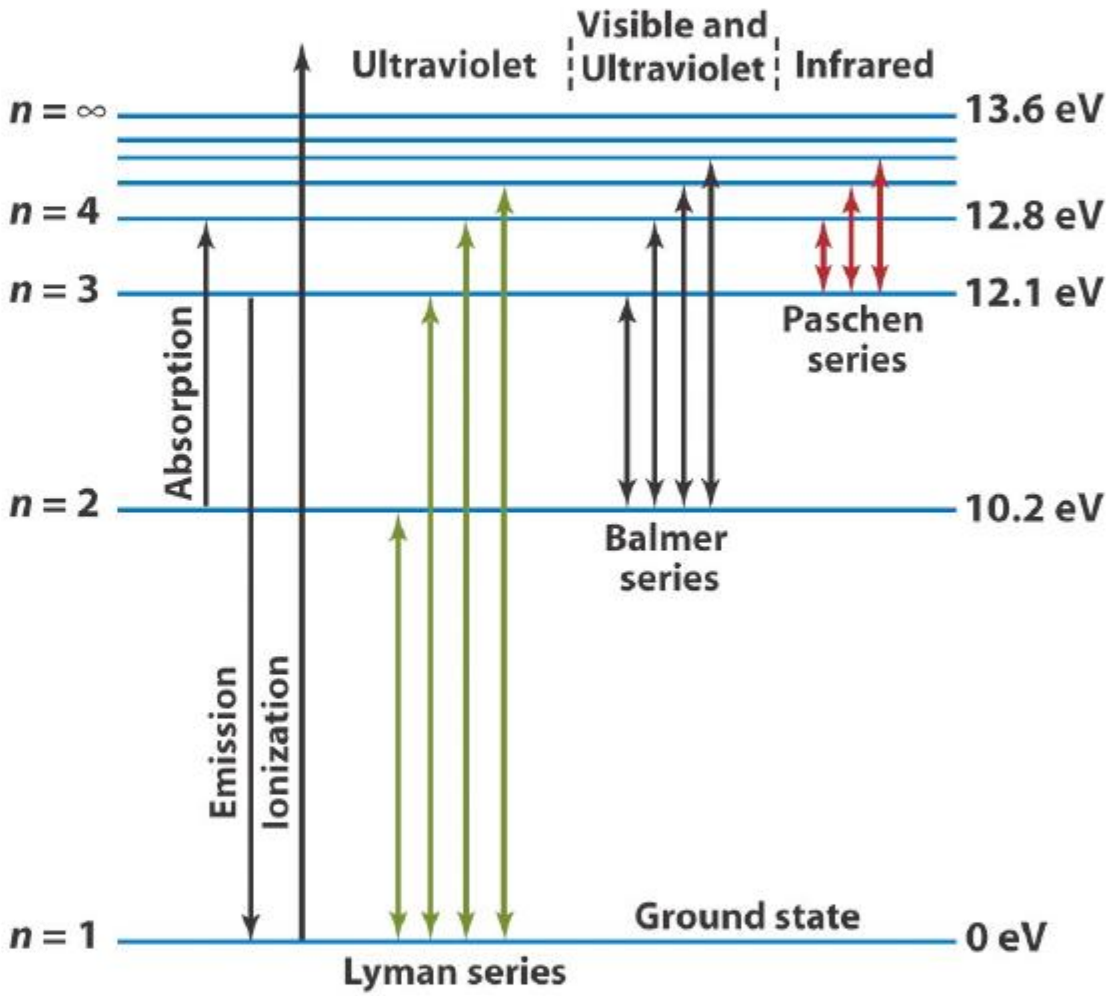
Lowest level, n=1, is the ground state.

$$E_n = -\frac{R}{n^2}$$

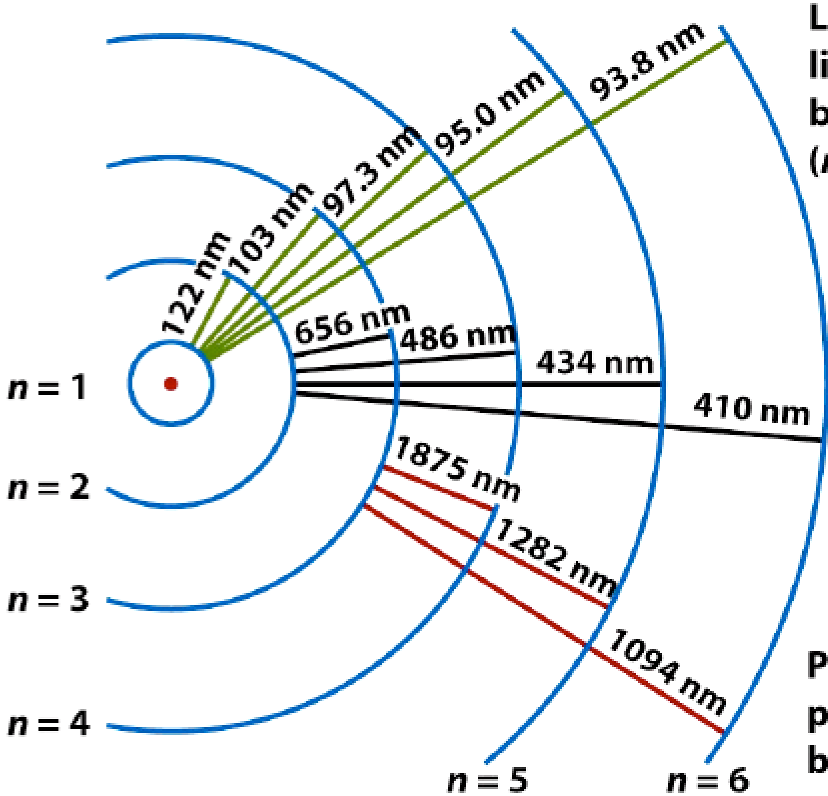
where R = 13.6 eV is a Constant (Rydberg)

n-th energy level has $2n^2$ quantum states, which are degenerate (same E).

Energy Levels in a Hydrogen Atom



Families of Energy Level Transitions Correspond to Spectroscopic Line Series

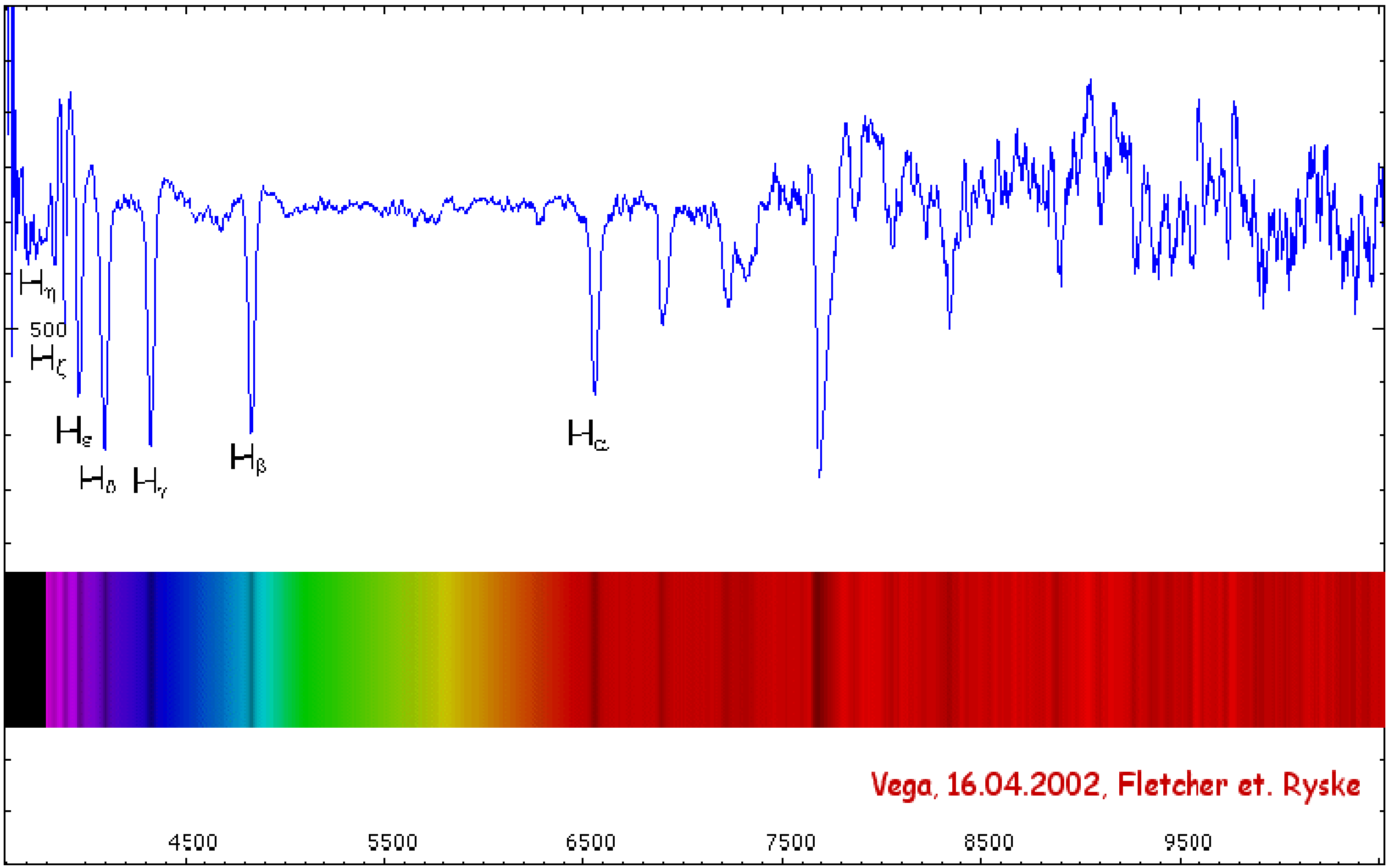


Lyman series (ultraviolet) of spectral lines: produced by electron transitions between the $n = 1$ orbit and higher orbits ($n = 2, 3, 4, \ldots$)

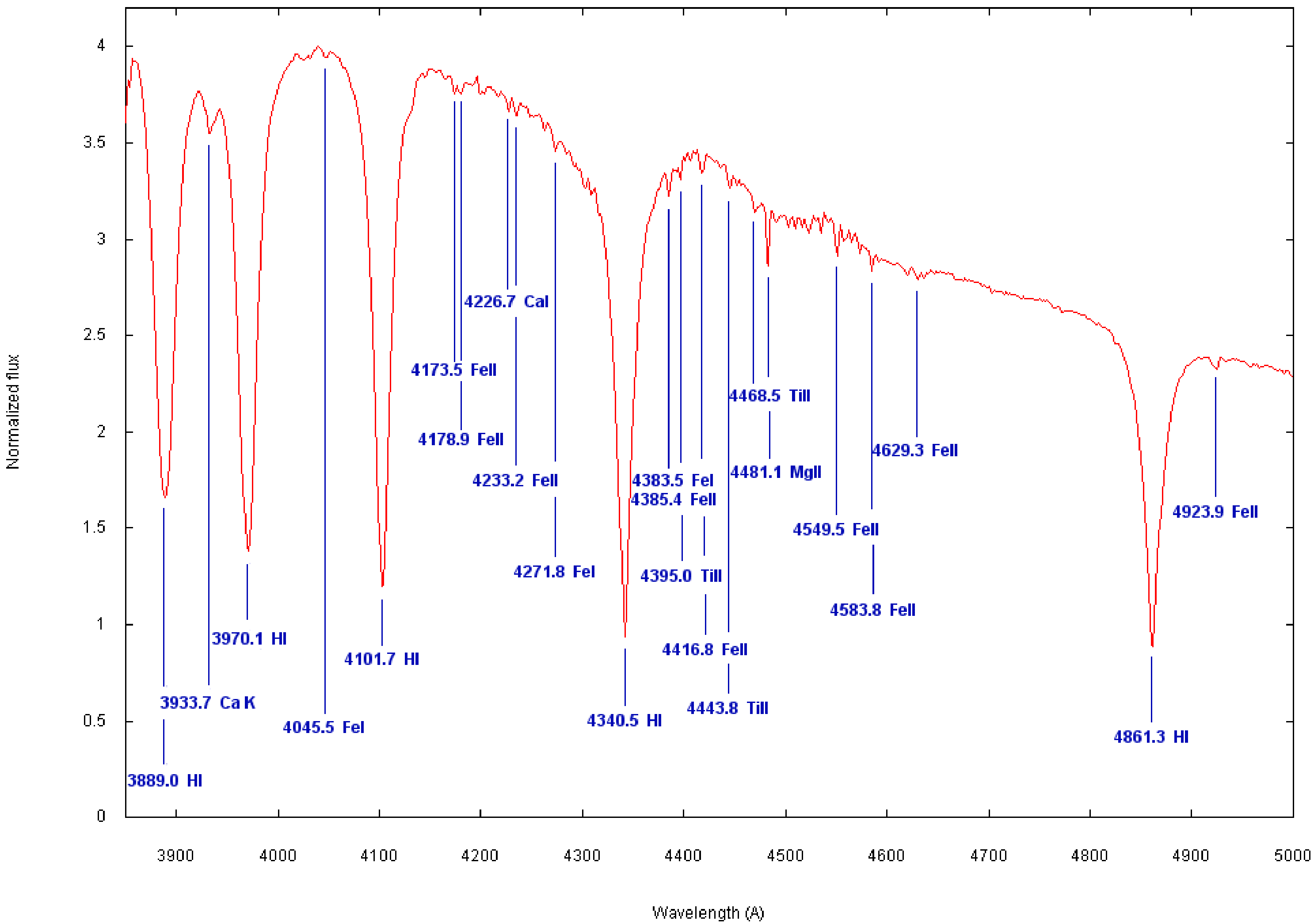
Balmer series (visible and ultraviolet) of spectral lines: produced by electron transitions between the $n = 2$ orbit and higher orbits ($n = 3, 4, 5, \ldots$)

Paschen series (infared) of spectral lines: produced by electron transitions between the $n = 3$ orbit and higher orbits ($n = 4, 5, 6, \ldots$)

Série de Balmer no espectro estelar

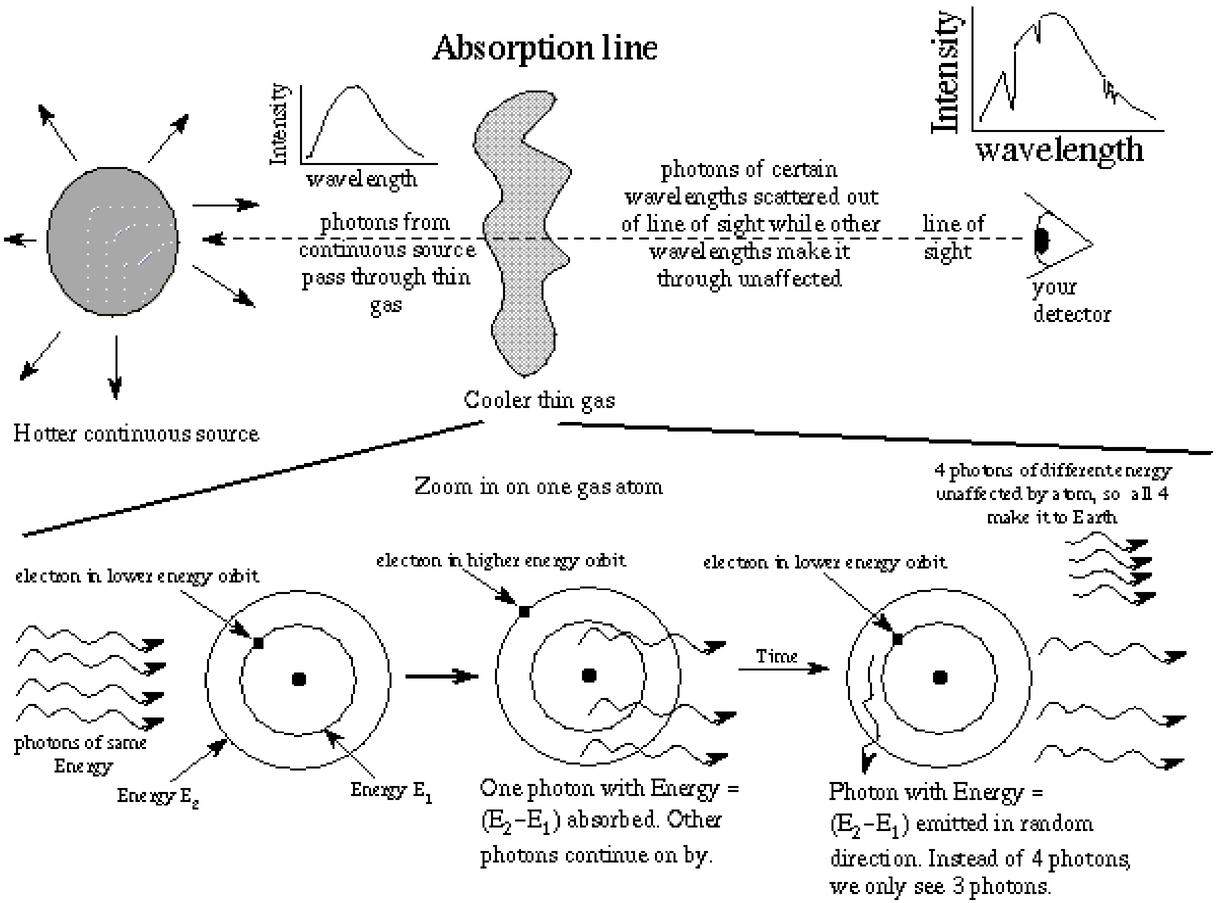


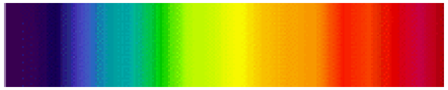
Balmer Series Lines in Stellar Spectra

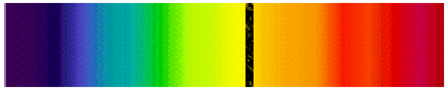


Radiação de átomos e moléculas

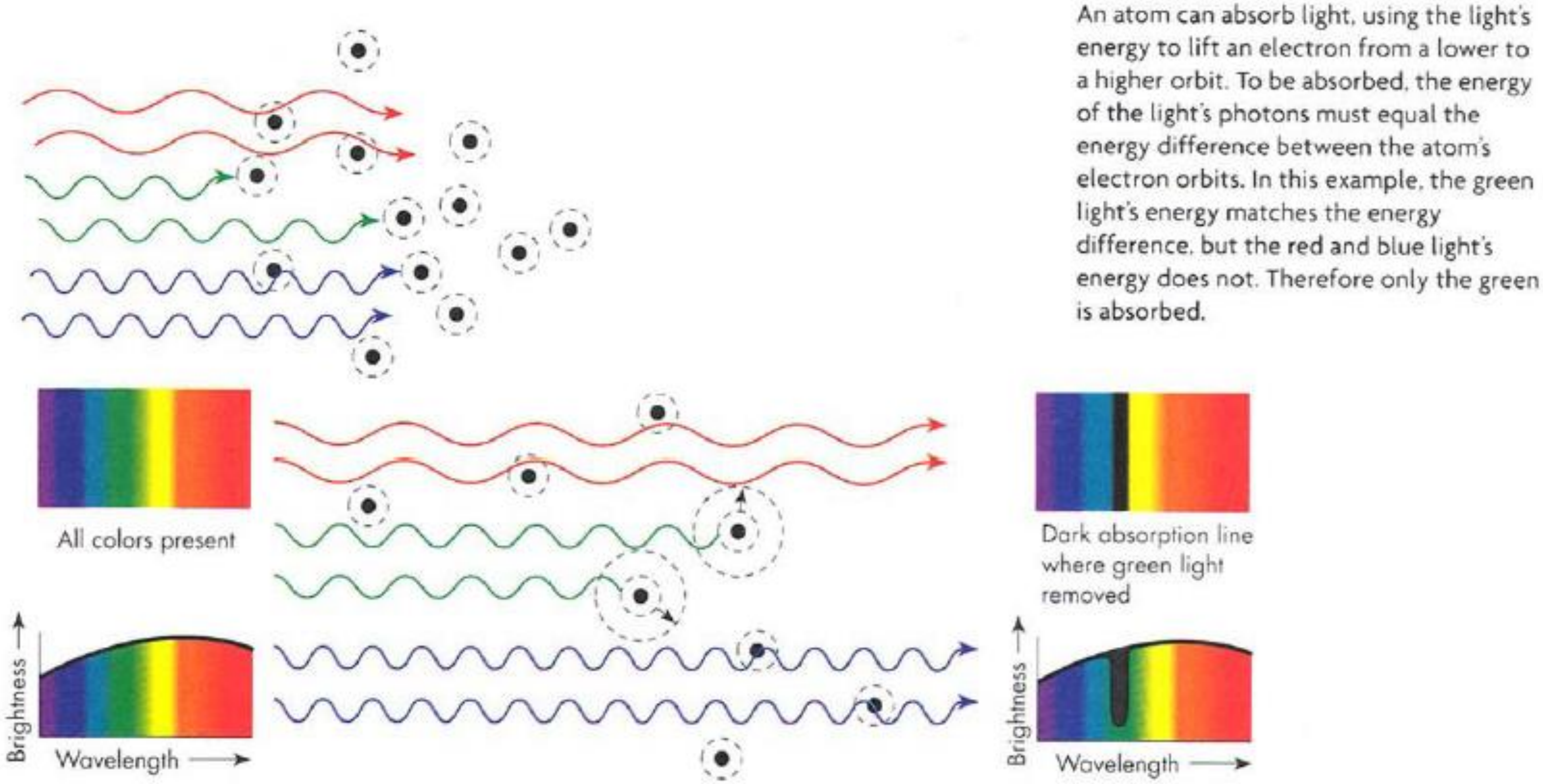
- **Formação de uma linha de absorção**







Absorption of light as it passes through an atmosphere



Which Energy Levels and Transitions?

Even for H - simplest atom - huge number of pairs of energy levels with different DE and hence different *n*. How do we decide which lines we will see?

• At particular T, some levels will have a higher probability of being occupied than others.

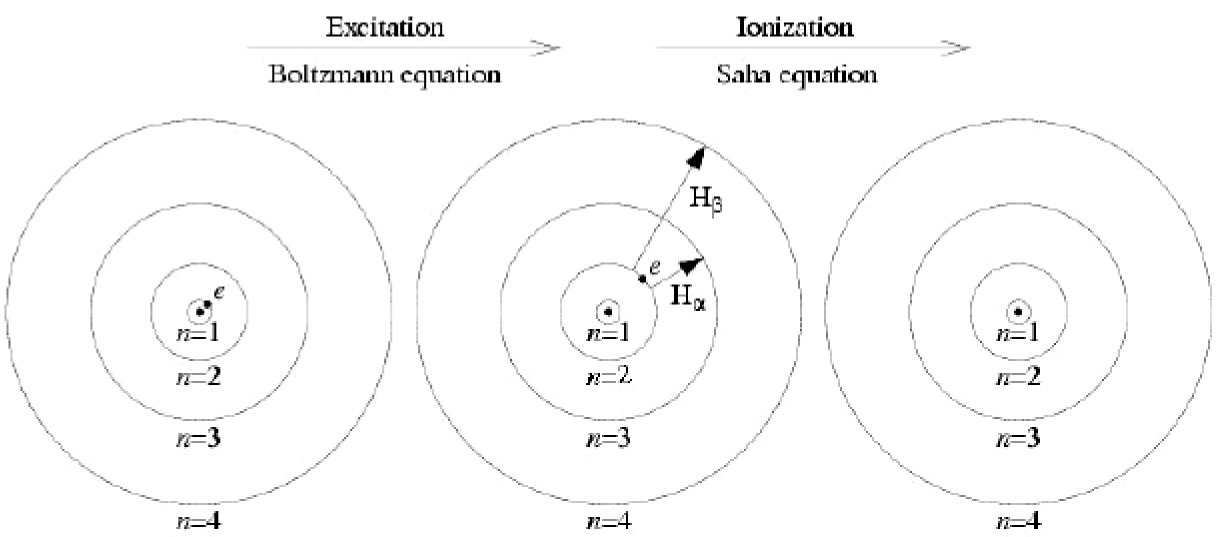
• Probability of some transitions is greater than others.

• Not all transitions are possible (*selection rules*).

Because of conservation laws - e.g. since a photon carries angular momentum cannot make a transition between two states with zero angular momentum by emitting one photon.

Computing the Occupation of Energy Levels

… from which we can then compute the relative intensities of spectroscopic lines



Need gas [ρ,T] and radiation spectrum and intensity (or just T_e, if it's a thermal spectrum). The key question is whether the gas and the radiation field are in a *thermal equilibrium*

Boltzmann's Law

Calculating the populations of energy levels is difficult if The gas is not in local thermodynamic equilibrium (LTE). In LTE, it is very easy. At temperature T, populations n_1 and n_2 of any two energy levels are:

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/kT}$$

g_1 and g_2 are the statistical weights of the two levels (allow for the fact that some energy levels are degenerate). For hydrogen:

$$g_n = 2n^2$$

Emission or Absorption?

It depends on whether the gas (plasma) is

Optically thick: short mean free path of photons, get absorbed and re-emitted many times, only the radiation near the surface escapes; or

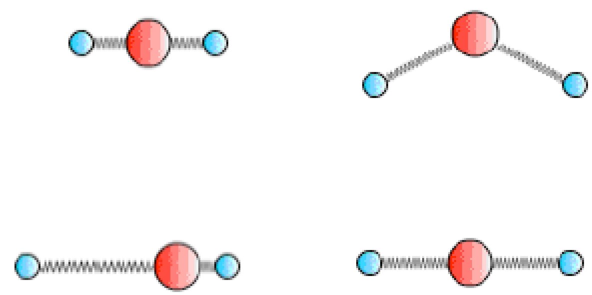
Optically thin: most photons escape without being reabsorbed or scattered

(Note that a medium can be optically thick or thin for either line or continuum photons. Optical thickness is generally proportional to density.)

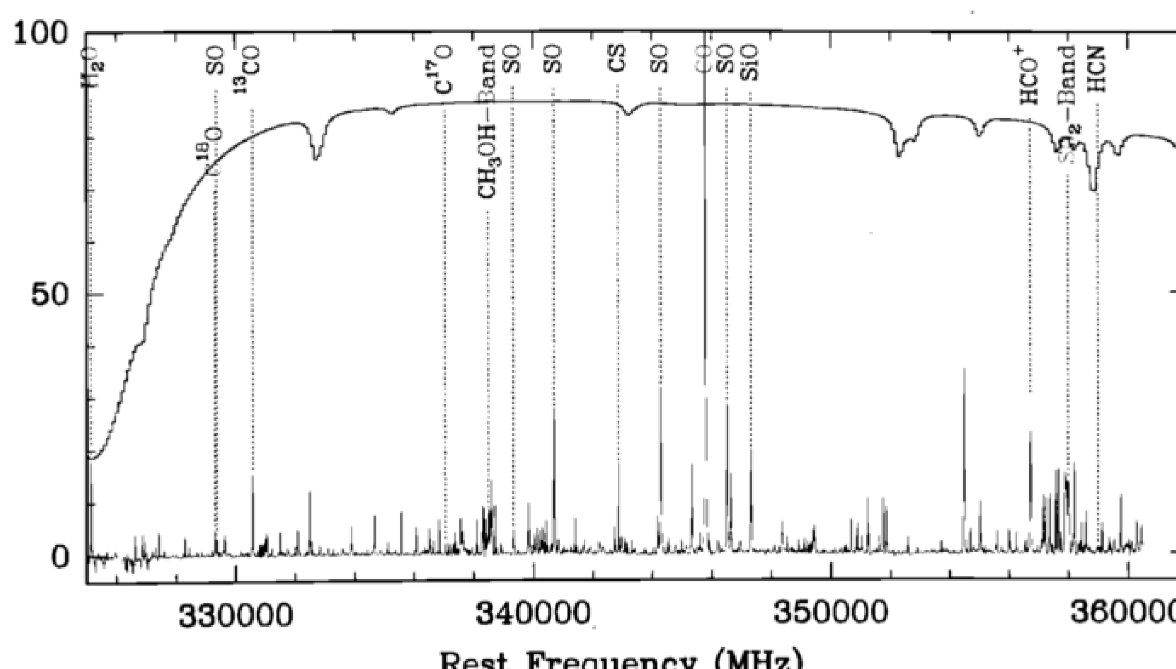
And then it depends on the **geometry:** if a continuum is seen though a cooler, optically thin gas, you will see an absorption spectrum; but if the gas is hotter, there will be an emission line spectrum superposed on the cont.

Spectral Line Emission: Molecular Rotational and Vibrational Modes

These transitions (or energy splittings) have generally lower energies (thus prominent in IR/sub-mm), but many more levels (thus complex spectra)



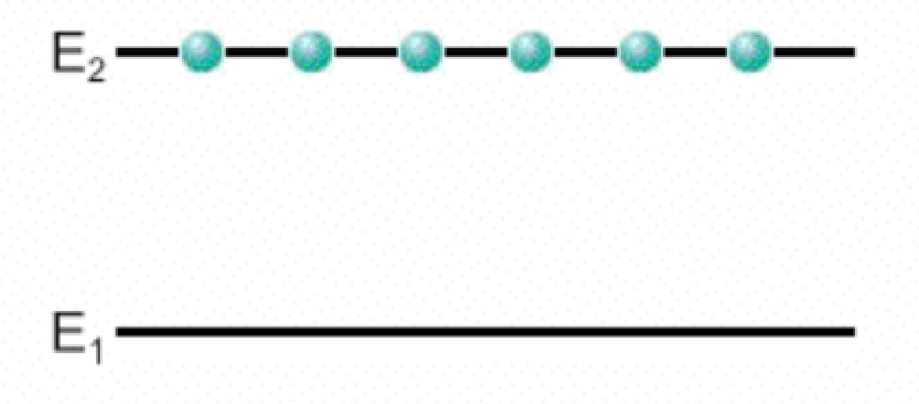
Example: Orion spectrum from CSO



Astrophysical Molecular Spectroscopy

- Because of the lower energy levels of molecular transitions, they are a good probe of colder gas ($T \sim 10 - 100$ K), e.g., star forming regions
- Commonly observed molecules in space include: hydrogen (H_2) carbon monoxide (CO), water (H_2O), OH, HCN, HCO^+, CS, NH_3, formaldehyde (H_2CO), etc. Less common molecules include sugar, alcohol, antifreeze (ethylene glycol), …
- As a bonus, longer wavelengths are not affected much by the interstellar extinction

Maser Emission



- Stimulated emission from overpopulated energy levels
- Sometimes seen in star-forming regions, or cold stellar envelopes
- Produces very sharp emission lines - an excellent tracer of velocity fields (e.g., for central massive black holes)

Hydrogen 21cm Line

Ground state of hydrogen (n=1) has 2 x 1^2 = 2 states.

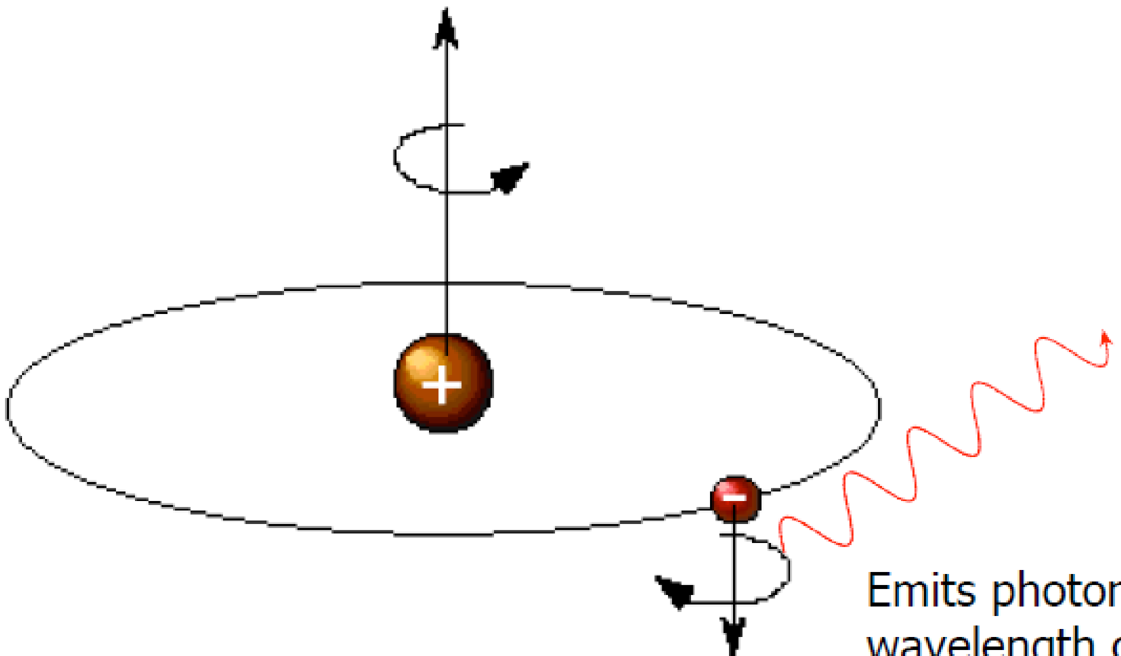
Correspond to different orientations of the electron spin relative to the proton spin.

Very slightly different energies - *hyperfine splitting*.

Energy difference corresponds to a frequency of 1.42 GHz, or 21cm wavelength.

Very important for radio astronomy, because neutral hydrogen is so abundant in the Universe. This is the principal wavelength for studies of ISM in galaxies, and their disk structure and rotation curves.

Spectral Line Emission: Hyperfine Transition of Neutral Hydrogen



Lower energy state: Proton and electron have opposite spins.

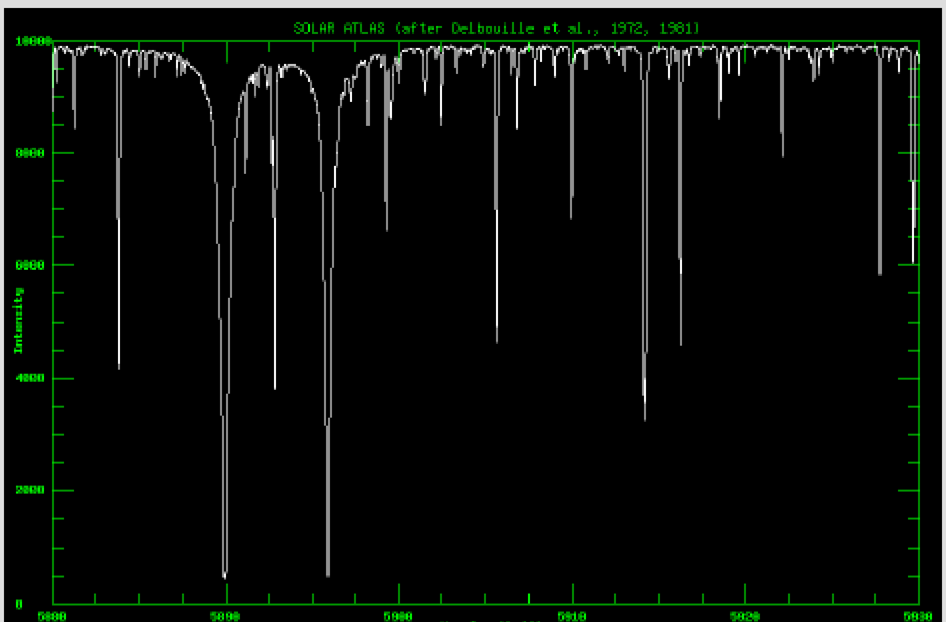
Emits photon with a wavelength of 21 cm (frequency of 1.42 GHz)

Transition probability=3×10^{-15} s^{-1} = once in 11 Myr

Absorption line and emission line spectra

Temperature of the Solar photosphere is ~6000K. Lots of spectral lines of different elements at this T.

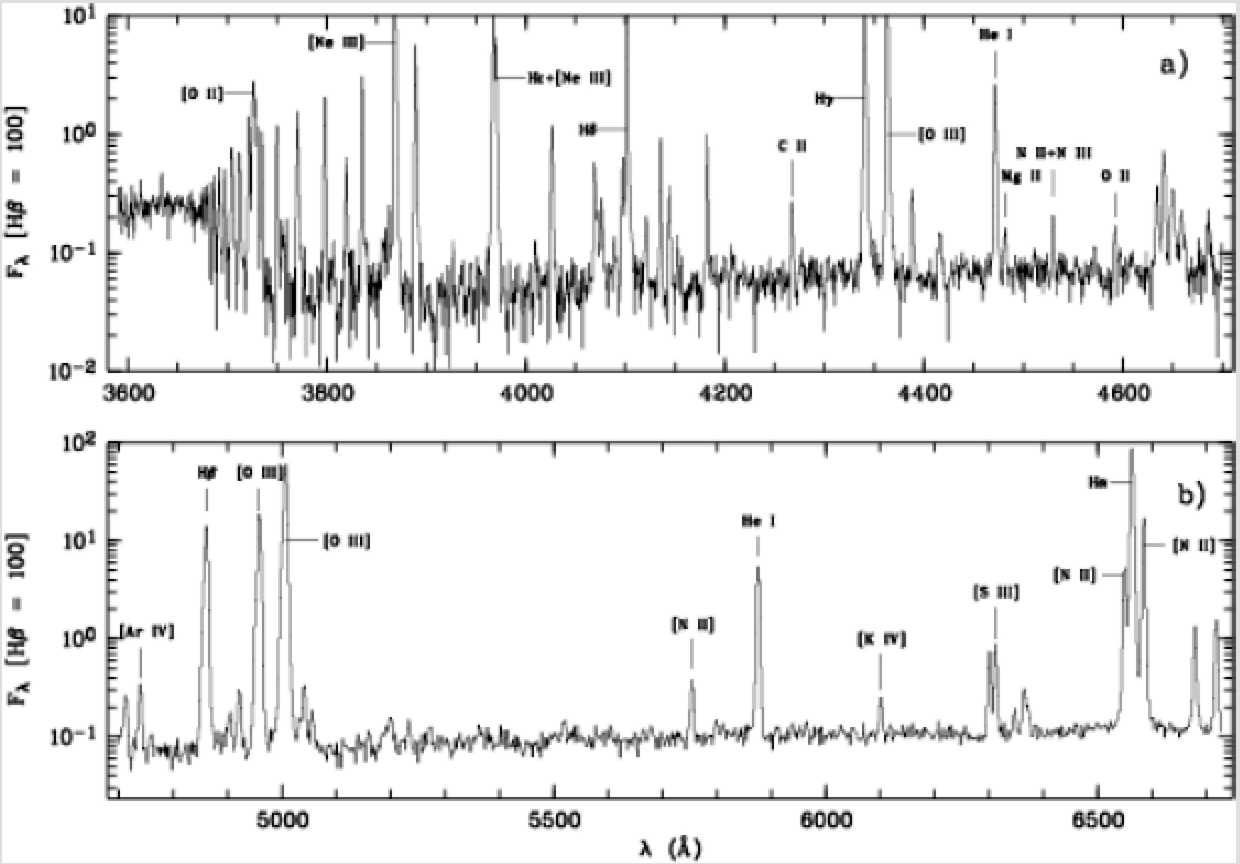
Optical spectrum is an **absorption line spectrum -** see dark absorption lines superimposed on a bright continuum.



Small section of the Solar spectrum showing two strong lines due to sodium.

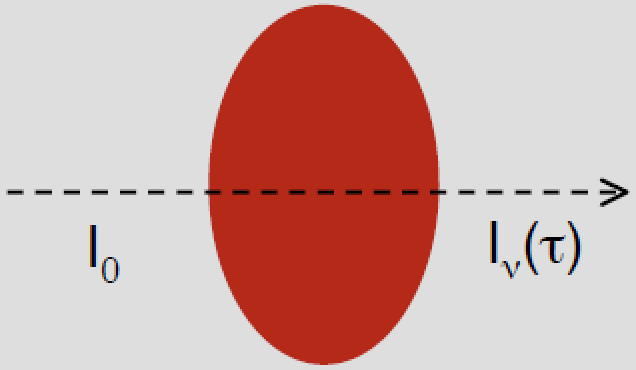
Nebulae of different sorts typically show **emission line spectra**:

Spectral lines are **stronger** than the continuum



Why this difference?

Use result we derived last time - consider radiation passing through a hot cloud of gas in thermal equilibrium:



Found:

$$I_\nu(\tau_\nu) = I_0 e^{-\tau_\nu} + B_\nu(1 - e^{-\tau_\nu})$$ (slide 43)

Suppose no intensity entering the cloud, $I_0 = 0$. If the cloud is very optically thin:

$$e^{-\tau_\nu} \approx 1 - \tau_\nu$$

$$I_\nu(\tau_\nu) = B_\nu(1 - 1 + \tau_\nu) = \tau_\nu B_\nu$$

Exponencial

- A função exponencial pode ser definida como uma série infinita

$$\boxed{e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots}$$

ou como limite de uma sequência

$$\boxed{e^x = \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n}$$

n! corresponde ao fatorial de *n* e *x* é qualquer número real ou complexo.

- O valor de e^{+1} é aproximadamente 2.718281828

Optical depth is related to the absorption coefficient via:

$$\tau_\nu = \alpha_\nu \Delta s \qquad \text{(for constant } \alpha\text{)}$$

Means that:

$$I_\nu = \tau_\nu B_\nu \propto \alpha_\nu B_\nu$$

Intensity is large at frequencies where the absorption coefficient is large.

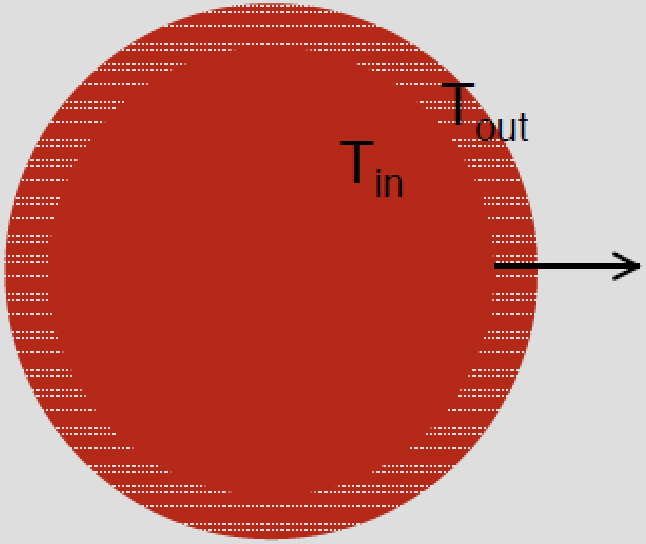
For a hot gas, absorption coefficient is large at the frequencies of the spectral lines.

⇨ For an optically thin medium such as a nebula, expect an emission line spectrum with large intensity at the frequencies where α_ν is large.

Next, consider an optically thick source:

- Already shown that in the interior, radiation will be described by the Planck function.
- Radiation escaping from the source will be modified because the temperature (and thus the Planck function) **varies** along the path.

Example: model a star using a two layer model:



Radiation starts from the inner layer as blackbody radiation at temperature T_{in}.

Escapes through an atmosphere of optical depth τ and temperature T_{out}.

Use same solution as before to describe change in intensity of the radiation:

$$I_\nu(\tau_\nu) = I_0 e^{-\tau_\nu} + B_\nu(1 - e^{-\tau_\nu})$$

Escaping radiation

$B_\nu(T_{in})$

$B_\nu(T_{out})$

Valid provided that all the gas is in thermal equilibrium (LTE).

Assume that optical depth of outer layer is small and use approximate expansion for the exponential as before:

$$I_\nu(\tau_\nu) = B_\nu(T_{in}) e^{-\tau_\nu} + B_\nu(T_{out})\left[1 - e^{-\tau_\nu}\right]$$

$$I_\nu(\tau_\nu) = B_\nu(T_{in})\left[1 - \tau_\nu\right] + B_\nu(T_{out}) \times \tau_\nu$$

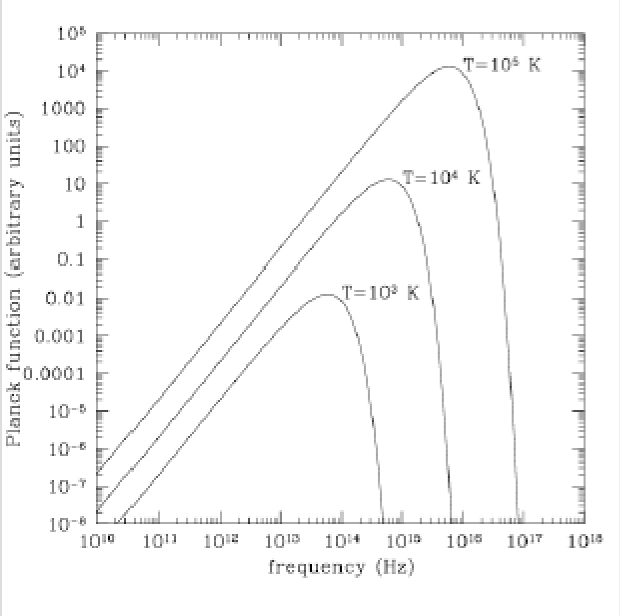
$$I_\nu(\tau_\nu) = B_\nu(T_{in}) + \tau_\nu\left[B_\nu(T_{out}) - B_\nu(T_{in})\right]$$

$$I_\nu(\tau_\nu) = \underbrace{B_\nu(T_{in})}_{\text{Initial radiation intensity}} + \tau_\nu \underbrace{\left[B_\nu(T_{out}) - B_\nu(T_{in})\right]}_{\text{Change in intensity caused by the outer layer. Depends upon frequency.}}$$

Initial radiation intensity

Change in intensity caused by the outer layer. Depends upon frequency.

Recall that intensity of blackbody radiation increases at **all** frequencies as the temperature goes up.



Sign of the second term depends upon whether $B_\nu(T_{out})$ is larger or smaller than $B_\nu(T_{in})$ - i.e. on whether $T_{out} > T_{in}$.

$$I_\nu(\tau_\nu) = B_\nu(T_{in}) + \tau_\nu\left[B_\nu(T_{out}) - B_\nu(T_{in})\right]$$

1) $T_{out} > T_{in}$: second term is positive:

Escaping intensity is **larger** at frequencies where τ_ν is greatest (frequencies corresponding to spectral lines). Expect emission lines on top of the continuum.

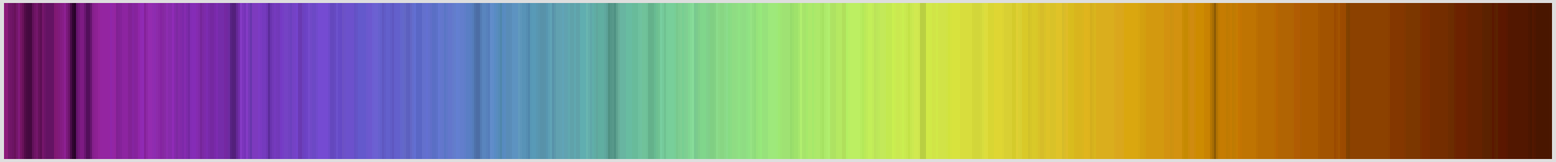
2) $T_{out} < T_{in}$: second term is negative:

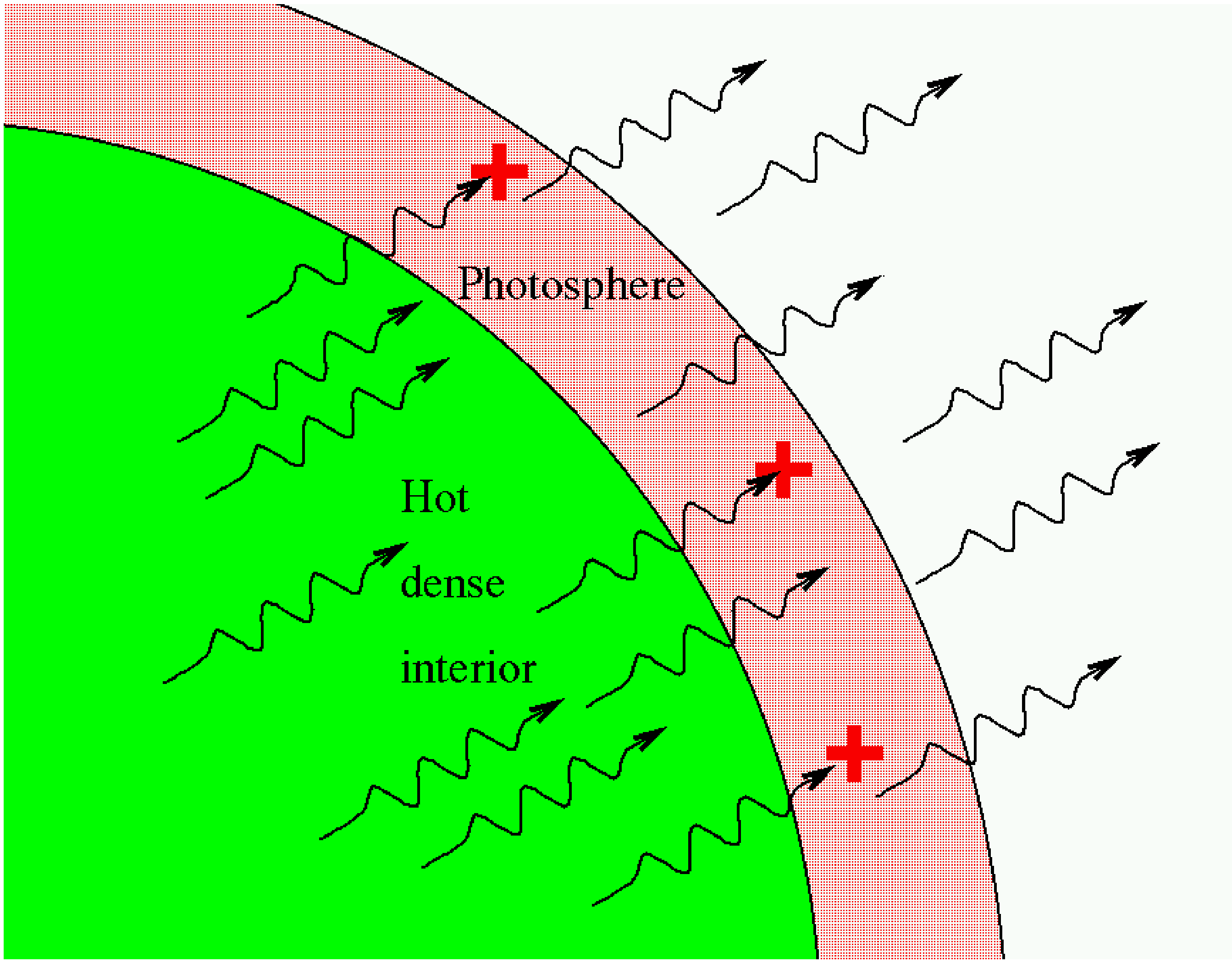
Escaping intensity is **reduced** at frequencies where τ_ν is greatest (frequencies corresponding to spectral lines). Expect absorption lines superimposed on the continuum.

For the Sun, temperature near the optical photosphere **decreases** outward (as it must since energy transport is from the center to the outside).

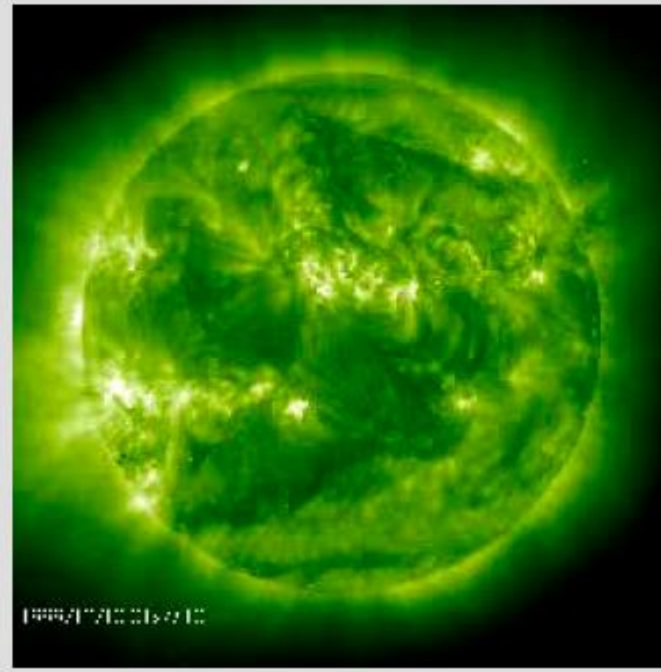
In second regime: $T_{out} < T_{in}$

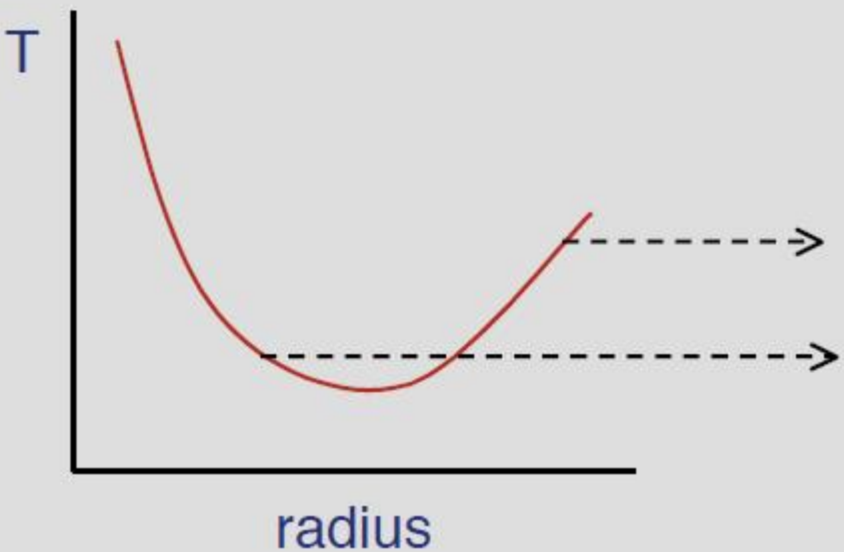
Expect to see an **absorption spectrum**, as observed:




Photosphere
Hot
dense
interior

Note: see strong UV and X-ray emission from the Solar corona, so obviously the temperature there is much hotter than that of the photosphere…





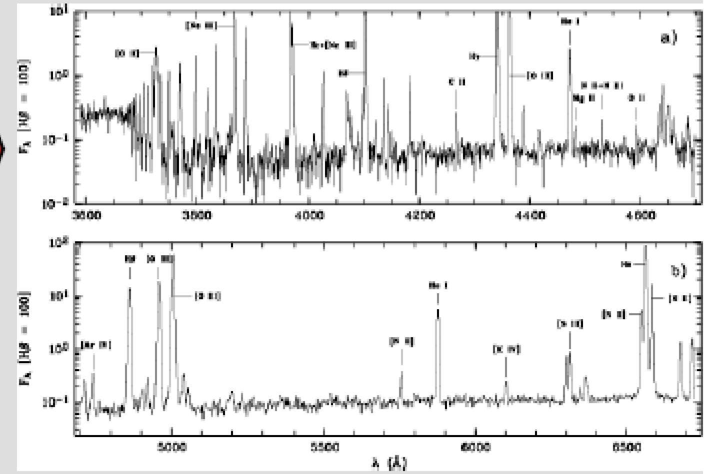
UV radiation comes from region where T increasing, so emission line spectrum

Optical radiation comes from region where T decreasing, so absorption spectrum

Summary:

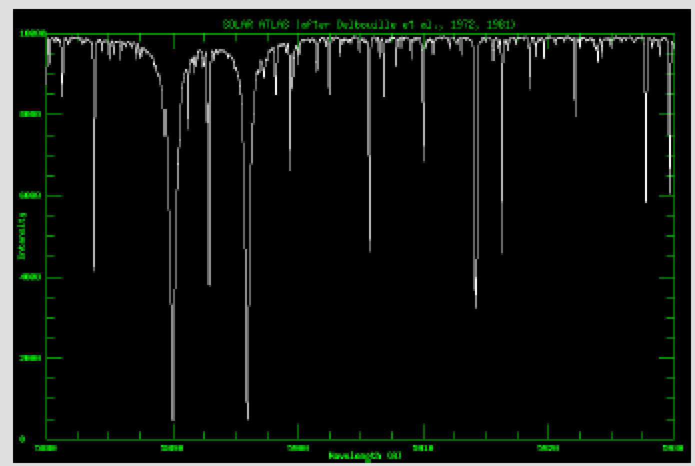
Emission line spectra:

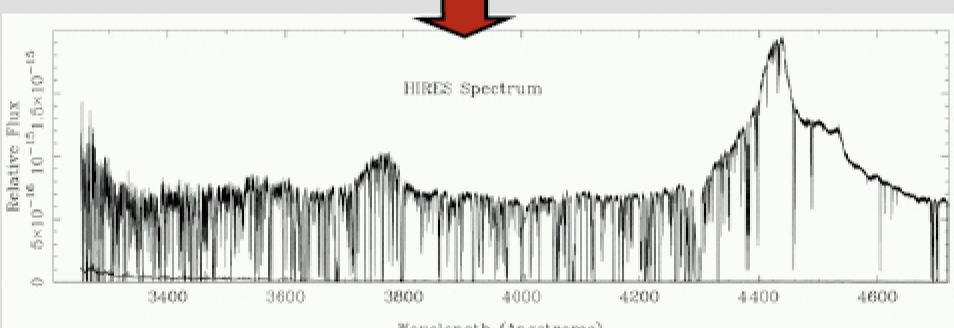
- Optically thin volume of gas with no background light
- Optically thick gas in which T increases outward



Absorption line spectra:

- Cold gas lies in front of a source of radiation at a higher temperature

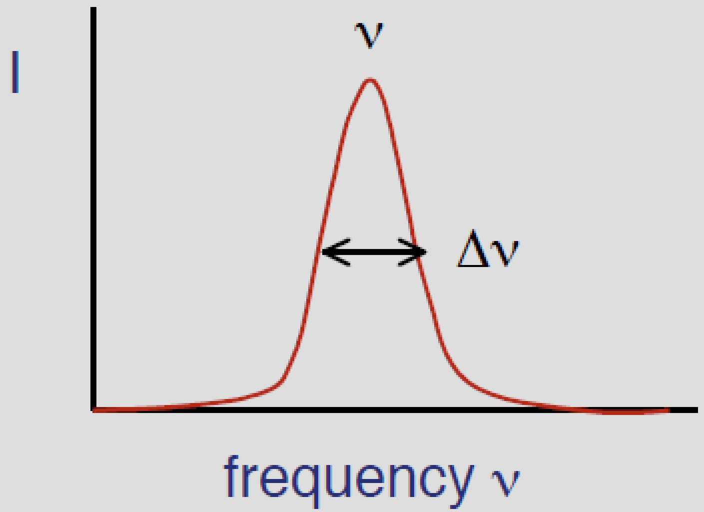




Broadening of spectral lines

An individual atom making a transition between energy levels emits one photon with a well-defined energy / frequency.

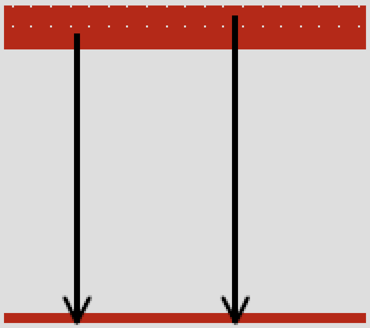
However, profiles of real spectral lines are not infinitely narrow.



e.g. for an emission line, width of the spectral line $\Delta\nu$ could be defined as the full width at half the maximum intensity of the line.

Details of definition don't matter - important to see what causes lines to have finite width.

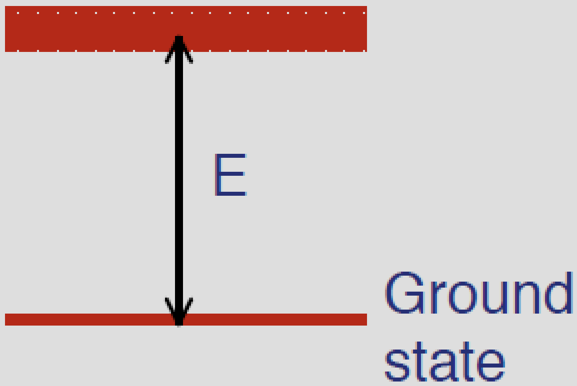
Two basic mechanisms:



1) Energy levels themselves are not infinitely sharp:
   emitted photons have a range of frequencies

2) Atoms in the gas are moving relative to the observer:
   observed photons don't have the same frequency as the emitted photons because of the Doppler effect.

Natural linewidth



Consider excited state with energy E above the ground state.

Electrons in excited state remain there for average time Δt before decaying to ground state.

Uncertainty principle: energy difference between states is uncertain by an amount ΔE given by:

$$\Delta E \Delta t \approx \frac{h}{2\pi}$$

But since $E = h\nu$, $\Delta E = h\Delta\nu$ $\Rightarrow$ $\Delta\nu \approx \frac{1}{2\pi\Delta t}$

Broadening due to this effect is called the natural linewidth.

Natural linewidth sets absolute minimum width of spectral lines. However, normally very small - other effects dominate.

e.g. for hydrogen n=2 to n=1 transition (Lyman α transition) the lifetime is of the order of 10^{-9} s.

Natural linewidth is $\sim 10^{8}$ Hz.

Compare to frequency of transition: $\frac{\Delta\nu}{\nu} \approx 10^{-7}$

In **astrophysical** situations, other processes will often give much larger linewidths than this.

Collisional broadening

In a dense gas, atoms are colliding frequently. This effectively reduces the lifetime of states further, to a value smaller than the quantum mechanical lifetime.

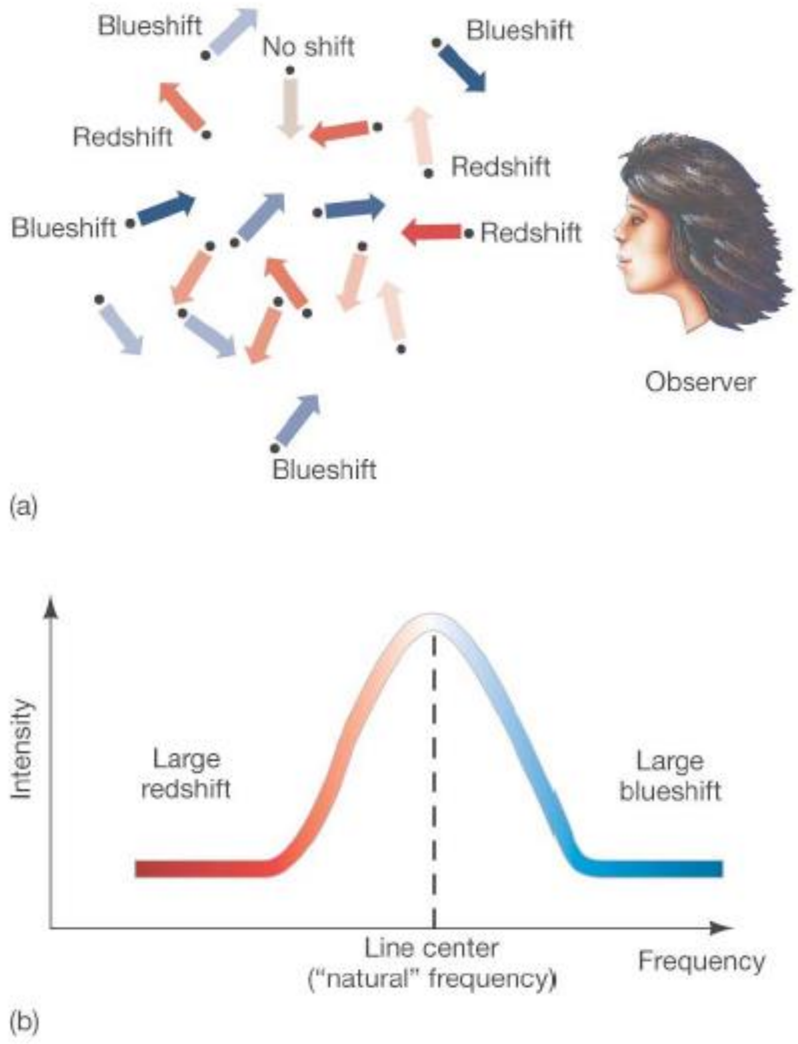
If the frequency of collisions is ν_{col}, then expect to get a collisional linewidth of about $\Delta\nu \sim \nu_{col}$.

Frequency of collisions increases with density - expect to see broader lines in high density regions as compared to low density ones.

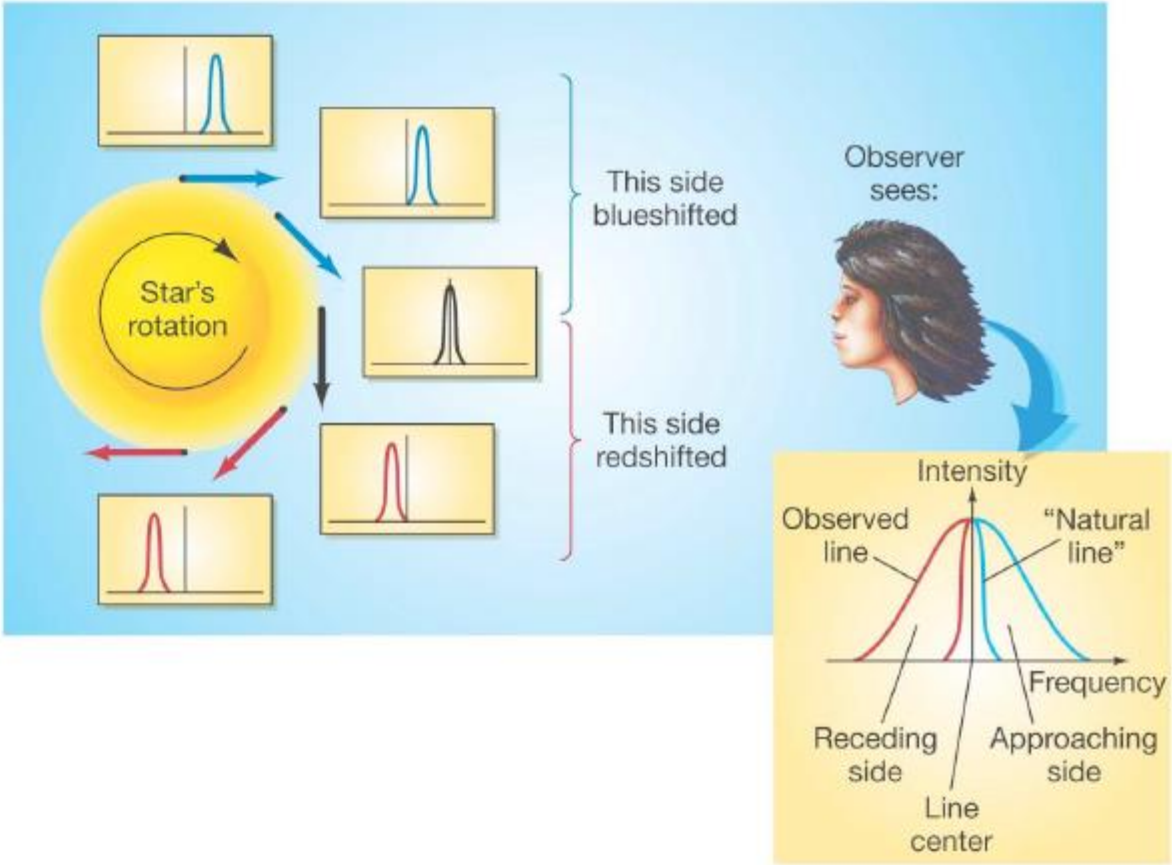
e.g. a main sequence star (small radius) has a higher density at the photosphere than a giant of the same surface temperature. Spectral lines in the main sequence star will be broader than in the giant.

Alargamento Doppler ou térmico

The Doppler shift may cause thermal broadening of spectral lines



Rotation will also cause broadening of spectral lines through the Doppler effect



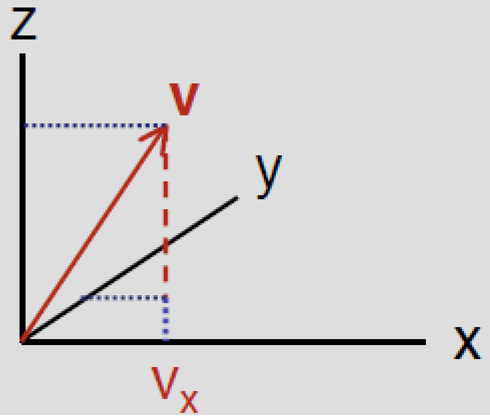
Doppler or thermal broadening

Atoms in a gas have random motions that depend upon the temperature. For atoms of mass m, at temperature T, the typical speed is obtained by equating kinetic and thermal energy:

$$\frac{1}{2}mv^2 = kT$$

k = Boltzmann's const

Number of atoms with given speed or velocity is given by **Maxwell's law**. Need to distinguish between forms of this law for speed and for any one velocity component:



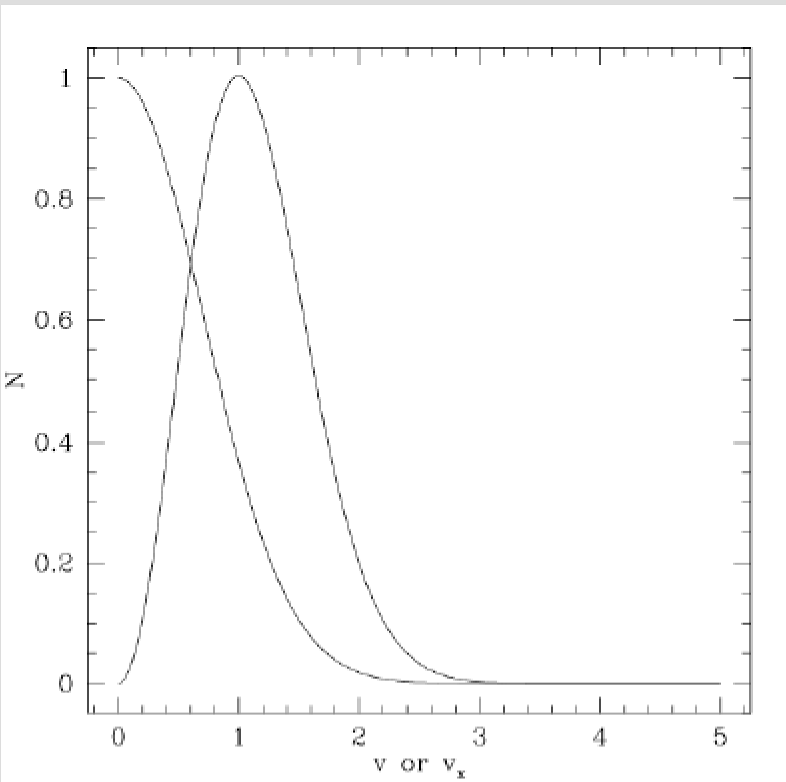
$$|\mathbf{v}|^2 = v_x^2 + v_y^2 + v_z^2$$

Distribution of *one component* of the velocity, say v_x, is relevant for thermal broadening - only care about motion along line of sight.

For one component, number of atoms dN within velocity interval dv_x is given by:

$$dN(v_x) \propto \exp\left(-\frac{mv_x^2}{2kT}\right)dv_x$$

Distribution law for speeds has extra factor of v^2:



$$dN(v) \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right)dv$$

Most probable speed:

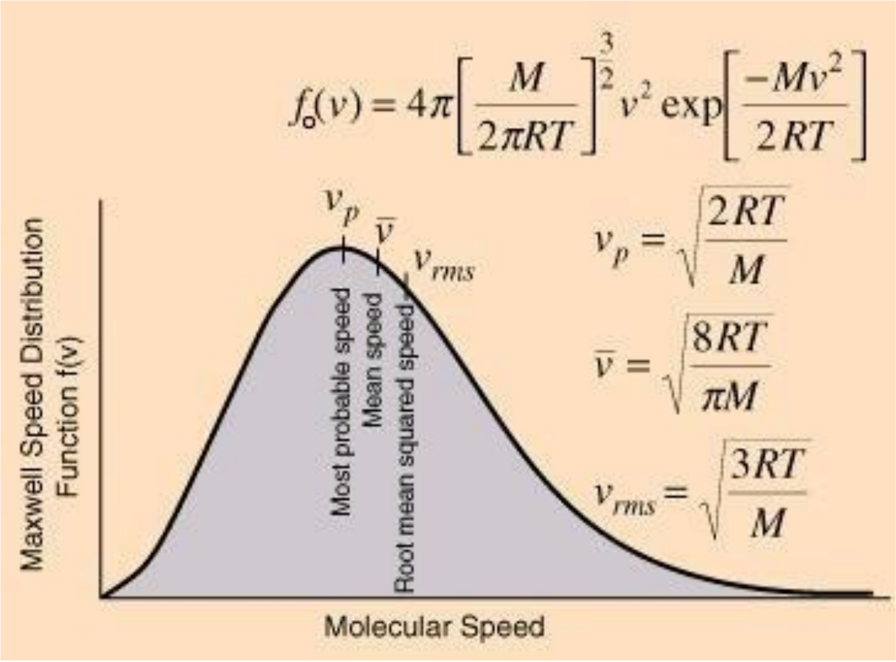
$$v_{peak} = \sqrt{\frac{2kT}{m}}$$

Average speed:

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

Velocidades – estatística

- The **Maxwell–Boltzmann distribution** or **Maxwell speed distribution** describes particle speeds in idealized gases where the particles move freely inside a stationary container without interacting with one another, except for very brief collisions in which they exchange energy and momentum with each other or with their thermal environment. Particle in this context refers to gaseous atoms or molecules, and the system of particles is assumed to have reached thermodynamic equilibrium.
- The distribution is a probability distribution for the *speed* of a particle within the gas - the magnitude of its velocity. This probability distribution indicates which speeds are more likely: a particle will have a speed selected randomly from the distribution, and is more likely to be within one range of speeds than another. The distribution depends on the temperature of the system and the mass of the particle.



Velocidade mais provável: v_p

- The most probable speed, v_p, is the speed most likely to be possessed by any molecule (of the same mass m) in the system and corresponds to the maximum value or mode of $f(v)$. To find it, we calculate df/dv, set it to zero and solve for v:

$$\frac{df(v)}{dv} = 0$$

- which yields:

$$v_p = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}}$$

where R is the gas constant and $M = N_A\, m$ is the molar mass of the substance.

Velocidade média $\bar{v}$ ou $\langle v \rangle$ e velocidade quadrática média v_{rms} *ou* $\sqrt{\langle v^2 \rangle}$

- The mean speed is the expected value of the speed distribution

$$\langle v \rangle = \int_0^{\infty} v\, f(v)\, dv = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \frac{2}{\sqrt{\pi}} v_p$$

- The root mean square speed is the second-order moment of speed:

$$\sqrt{\langle v^2 \rangle} = \left(\int_0^{\infty} v^2\, f(v)\, dv \right)^{1/2} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3}{2}} v_p$$

Emits at frequency ν_0

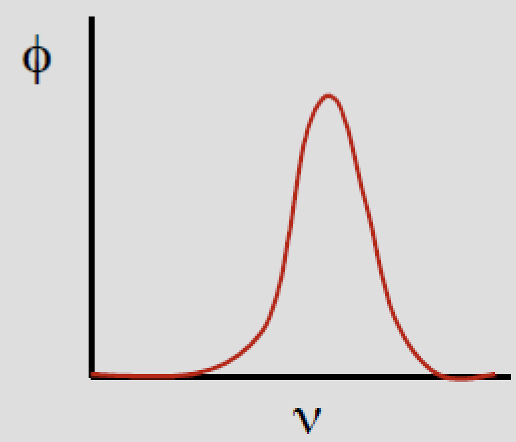
Observed at frequency ν

Consider atom moving with velocity v_x along the line of sight to the observer.

Doppler shift formula: $$\frac{\nu - \nu_0}{\nu_0} = \frac{v_x}{c}$$

Combine this with the thermal distribution of velocities:

$$\phi(\nu) = \frac{1}{\Delta\nu_D \sqrt{\pi}} \exp\left[-\frac{(\nu - \nu_0)^2}{(\Delta\nu_D)^2} \right]$$



…where the **Doppler width** of the line:

$$\Delta\nu_D = \frac{\nu_0}{c} \sqrt{\frac{2kT}{m}}$$

If the gas also has large-scale (i.e. not microscopic) motions due to turbulence, those add to the width:

$$\Delta\nu_D = \frac{\nu_0}{c}\left(\frac{2kT}{m} + v_{turb}^2\right)^{1/2}$$

v_{turb} is a measure of the typical turbulent velocity (note: really need same velocity distribution for this to be strictly valid).

Some numbers for hydrogen:

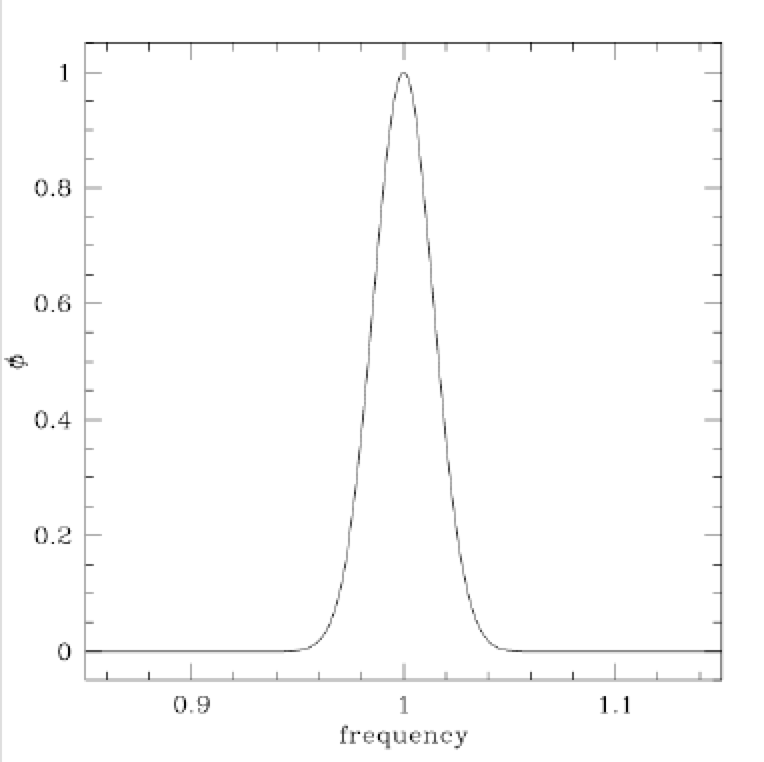
$$\frac{\Delta\nu_D}{\nu_0} \approx 4.3\times10^{-5}\left(\frac{T}{10^4\ \mathrm{K}}\right)^{1/2}$$

larger than natural linewidth

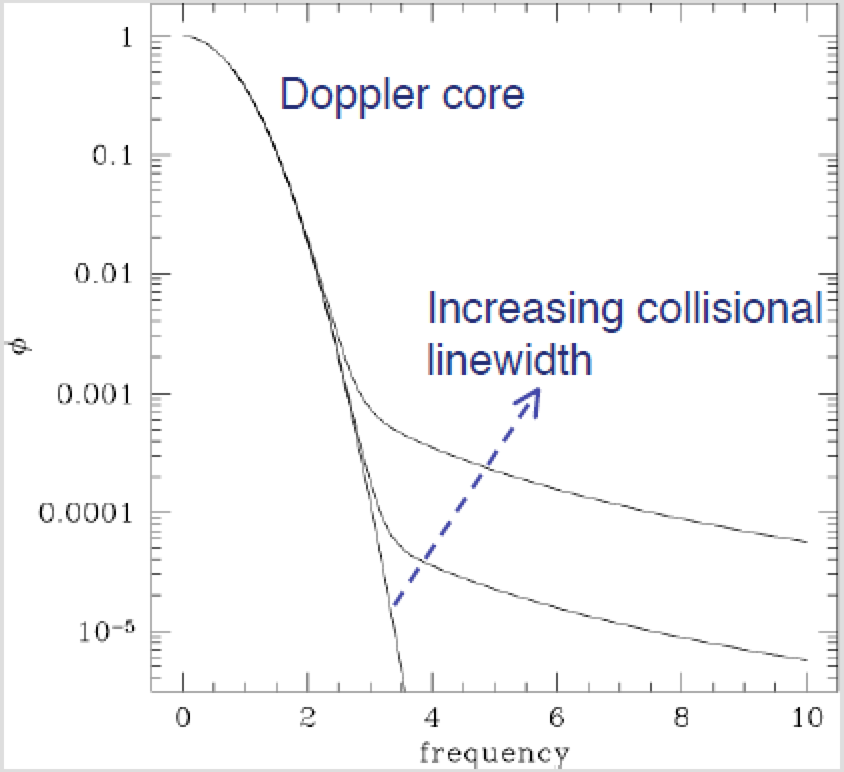
$$\frac{\Delta\nu_D c}{\nu_0} \approx 13\left(\frac{\mathrm{T}}{10^4\ \mathrm{K}}\right)^{1/2}\ \mathrm{km\ s^{-1}}$$

measured in velocity units, comparable to the sound speed in the gas

Thermal line profile



Voigt profile: combination of thermal and natural (or collisional) broadening



Gaussian: falls off very rapidly away from line center

Natural line profile falls off more slowly - dominates wings of strong lines

Summary:

- **Strength** of different spectral lines depends upon the abundance of different elements, and on the excitation / ionization state (described in part by the Boltzmann formula).
- **Width** of spectral lines depends upon:
  - *Natural linewidth* (small)
  - *Collisional linewidth* (larger at high density)
  - *Thermal linewidth* (larger at higher temperature)

➡ High quality spectrum gives information on composition, temperature and density of the gas.

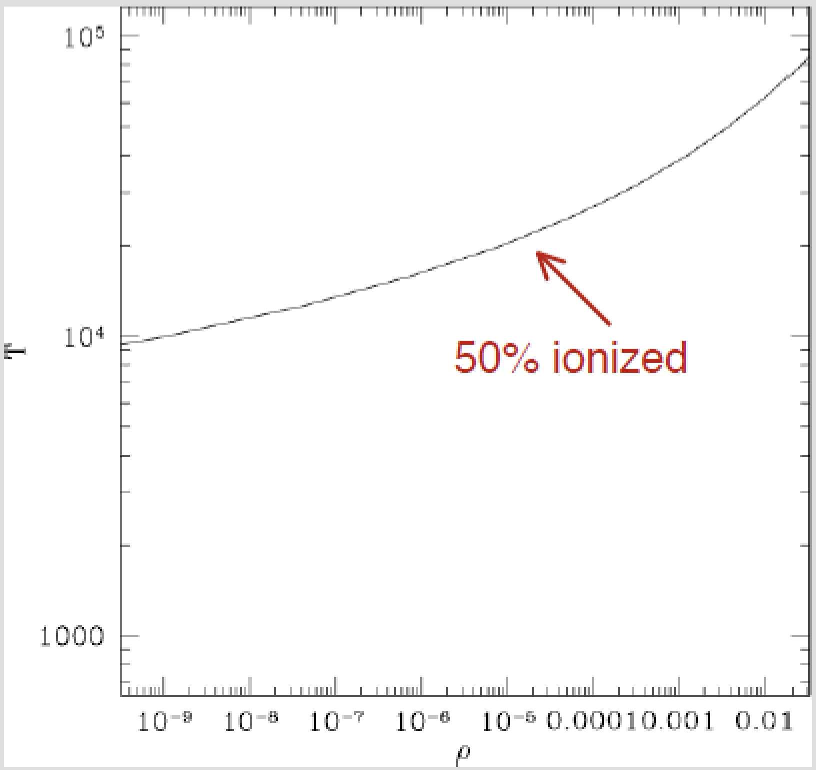
---

c.f. `Modern Astrophysics' section 8.1: more on thermal broadening, Boltzmann law, and Saha equation (version of Boltzmann law for ionization).

Free-free radiation: Bremsstrahlung

Hydrogen is ionized at T ~ 10^4 K at low density.

For the same mixture of chemical elements as the Sun, maximum radiation due to spectral lines occurs at T ~ 10^5 K.

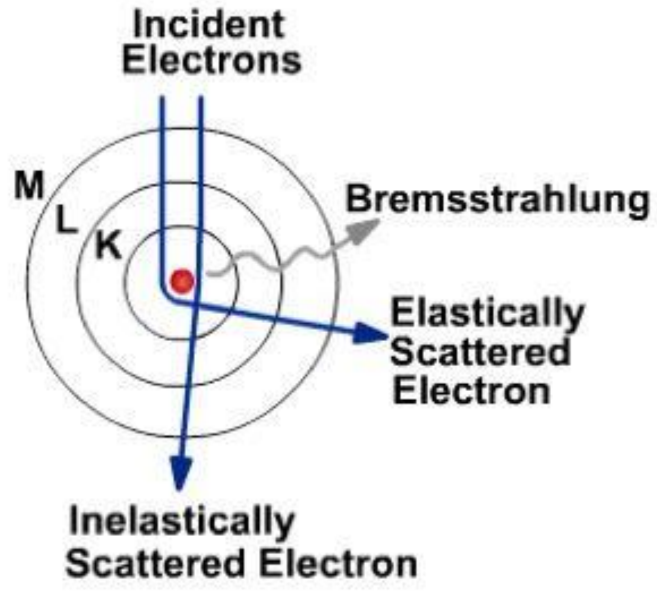


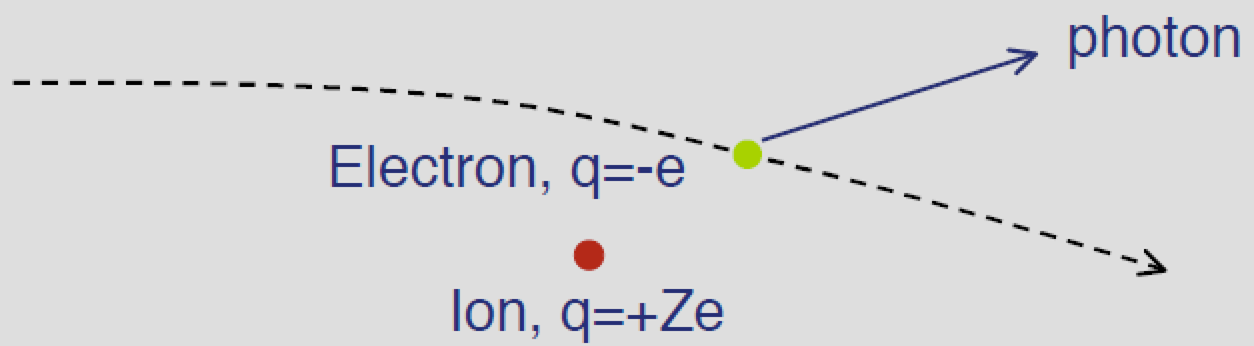
At higher T, radiation due to acceleration of unbound electrons becomes most important.

Free-free radiation or **bremsstrahlung**.

Bremsstrahlung Radiation

- German for "Braking"
- Is caused when a free electron interacts with a nucleus
- Gives a continuous energy spectrum





`Collisions' between electrons and ions accelerate the electrons. Power radiated by a single electron is given by **Larmor's formula:**

$$P = \frac{2q^2}{3c^3}|\mathbf{a}|^2$$

c.g.s. units: q is the charge, where electron charge = 4.80 x 10^{-10} esu. **a** is the acceleration, c is speed of light.

Prefer to work in SI? Larmor's formula:

…with q in Coulombs, ε_0 is the permittivity of the vacuum [10^7 / ($4\pi c^2$) C^2 N^{-1} m^{-2}]

$$P = \frac{q^2}{6\pi\varepsilon_0 c^3}|\mathbf{a}|^2$$

Power is proportional to the square of the charge and the square of the magnitude of the acceleration.

To derive spectrum of bremsstrahlung, and total energy loss rate of the plasma, need to:

- Calculate acceleration and energy loss for one electron of speed v, passing ion at impact parameter b.
- Integrate over all collisions, assuming a distribution of encounter speeds (normally a thermal / Maxwellian distribution).

Total energy loss rate from Bremsstrahlung

Plasma at:

- Temperature T
- Electron number density n_e (units: cm^{-3})
- Ions, charge Ze, number density n_i

Rate of energy loss due to bremsstrahlung is:

$$\varepsilon^{ff} = 1.4 \times 10^{-27} T^{1/2} n_e n_i Z^2 \text{ erg s}^{-1} \text{ cm}^{-3}$$

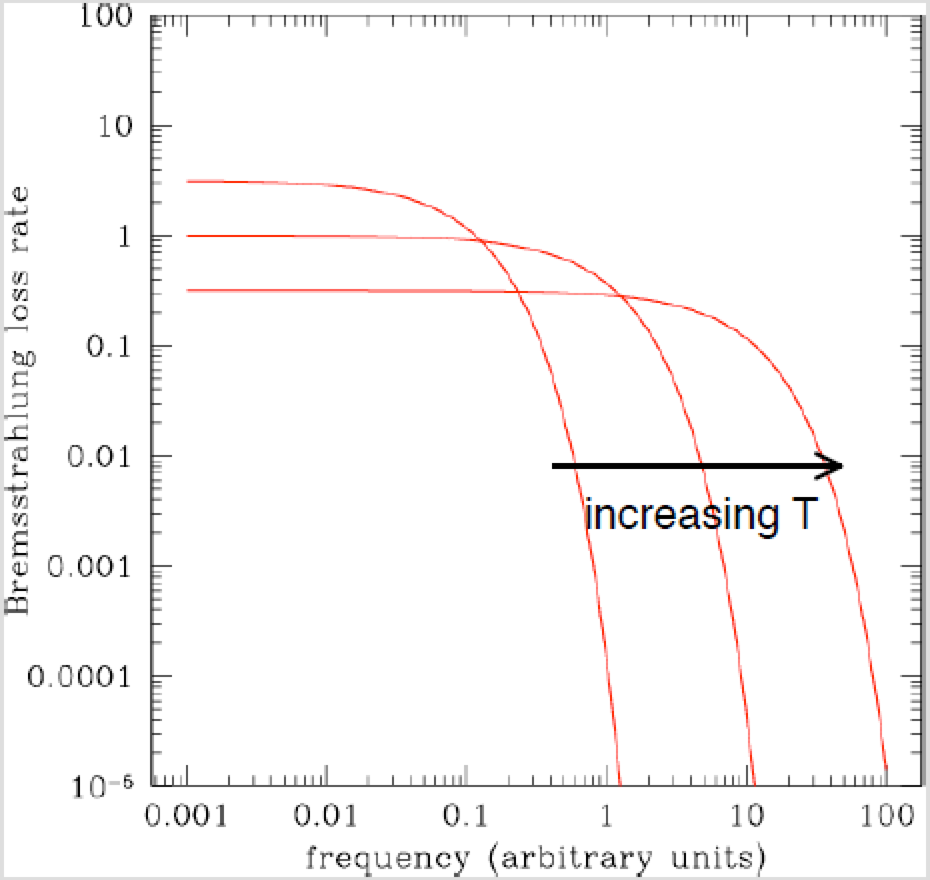
For pure hydrogen, Z=1 and $n_e = n_i$:

$$\varepsilon^{ff} = 1.4 \times 10^{-27} T^{1/2} n_e^2 \text{ erg s}^{-1} \text{ cm}^{-3}$$

Note: this is the energy loss rate per unit volume (1 cm^3) of the gas.

Spectrum of bremsstrahlung

$$\varepsilon_\nu^{ff} = 6.8\times10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \text{ erg s}^{-1}\text{ cm}^{-3}\text{ Hz}^{-1}$$



Flat spectrum up to an exponential cut off, at $h\nu = kT$.

Energy loss rate (overall and per Hz) depends on the **square** of the density.

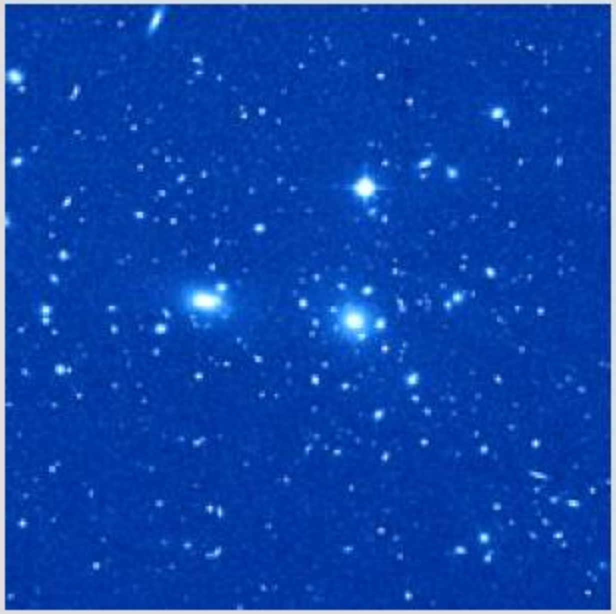
Continuous spectrum.

Shape of bremsstrahlung spectrum

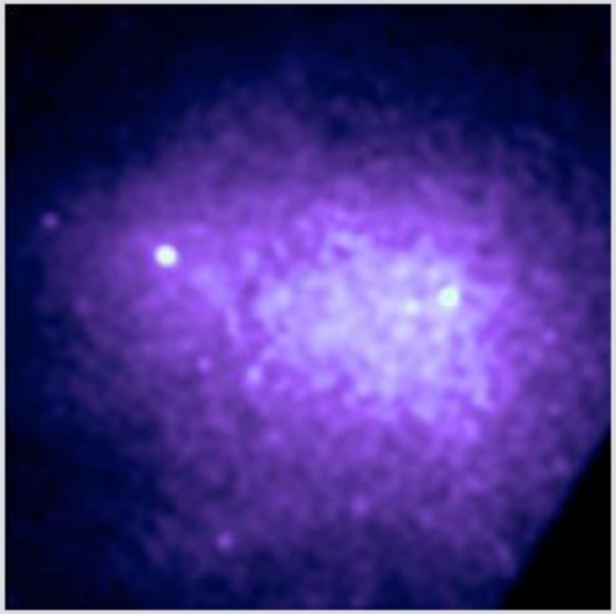
When is bremsstrahlung important?

$$\left.\begin{array}{l}\text{Bremsstrahlung loss rate increases with temperature} \\ \text{Atomic processes become less important as the gas} \\ \quad\quad\text{becomes fully ionized}\end{array}\right\}\text{high T}$$

Example: gas in the Coma cluster of galaxies

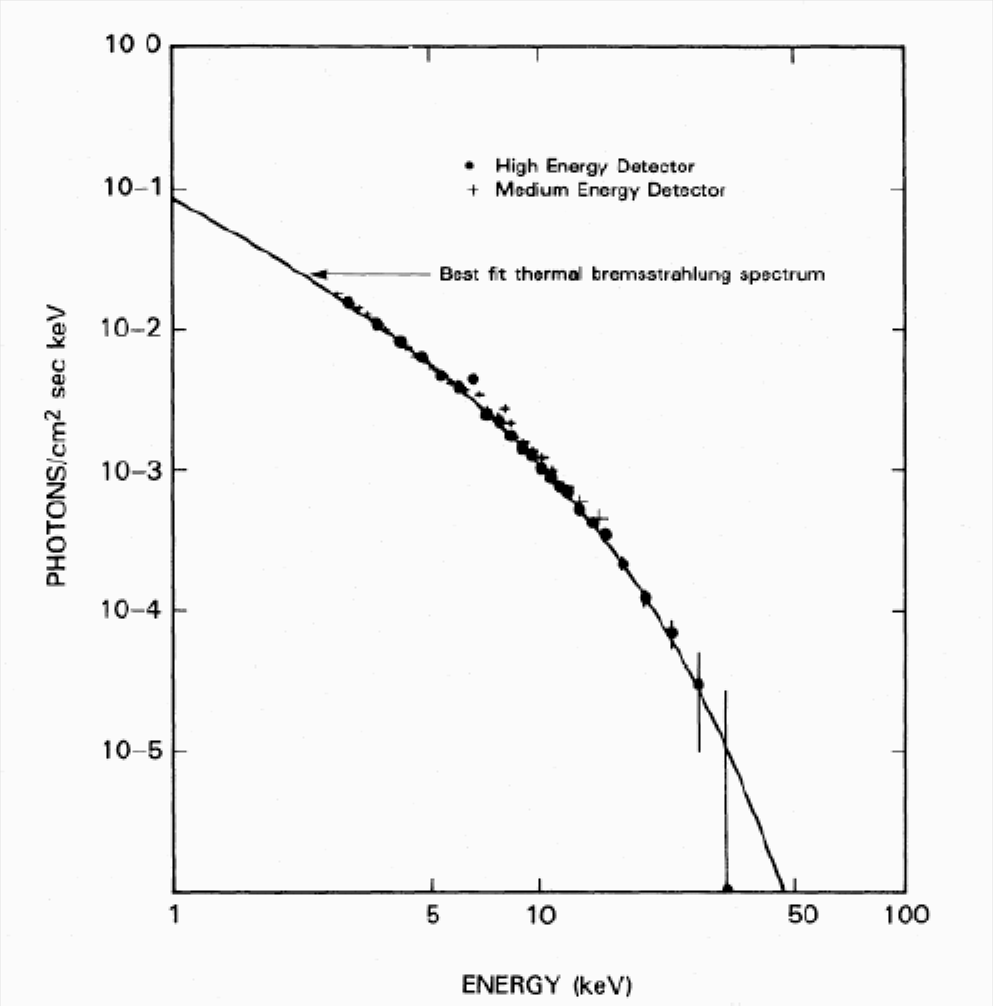


Optical



X-ray

X-ray spectrum of Coma



Shape of spectrum gives the temperature.

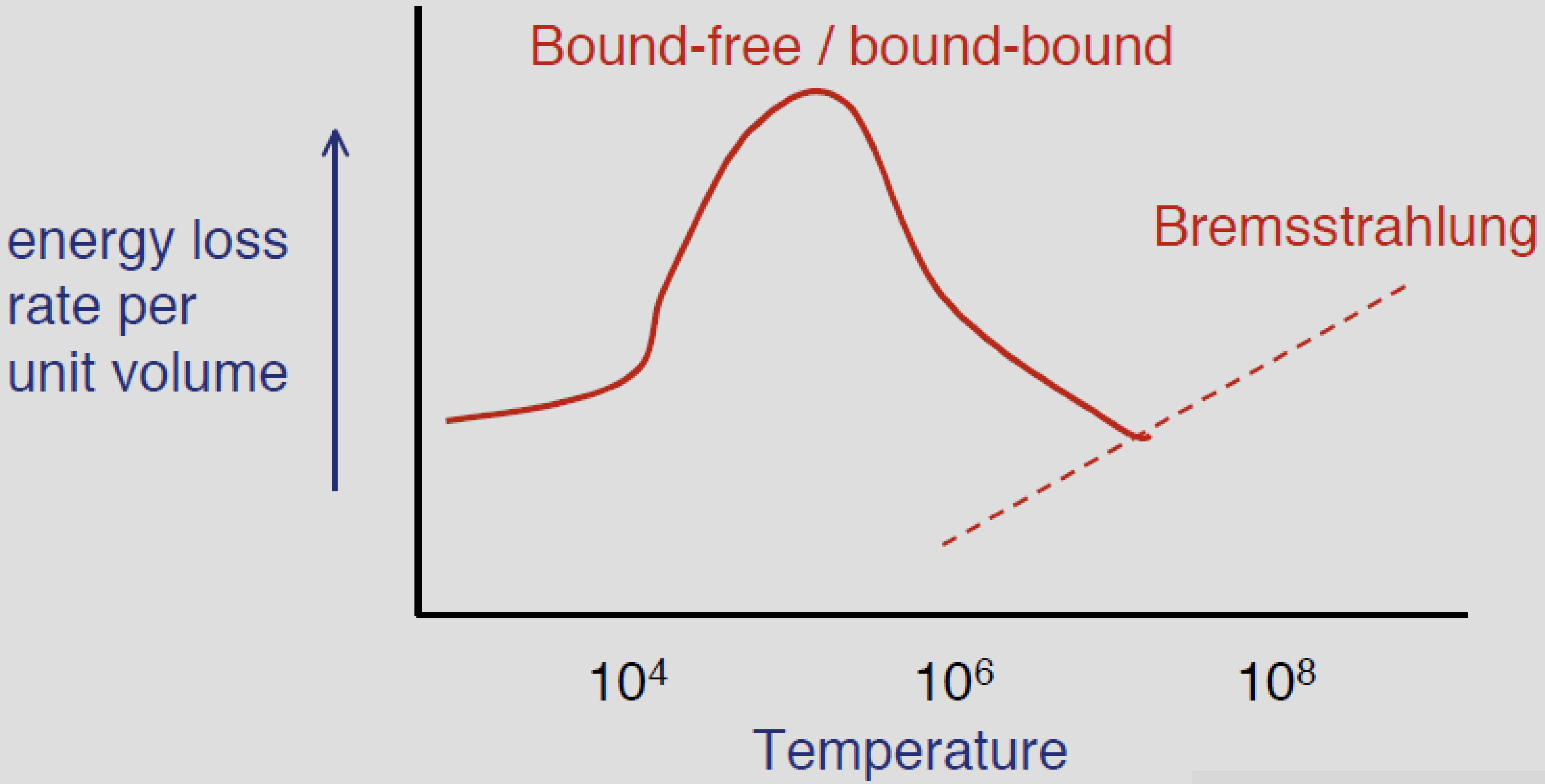
Intensity (for a known distance) gives the density of the gas.

Galaxy cluster: find T = 10 - 100 million K.

Overall energy loss rate from a gas

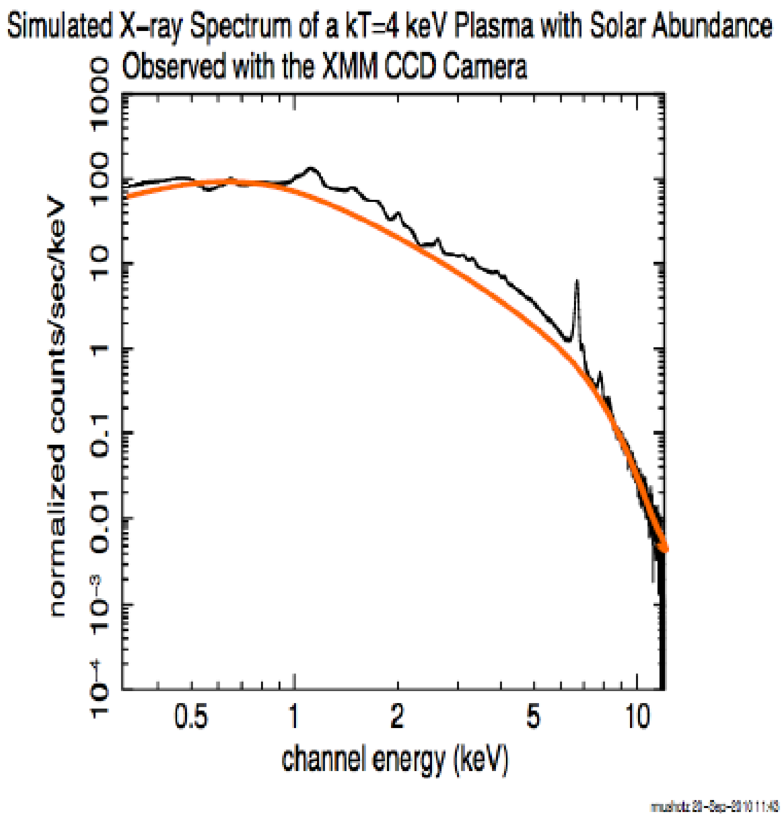
Sum up various processes: bound-bound, bound-free, free-free. Depend upon the square of the density.

Results of a detailed calculation:



X-ray Spectrum of a Hot Plasma

- Continuum is due to thermal bremmstrahlung (see Longair figure 6.2)
- Emission lines are due to recombination of H and He-like ions (more later)
- Curvature of spectrum gives temperature- amplitude gives emission measure (n^2V)
- Detailed fit to shape confirms physical mechanism of radiation

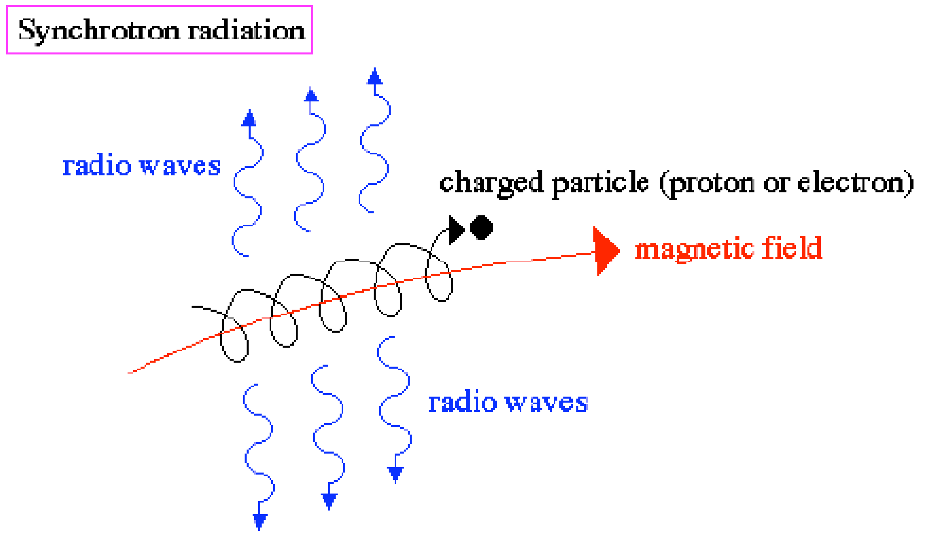


Conclude:

- gas of Solar composition cools most efficiently at temperatures $\sim 10^5$ K - lots of atomic coolants.
- cold gas cools further inefficiently - have to rely on molecules at very low T
- gas at $T \sim 10^7$ K also cools slowly - all atoms are ionized but bremsstrahlung not yet very effective.

Use this plot when we consider why gas in the galaxy comes in different phases.

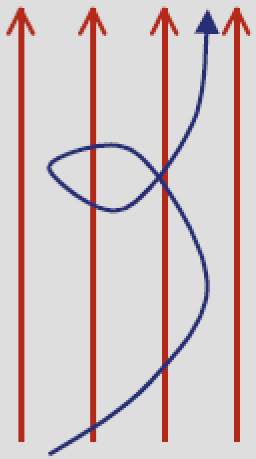
Synchrotron Radiation



synchrontron radiation occurs when a charged particle encounters a strong magnetic field – the particle is accelerated along a spiral path following the magnetic field and emitting radio waves in the process – the result is a distinct radio signature that reveals the strength of the magnetic field

- Polarization properties of light provides information on magnetic field geometry

Cyclotron and synchrotron radiation



Electron moving perpendicular to a magnetic field feels a Lorentz force.

⇒ Acceleration of the electron.

⇒ Radiation (Larmor's formula).

Define the Lorentz factor: $\gamma \equiv \dfrac{1}{\sqrt{1 - v^2/c^2}}$

<u>Non-relativistic electrons</u>: ($\gamma \sim 1$) - **cyclotron radiation**

<u>Relativistic electrons</u>: ($\gamma >> 1$) - **synchrotron radiation**

Same physical origin but very different spectra - makes sense to consider separate phenomena.

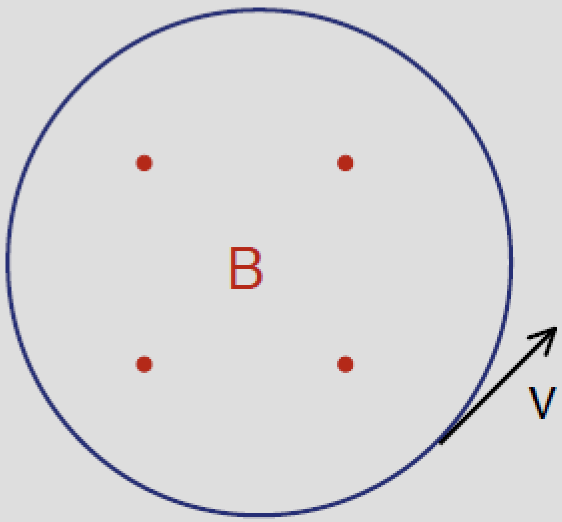
Cyclotron

Start with the non-relativistic case:

Particle of charge q moving at velocity **v** in a magnetic field **B** feels a force:

$$\mathbf{F} = \frac{q}{c}\mathbf{v} \times \mathbf{B}$$

Let v be the component of velocity perpendicular to the field lines (component *parallel* to the field remains constant). Force is constant and normal to direction of motion.



Circular motion: acceleration -

$$a = \frac{qvB}{mc}$$

…for particle mass m.

Let **angular velocity** of the rotation be ω_B. Condition for circular motion:

$$m\frac{v^2}{r} = \frac{qvB}{c}$$

$$m\omega_B v = \frac{qvB}{c}$$

$$\omega_B = \frac{qB}{mc}$$

Use c.g.s. units when applying this formula, i.e.

- electron charge = 4.80 x 10^{-10} esu
- B in Gauss
- m in g
- c in cm/s

Power given by Larmor's formula:

$$P = \frac{2q^2}{3c^3}|\mathbf{a}|^2 = \frac{2q^2}{3c^3} \times \left(\frac{qvB}{mc}\right)^2 = \frac{2q^4\beta^2B^2}{3c^3m^2}$$

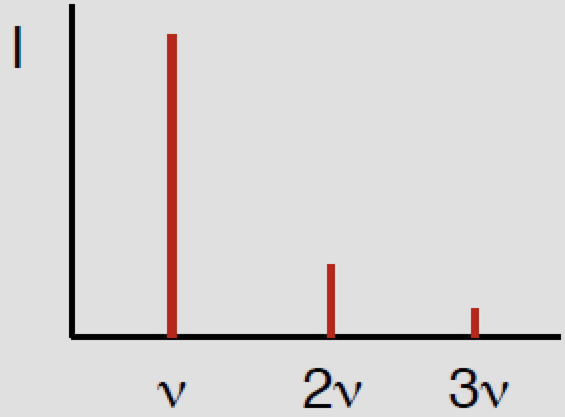
where β=v/c

$$P = \frac{2q^4\beta^2B^2}{3c^3m^2}$$

Magnetic energy density is $B^2 / 8\pi$ (c.g.s.) - energy loss is proportional to the energy density.

Energy loss is largest for low mass particles, electrons radiate much more than protons (c.f. highest energy particle accelerators are proton / antiproton not electron / positron).

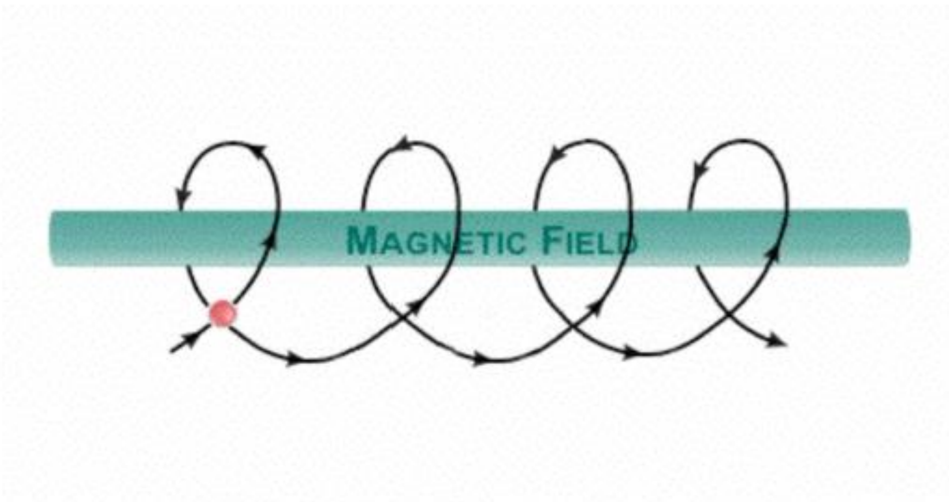
Cyclotron spectrum: ideally a single line at: $\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi mc}$



Actually get some emission at the harmonics too - 2ν, 3ν etc.

Very nice way to measure the magnetic field.

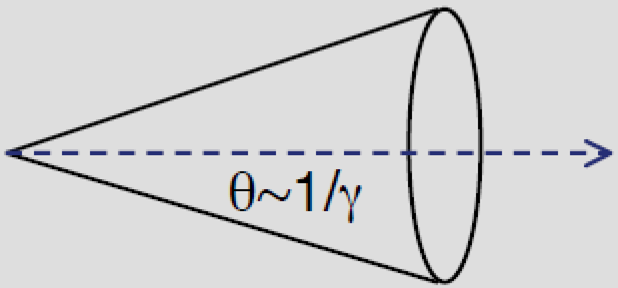
Syncrotron Emission



Synchrotron radiation

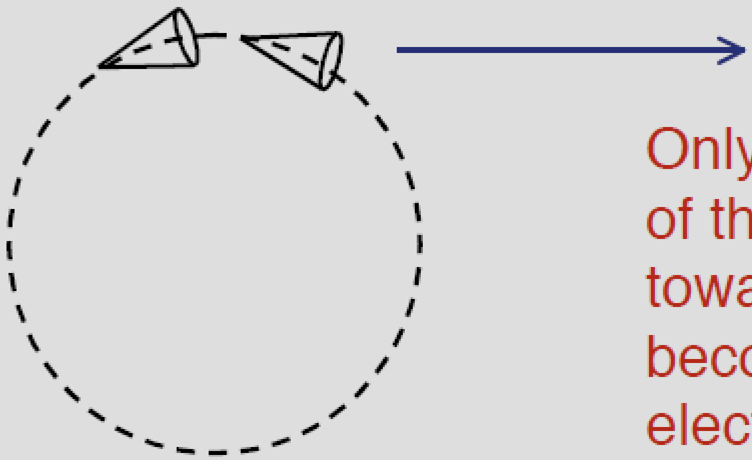
If the electrons are moving at close to the speed of light, two effects alter the nature of the radiation.

1) Radiation is **beamed**:

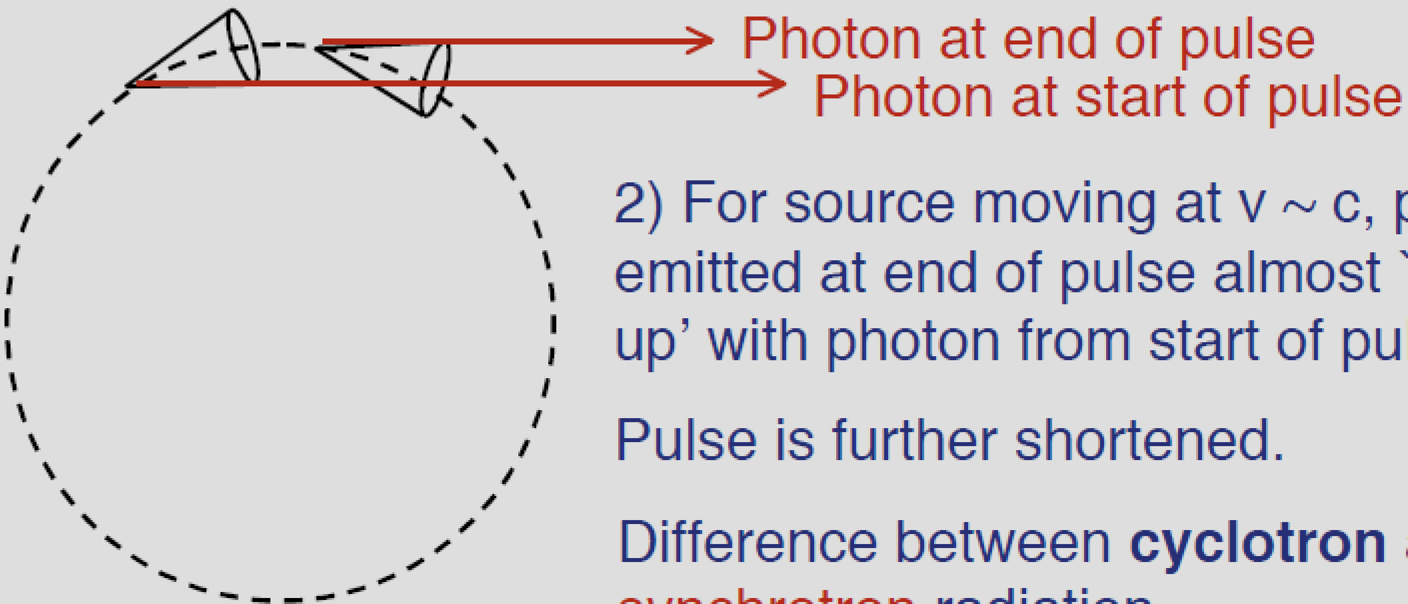


Particle moving with Lorentz factor γ toward observer emits radiation into cone of opening angle:

$$\theta \approx \gamma^{-1}$$



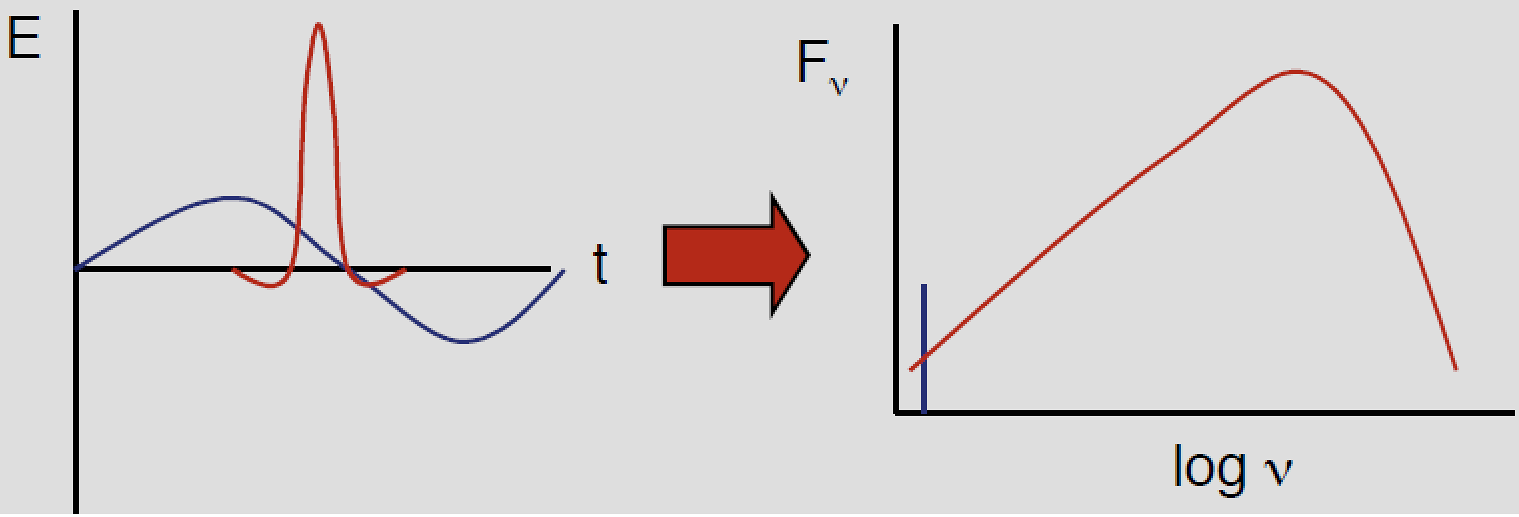
Only see radiation from a small portion of the orbit when the cone is pointed toward us - pulse of radiation which becomes shorter for more energetic electrons.



2) For source moving at v ~ c, photon emitted at end of pulse almost `catches up' with photon from start of pulse.

Pulse is further shortened.

Difference between **cyclotron** and synchrotron radiation.



Useful formulae for synchrotron radiation

For a **single particle**, spectrum extends up to a peak frequency roughly given by:

$$\nu \sim \underbrace{\gamma^2 \nu_c}_{\text{cyclotron frequency}} \sim \frac{\gamma^2 qB}{2\pi mc}$$

Can produce very high frequency radiation, with a continuous spectrum (no lines).

Normally, the electrons which produce synchrotron radiation have a (wide) range of energies. If number of particles with energy between E and E+dE can be written as:

$$N(E)dE = CE^{-p}dE$$

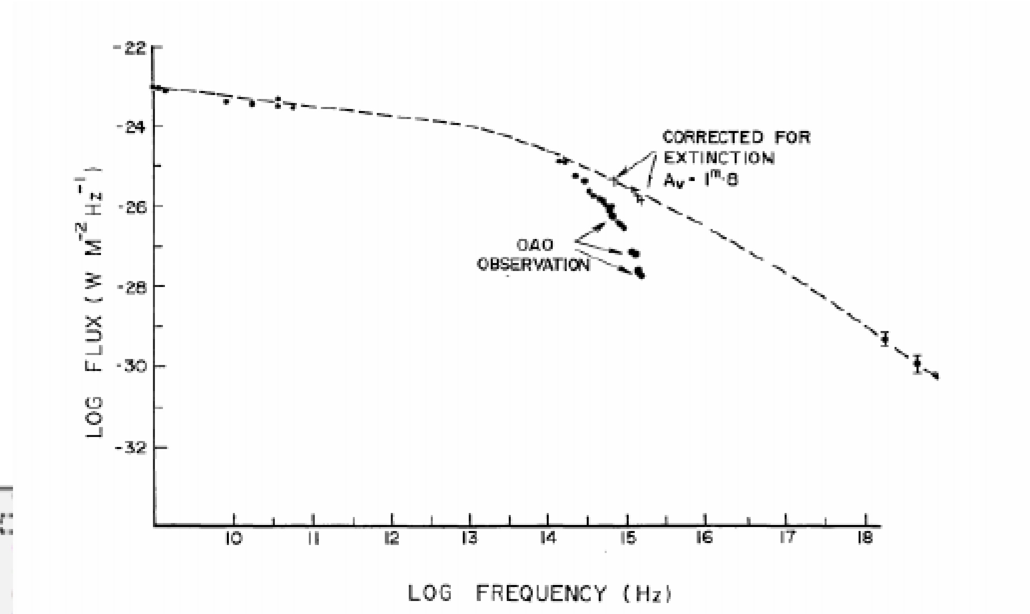
i.e. as a power-law in energy, then it turns out that the spectrum of the resulting synchrotron radiation is *also* a power-law, but with a different index:

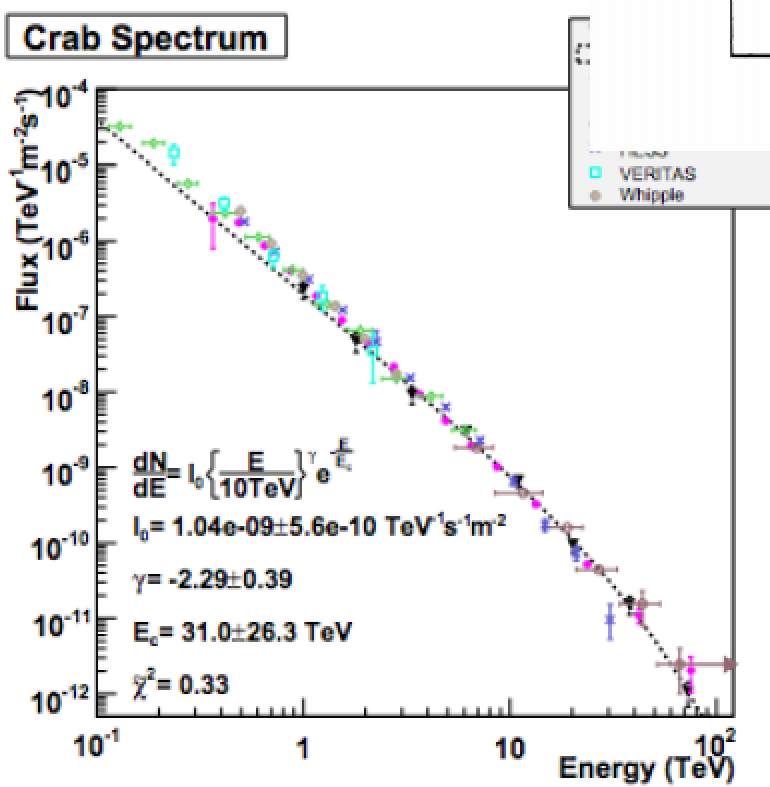
$$P(\nu) \propto \nu^{-s} \propto \nu^{-(p-1)/2}$$

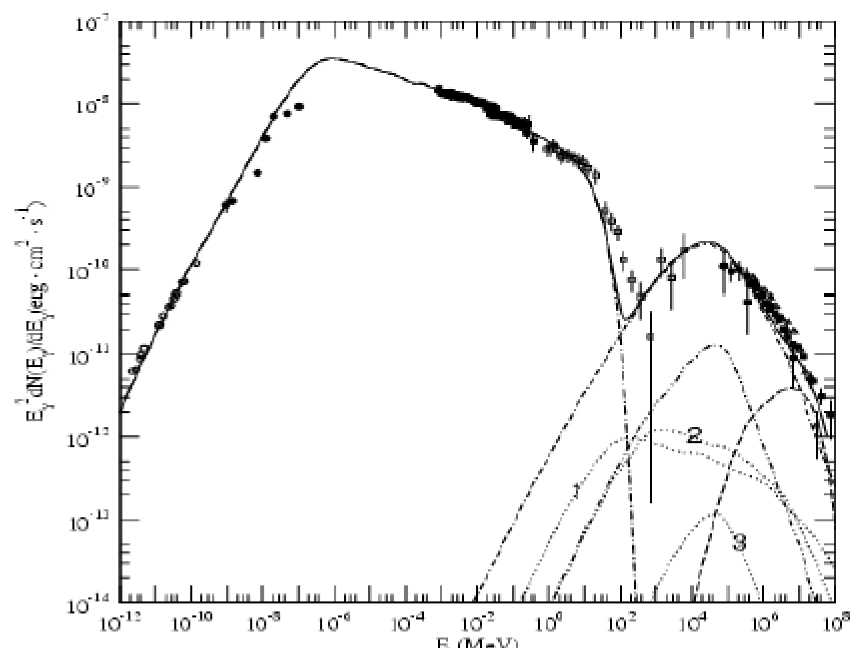
Measure the spectral index of the radiation (s), this then gives an indication of the distribution of particle energies (p)!

$$s = \frac{p-1}{2}$$

- Crab is the often used example of a 'pure' synchrotron emitter

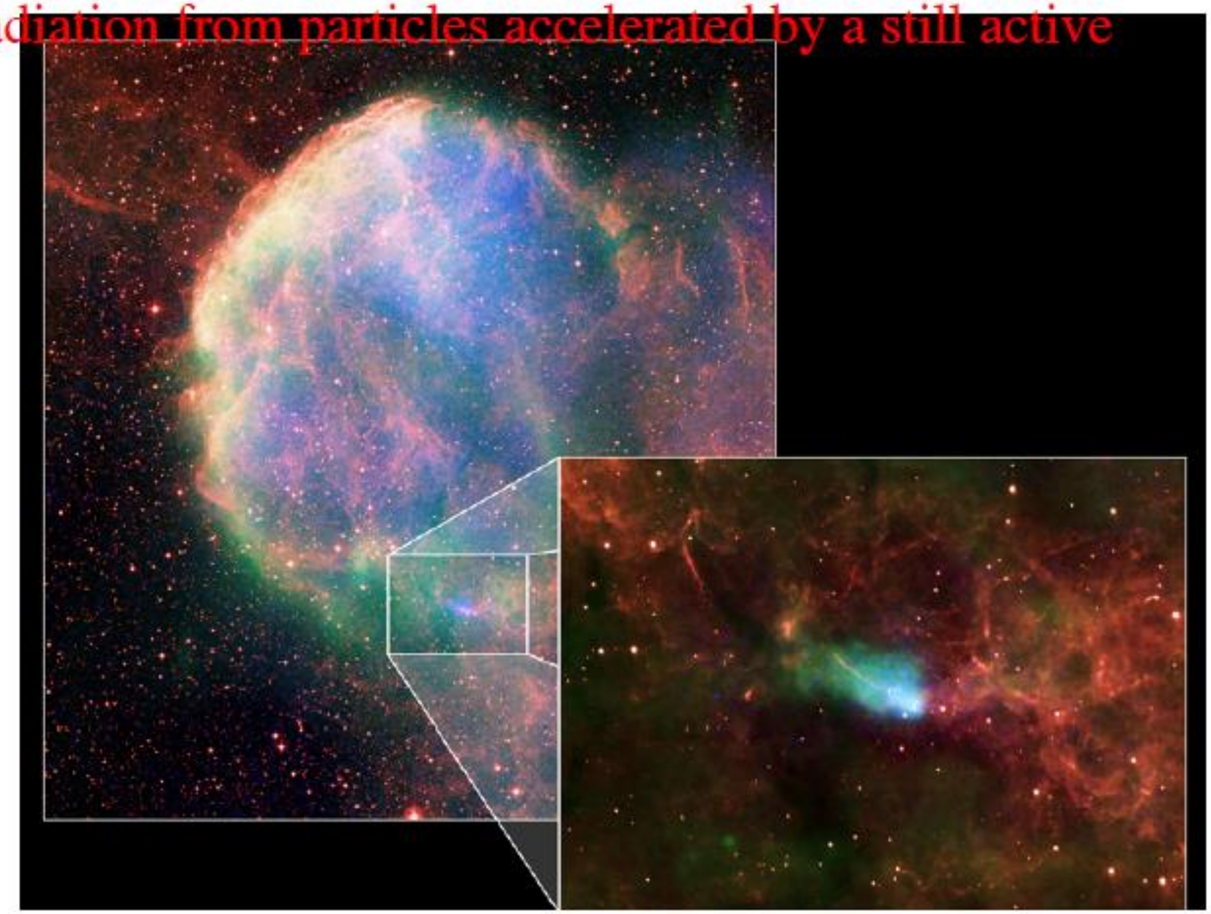


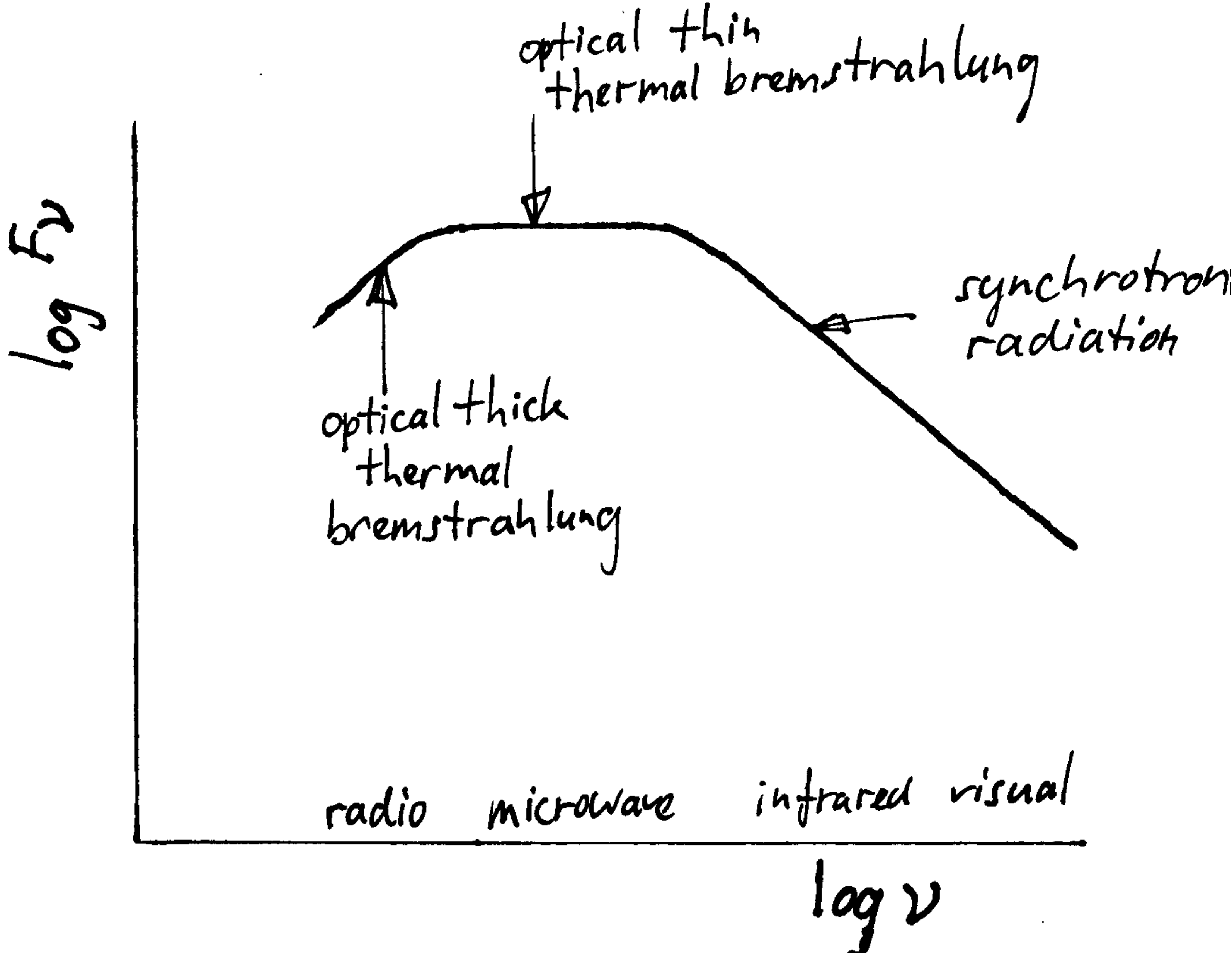




Combining Bremmstrahlung and Synchrotron Radiation

- In some supernova remnants one sees both processes at work
  - Bremmstrahlung from electrons that are shock heated by the SN blast wave
  - Synchrotron radiation from particles accelerated by a still active pulsar





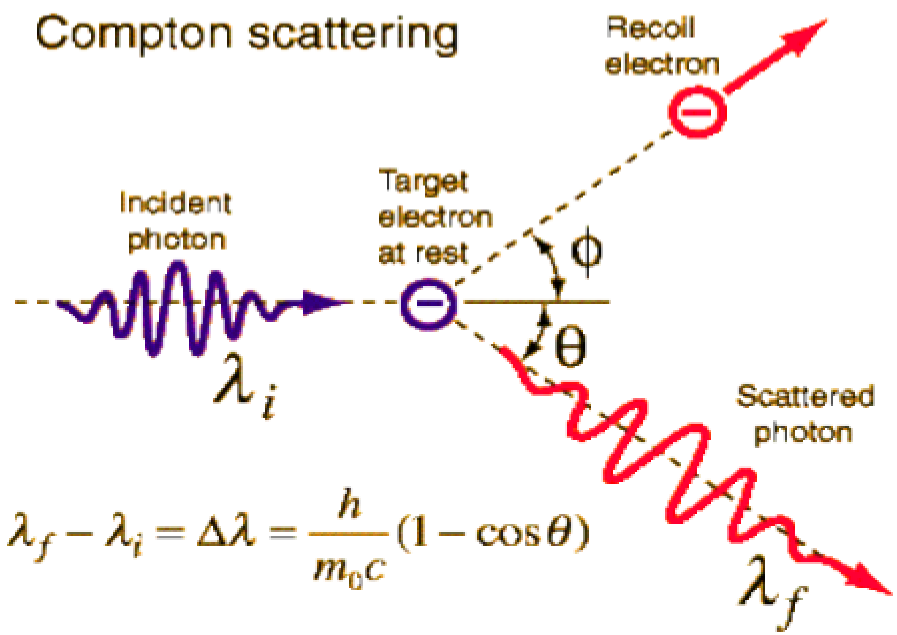
optical thin
thermal bremstrahlung
log Fν
synchrotron
radiation
optical thick
thermal
bremstrahlung
radio
microwave
infrared
visual
log ν

Thompson/Compton Scattering

- Thomson scattering: elastic scattering of low-energy photons from low-energy electrons, with cross-section $\sigma_T = (8\pi/3)\,(e^2/mc^2) = 0.665 \times 10^{-24}$ cm^2
- Compton scattering: low-energy photon inelastically scatters off non-relativistic electron, photon ends up with lower energy
- Inverse Compton scattering: low-energy photon inelastically scatters off relativistic electron, photon gains energy in observer rest frame

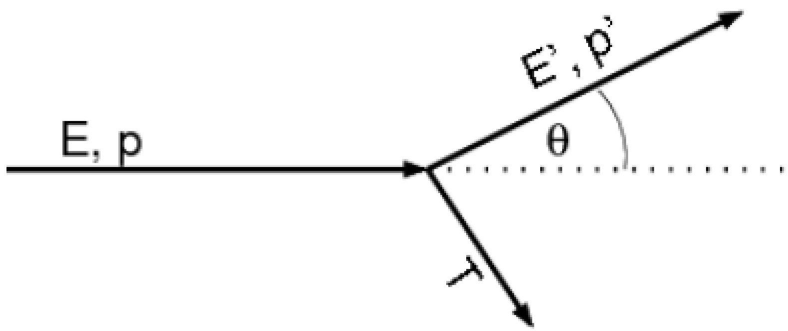
Whether the photon gives energy to the electron of vice versa

Compton Wavelength=h/mc=0.00243 nm for an electron



http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/compton.html

Compton Scattering



Thomson scattering: initial and final wavelength are identical.

But: in reality: light consists of photons

$\Longrightarrow$ Scattering: photon changes direction

$\Longrightarrow$ Momentum change

$\Longrightarrow$ Energy change!

This is a quantum picture

$\Longrightarrow$ Compton scattering.

Dynamics of scattering gives energy/wavelength change:

$$E' = \frac{E}{1 + \frac{E}{m_e c^2}(1 - \cos\theta)} \sim E\left(1 - \frac{E}{m_e c^2}(1 - \cos\theta)\right) \tag{7.14}$$

and

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{7.15}$$

where $h/m_e c = 2.426 \times 10^{-10}$ cm (Compton wavelength).

Averaging over θ, for $E \ll m_e c^2$:

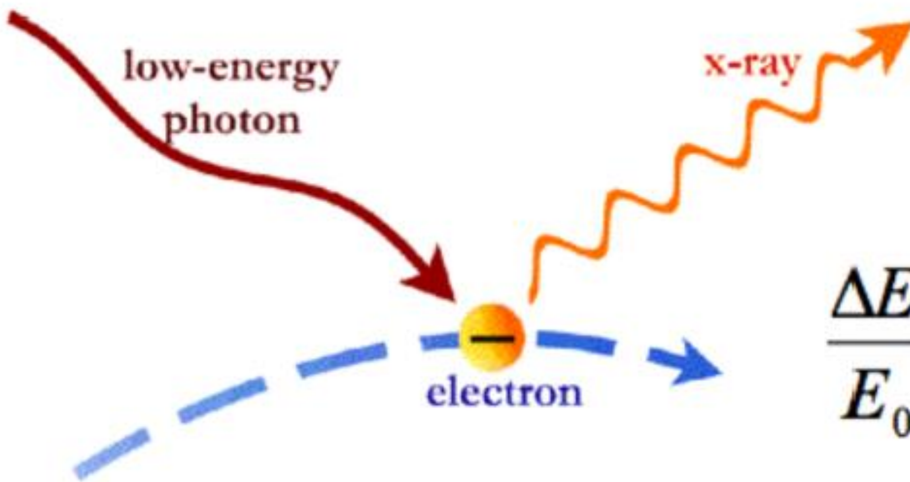
$$\frac{\Delta E}{E} \approx -\frac{E}{m_e c^2} \tag{7.16}$$

- http://pulsar.sternwarte.uni-erlangen.de/wilms/teach/radproc/radproc0177.html

INVERSE COMPTON EMISSION

Compton scattering

- Photon $E_0 = h\nu$ boosted in energy by hot e^- at kT to e.g. X-rays



$$\frac{\Delta E}{E_0} = \frac{1}{m_e c^2}(4kT - E_0)$$

Examples in X-ray astronomy: active galactic nuclei (AGN), X-ray binaries

INVERSE COMPTON EMISSION

- Results depend on source geometry

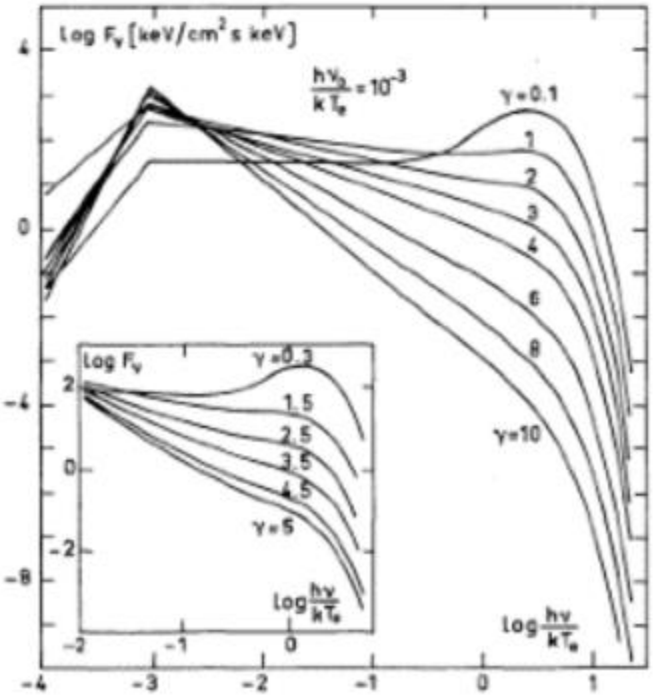


Fig. 5. The spectrum resulting from comptonization of low-frequency photons ($h\nu_0 = 10^{-3}$ kT$_e$) in a high temperature plasma clouds with different parameters γ (14)

- ***Power law***

$$F(E) = AE^{-\Gamma}e^{-E/E_c}$$

$$I(E) = BE^{-\alpha}e^{-E/E_c}$$

A,B normalizations

F,Γ ***photon*** flux photon index

I,α ***energy*** flux, index (α=Γ-1)

E_c=*kT*=cutoff energy

Sunyaev & Titarchuk 1980

Resumo: Como os fotons são gerados/absorvidos

- Processos físicos:

  - **Black body radiation**- system is in equilibrium and all electromagnetic radiation falling on it is absorbed. At a particular temperature a black body emits the maximum amount of energy possible for that temperature.

  - **Synchrotron radiation**
    High energy (relativistic) particles 'spiraling' in a magnetic field (accelerated electrons)

Compton scattering
Electrons scattering of photons/photons scattering off electrons

Line Emission and absorption
Atomic transitions in atoms- x-rays mostly from K, L shell transitions

Photoelectric Absorption
Photons are absorbed by atomic transitions

- continuum
  - blackbody
  - synchrotron & bremsstrahlung
  - scattering
  - radiative recombination
- lines
  - charge exchange
  - fluorescence
  - thermal

– Continuum

Thermal emission processes

Blackbody radiation
Bremsstrahlung

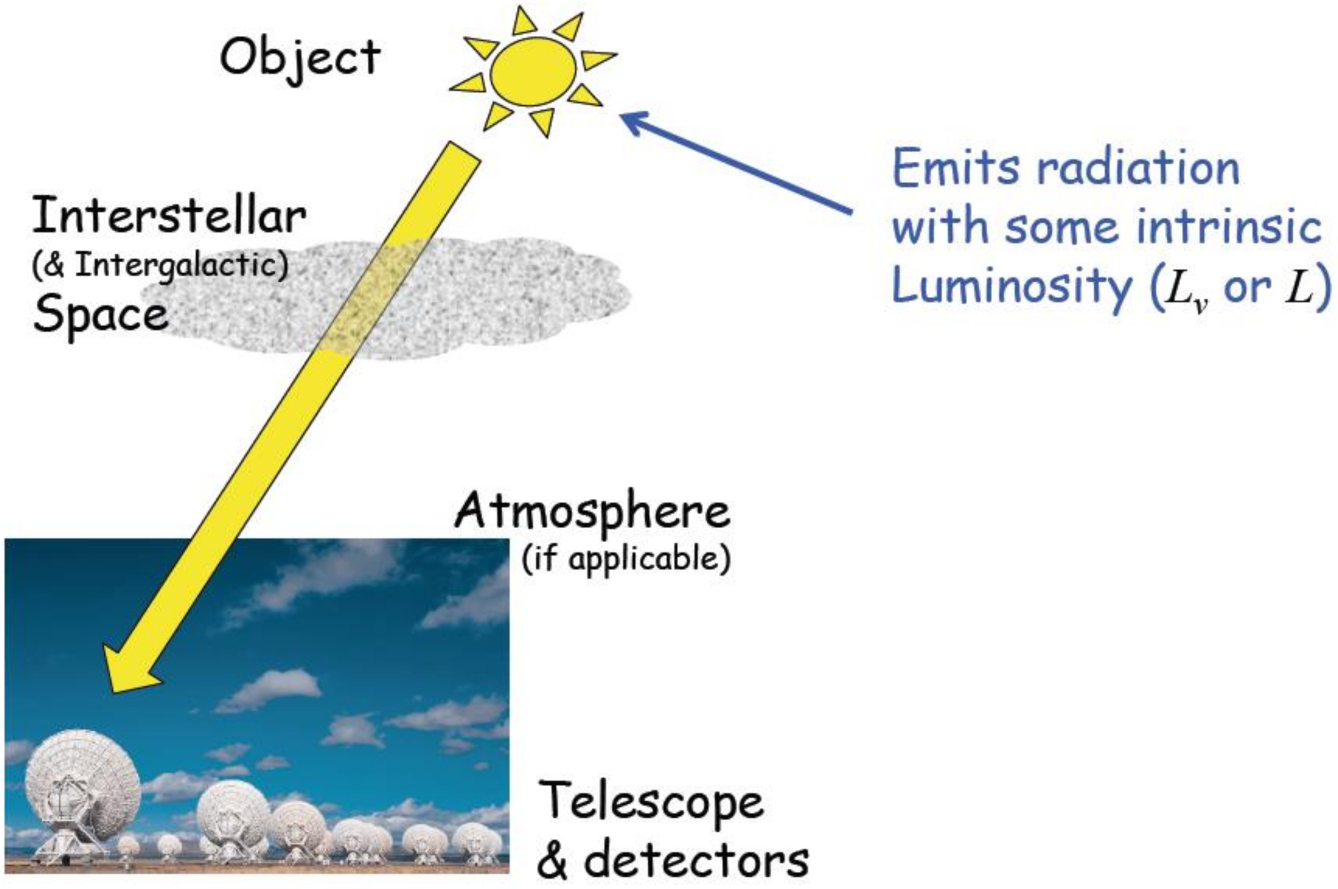
Non-thermal processes

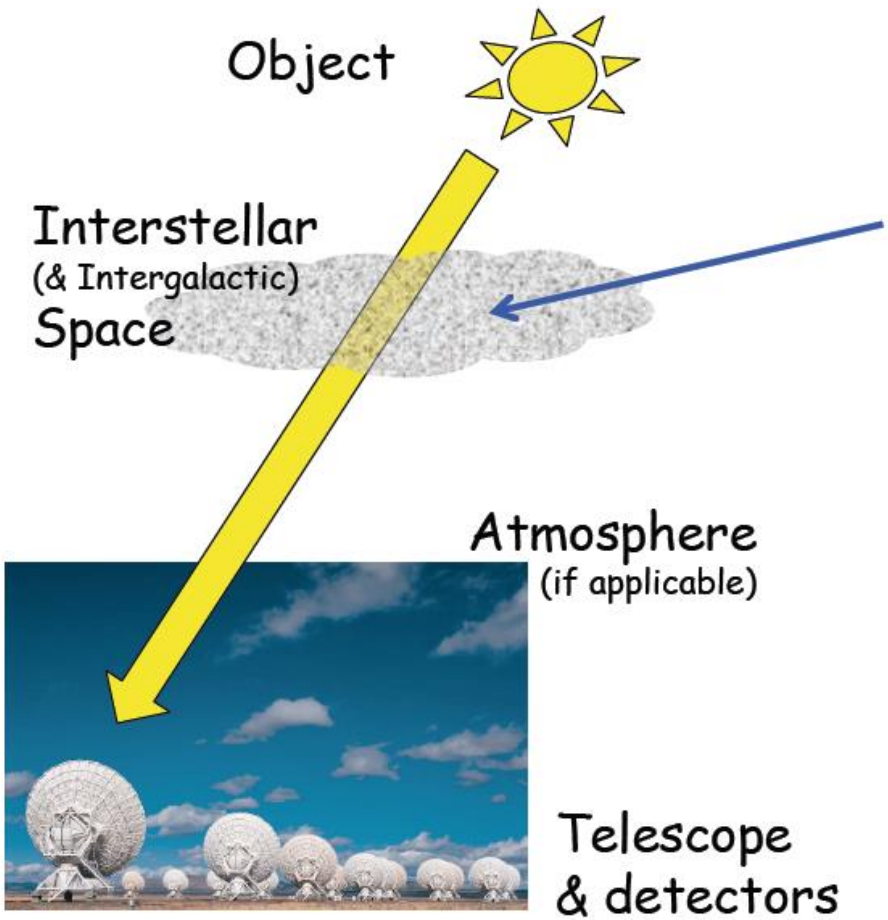
Synchrotron radiation
Inverse Compton emission
Non-thermal bremms

In "thermal" processes the electrons are in a Maxwell-Boltzman distribution- the system has a 'temperature'

In non-thermal the electron distribution is often a power law-no temperature

Tornando os conceitos realidade! (?)



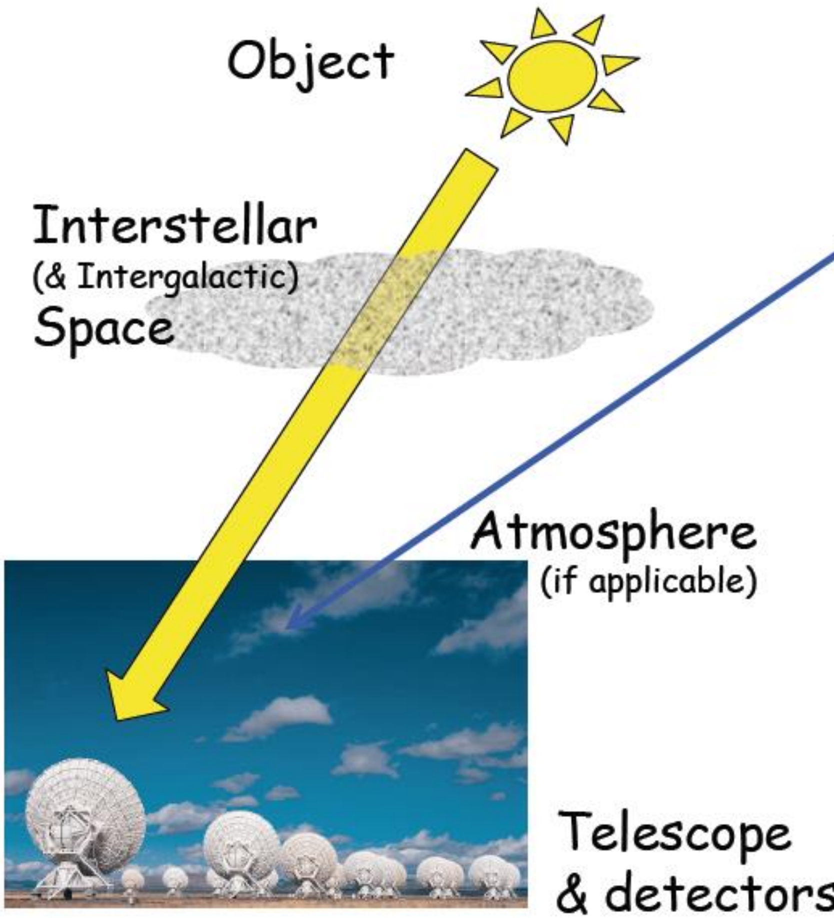


Radiation travels through space to Earth

... some of the radiation may be "lost" en route

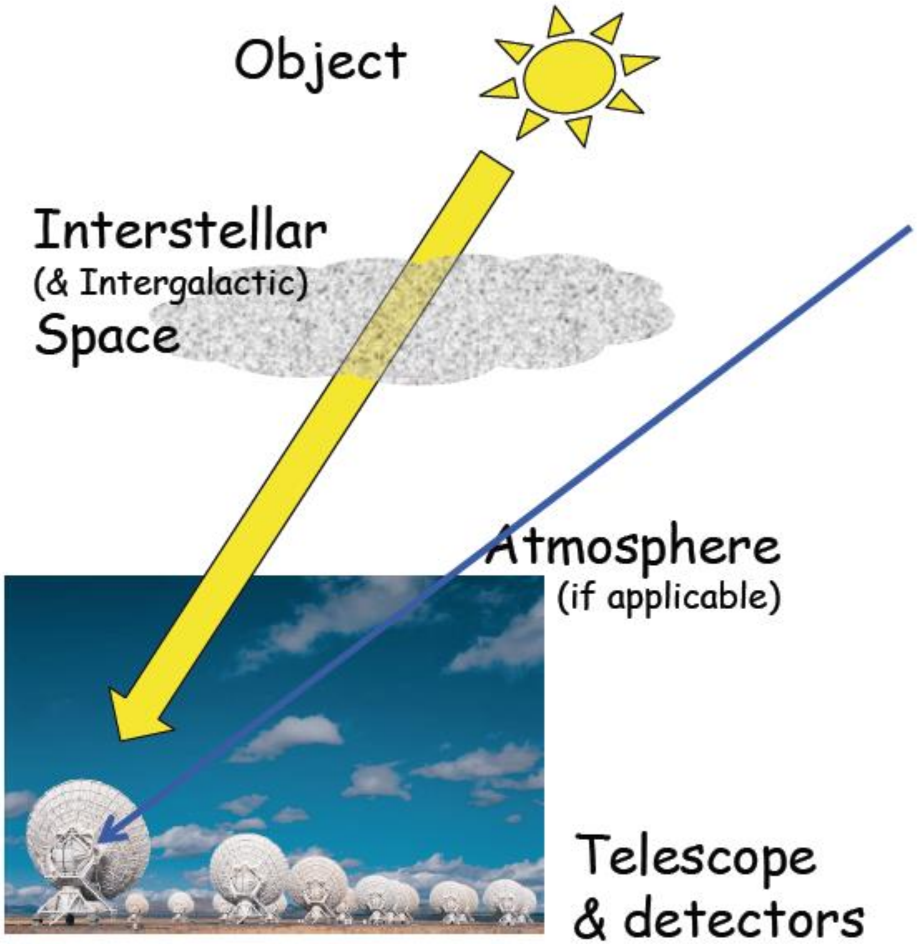
-But we may be able to account for this

(more later)



Radiation travels
through Earth's
Atmosphere
(if applicable)
... some of the
radiation may be
"lost" here too

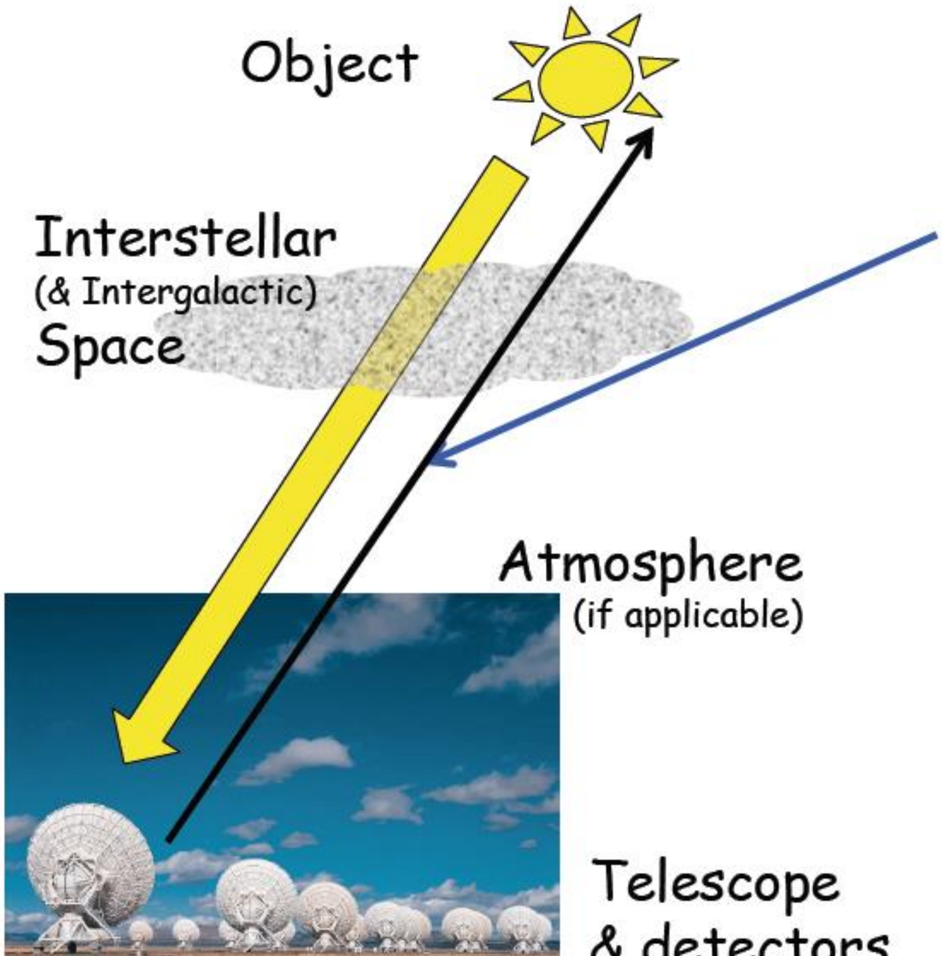
-But we may be
able to account
for this too
(more later)



Radiation enters our instruments

… some of the signal may be also "lost" here

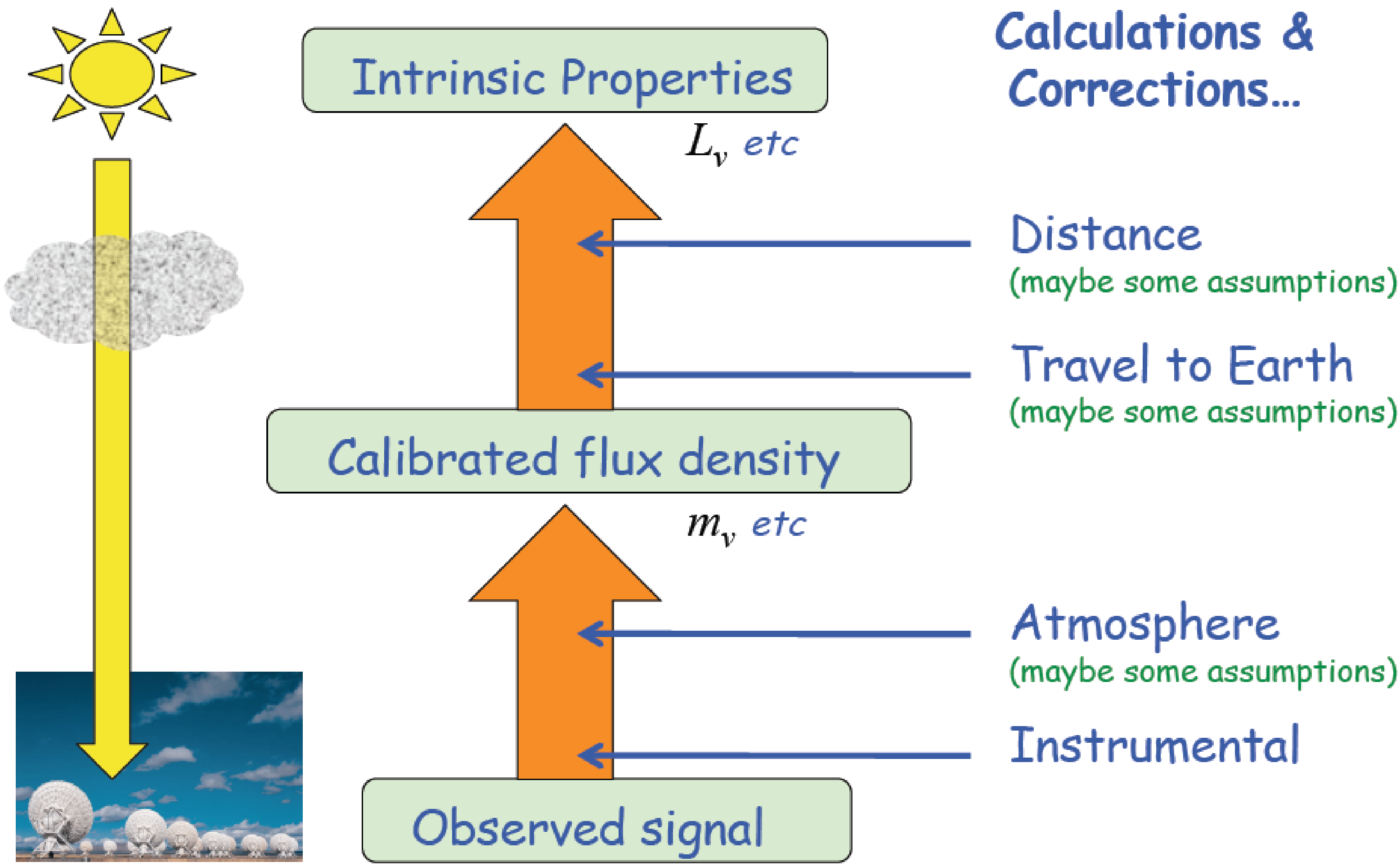
-But we certainly *can* account for this ("calibration")



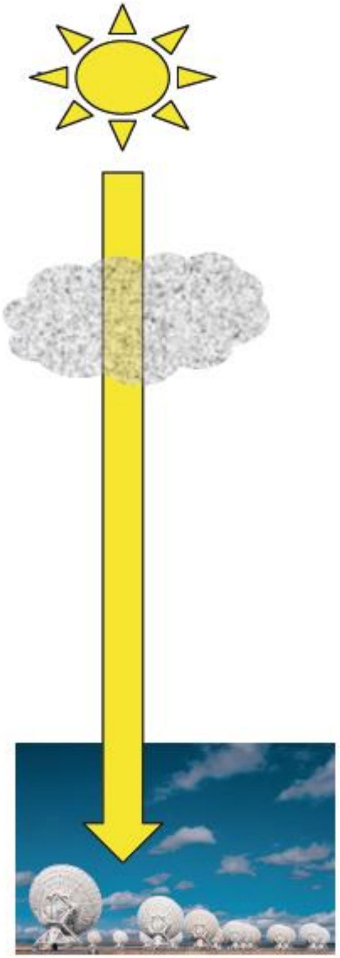
Lets assume we can measure or estimate the distance to the object

then…

Fluxograma Genérico!



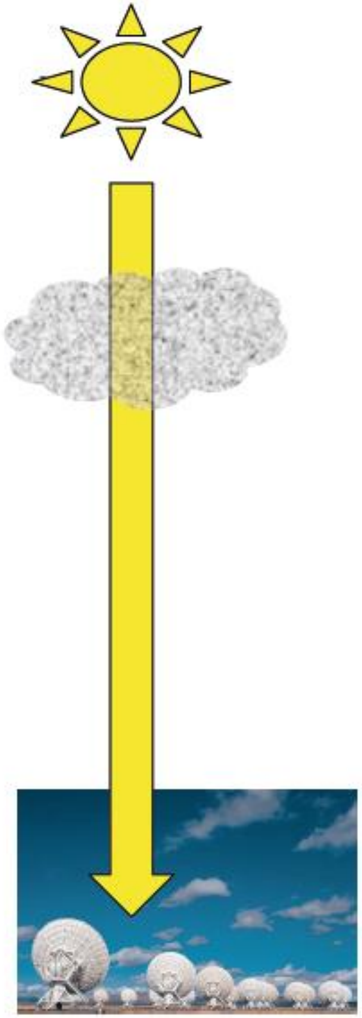
Introdução à extinção



... en route to our detectors,
some of the radiation can be
absorbed,
or scattered out of the path

(by various physical processes)

referred to as "extinction"
or "reddening"
or simply "absorption"



… the amount of extinction

is **wavelength-dependent**

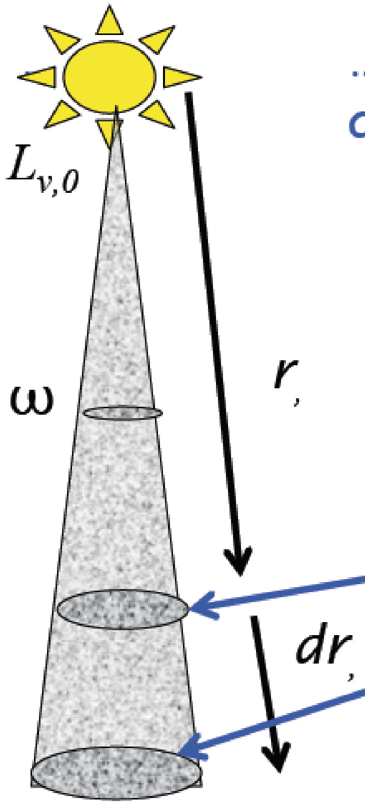
and depends on

-how much "stuff" is in the way

-the composition of the "stuff"

…. & sometimes other properties such as the temperature of the "stuff"

Opacidade....



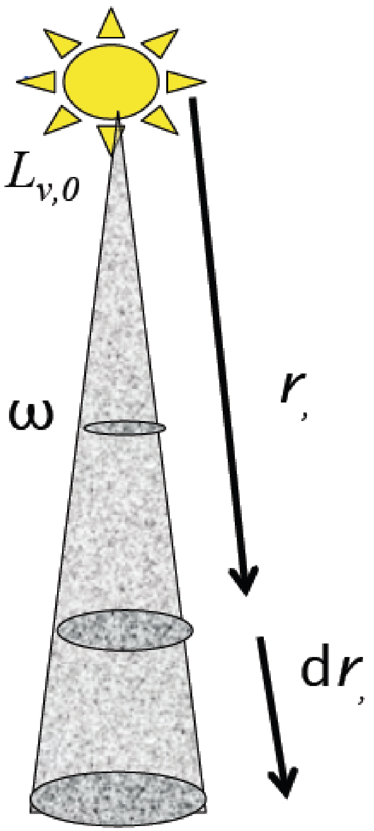
... consider an object emitting radiation with a specific energy flux $L_{\nu,0}$ into a solid angle ω

... this radiation travels through a medium
(say an interstellar cloud)
hence some of the radiation will be absorbed and/or scattered

define energy flux F_ν at radius r
& $F_\nu + dL_\nu$ at radius $r+dr$

then it turns out $dF_\nu = -\alpha\, F_\nu\, dr$

where α is known as the **opacity** of the medium



$$dF_v = -\alpha\, F_v\, dr$$

note opacity α has units of (length)$^{-1}$

Can now define a dimensionless quantity
Optical thickness τ
such that

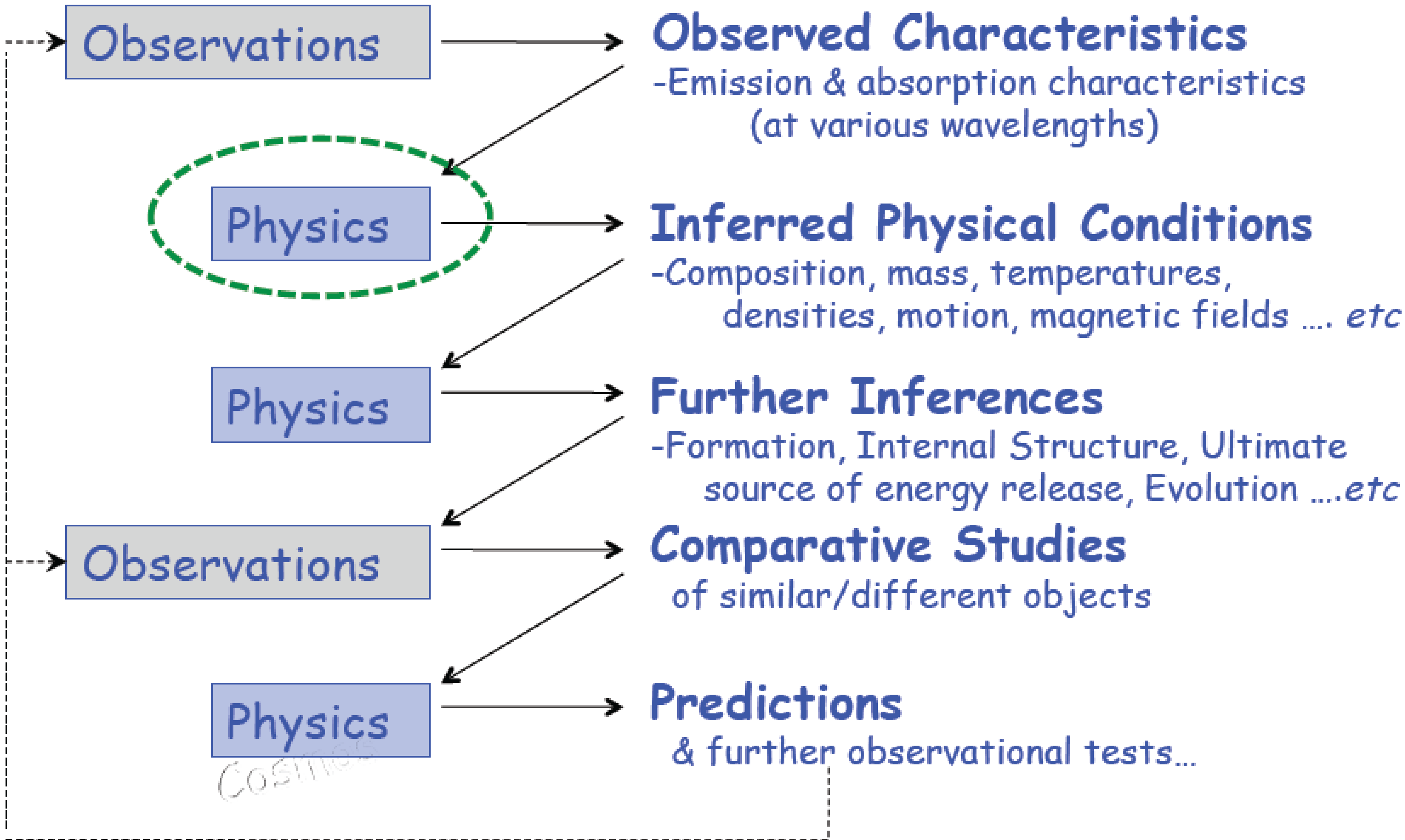
$$d\tau = \alpha\, dr$$

hence

$$dF_v = -F_v\, d\tau$$

integration of which ($F_{v,0}$ at τ=0 to F_v at τ)

gives $F_v = F_{v,0}\, e^{-\tau}$



Observations
Observed Characteristics
-Emission & absorption characteristics
(at various wavelengths)
Physics
Inferred Physical Conditions
-Composition, mass, temperatures,
densities, motion, magnetic fields …. etc
Physics
Further Inferences
-Formation, Internal Structure, Ultimate
source of energy release, Evolution ….etc
Observations
Comparative Studies
of similar/different objects
Physics
Predictions
& further observational tests…

COMPARISON OF TWO CANONICAL SOURCE TYPES

We discuss two canonical types of astrophysical sources: **STARS & AGNs** and compare them in the following characteristics:

- Structures
- Spectra: Continua
- Spectra: "Lines" (fine structure)
- Information carried by their spectra

STRUCTURES

STARS:

- Dense spheres, $r \sim 10^{11\text{-}13}$ cm
- Stars are dynamically stable, apart from convection and surface phenomena, throughout most of their lifetimes
- $\sim$ Isotropic radiators.
- Nuclear reactions maintain high-temperature ($>10^7$ K) core.
- XR and GR radiation transferred slowly through high optical depth envelope. Degraded to UVOIR band.
- Observed radiation escapes from a very thin (300 km in Sun) layer = "photosphere"

  NB: photospheres ($\sim 10^{17}$ particles/cm^3 in the Sun) are DENSE by the standards of AGN emitting regions

- Subsidiary processes (e.g. driven by magnetic fields) radiate small amounts in non-UVOIR bands.

AGNs ("Active Galactic Nuclei"):

- Supermassive black hole ($M \sim 10^{7-9} M_{sun}$; $R_{Schw} \sim 3 \times 10^{13} M_8$ cm) at center of an accretion disk with r $\gtrsim 10^{16}$ cm.
- Accretion disk is fed by infalling material; matter is continuously transported through disk
- Dissipation and magnetic processes near center of disk generate relativistic particles, $\gamma \sim 10^3$. These can generate relativistic jets.
- Observed radiation emerges from a large volume with nonuniform properties
- Relativistic jets and disk confinement produce anisotropic radiation.
- Relativistic particles produce broad-band non-thermal synchrotron radiation, directly observed at radio wavelengths.
- Direct thermal radiation from denser, hot inner disk (UV).
- Photon boosting of low-energy photons by inverse Compton scattering to UV/XR bands.

  Compton boost: $\nu_2 \sim \nu_1 \gamma^2$

- Hard radiation field produces strong ionization of gas in a large, low-density volume around disk, ===> UVOIR, XR emission lines.
- Strong heating of surrounding dust grains ===> IR continuum & emission lines (3-500μ). (But grains vaporized near center.)
- Star-formation regions often associated with AGN in disk galaxies (chicken or egg?).

CONTINUUM SPECTRA

STARS

- Primary component is UVOIR radiation from photosphere.
  - Thermal source; emergent spectrum $\sim$ Planck function
  - Small spread of T around characteristic effective T_e of photosphere, approximately depth where $\tau \sim 1$ at any wavelength.
  - F_λ peak at $\sim 2900/T_4$ Å.
  - Photospheric temperatures $\sim$ 1000-100000 K.
  - Strong time variation only in minority of cases
- Strong concentration to UVOIR, with rise $\nu f_\nu \sim \nu^3$ to peak, then dropoff.
- Spectral slope at higher ν allows estimate of mean T_e.
- Major absorption discontinuities from ionization edges of abundant ions (e.g. H: Lyman edge 912 Å, Balmer edge 3646 Å).
- Low level radiation in other bands from high temperature corona, synchrotron radiation, etc.

AGNs:

- Primary component is <u>very broad-band, nonthermal radiation</u> extending from radio to XR.
- Shape: <u>no simple parameterization</u>.
- Thermal components in IR (dust grains) & UV ("UV bump" from inner accretion disk).
- Continuum spectrum <u>depends on viewing angle</u> because of thick obscuring tori
- Strong time variation common

LINE SPECTRA

STARS:

- Complex, narrow, absorption lines
  - Produced by transitions in those atoms, ions, and molecules which are prevalent at characteristic T_e and pressure.
  - From thin layers of cooler gas projected against higher T continuum of inner photosphere.
- Doppler widths small (thermal), ~ few km s^{-1}
- Line spectrum reflects physical state (composition, temperature, pressure) of photosphere
- Lines and local continuum usually coupled (imply ~ same T)

AGNs:

- Emission lines since generally not viewed against continuum source.
- Wide range of ionizations (e.g. neutral to Fe XIV)
- Wide range of Doppler widths in different galaxies, to $>10^4$ km s^{-1}
- Lines and local continuum decoupled since originate in different volumes
- Doppler widths reflect kinematic motions of gas clouds
  - "Broad" lines from $r \lesssim$ 1 pc
  - "Narrow" lines to $\gtrsim$ 100 pc from BH
- Line spectrum depends on viewing angle (inner regions concealed by thick tori, dust clouds). Polarization.

DEDUCTIONS FROM OBSERVATIONS

STARS:

- Continuum slope & structure: T
- Ionization edge discontinuities: pressure/gravity
- Line widths: pressure/gravity, rotation, outflows
- Line strengths: T, pressure/gravity, chemical abundances. E.g.:
  - Classic "spectral-type" sequence is a T-sequence
  - Abundances: selected species easy to measure: e.g. Ca/H, Mg/H, Fe/H. He/H (hot stars only)
  - Light element (e.g. C,N,O) abundances more difficult: C IV (UV) in hot stars; various atomic lines in cool stars require high spec resol; molecules in cool stars (CH, CN, NH, etc)
  - Ionization decreases with increasing gas pressure: e.g. use Mg I 5175 Å strength as dwarf/giant discriminant for Galactic structure studies
  - NB: Derived abundances are sensitive to proper T,P estimation
- Integrated light of stars allows inferences concerning ages, abundances of distant stellar populations. "Population synthesis":
  - Fit full energy distribution with combinations of single generation models to determine star formation history & abundances
  - Use selected lines for age-abundance separation, e.g. $H\beta$, Mg,Fe for old pops

AGNs:

- Slope & structure of continuum related to energy distribution of electrons, importance of Compton scattering, accretion disk structure, dust emission/absorption, etc.
    - Less definitive interpretation than stellar continua because of multiple components, complex generation mechanisms, absence of near-TEQ.
    - One test for a nonthermal source: polarization. Another: compare its mean surface brightness to the Planck function and derive corresponding T:

      $$B_\nu(T) = f_\nu/\Omega$$

      where f is the flux, B is Planck function and Ω is the angular area (or upper limit) of the source. Is T "unphysically" high?
- (Emission) line strengths yield electron temperature & gas density;
- Line ionization level constrains far-UV continuum ("hardness" of ionizing radiation).
- Line widths, positions probe kinematics of turbulent gas near BH, outflows, etc.
- Line strengths yield abundances
    - ...although of different species than in stars: e.g. O, N, He, S, Fe (uncommon)...but not Ca, Mg (unless UV access), etc.
- ID's, Surveys: strong emission line sources (photons concentrated to narrow bands) easier to detect than pure continuum sources